Structural Safety 45 (2013) 48–58

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Gamma degradation models for earthquake-resistant structures

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ARTICLE INFO

Article history:
Received 28 December 2012
Received in revised form 30 August 2013
Accepted 3 September 2013

Keywords:
Stochastic process
Life-cycle
Performance-based earthquake engineering
Cumulative damage
Aging

ABSTRACT

Stochastic modeling of deterioration of structures at the scale of the life of the construction is the subject of this study. The categories of degradation phenomena considered are those two typical of structures, that is progressive degradation of structural characteristics and cumulative damage due to point overloads; i.e., earthquakes. The wearing structural parameter is the seismic capacity expressed in terms of kinematic ductility to conventional collapse, as a proxy for a dissipated hysteretic energy damage criterion. The gamma distribution is considered to model damages produced by earthquakes. The exponential distribution is also addressed as a special case. Closed-form approximations, for life-cycle structural assessment, are obtained in terms of absolute failure probability, as well as conditional to different knowledge about the structural damage history. Moreover, the gamma stochastic process is considered for continuous deterioration; that is *aging*. It is shown that if such probabilistic characterizations apply, it is possible to express total degradation (i.e., due to both aging and shocks) in simple forms, susceptible of numerical solution. Finally, the possible transformation of the repeated-shock effect due to earthquakes in an equivalent aging (*forward virtual age*) is discussed. Examples referring to simple bilinear structural systems illustrate potential applicability and limitations of the approach within the performance-based earthquake engineering framework.

1. Introduction and formulation

Dependency on history (e.g., number of occurred earthquakes, time elapsed since the last seismic event, or structural repair, etc.) of seismic structural risk may involve all the three elements constituting the performance-based earthquake engineering framework (PBEE) [1], that is, hazard, vulnerability, and loss. History-dependency of seismic hazard is often considered to be related to occurrence of characteristic earthquakes on individual faults, clustering of earthquake sequences, and fault interactions. All of these may be directly linked to the common cause of stress accumulation on the fault, which triggers seismic events. On the other hand, when many independent sources contribute to hazard, in classical probabilistic seismic hazard analysis (PSHA) familiar to engineers, it is customary to stochastically model earthquake arrivals via a homogeneous Poisson process (HPP); i.e., a process with independent and stationary increments [2].

Earthquake loss may be time-dependent mainly because of investment costs, which require financial discounting (e.g., [3]), or time-variant occupancy issues. Seismic structural vulnerability, finally, is commonly considered affected by two categories of phenomena that may lead it to vary with time: (1) continuous deterioration of material characteristics, or *aging*, and (2) cumulating damage because of repeated overloading due to earthquake shocks [4]. Both of them are damage accumulation processes; in the following, aging will be referred to as progressive damage, while damage due to earthquakes will be referred to as shock-based.

Aging, which in some cases may show an effect in increasing seismic structural fragility [5], is often related to an aggressive environment which worsens mechanical features of structural elements, for example: corrosion of reinforcing steel due to chloride attack, or carbonation in concrete (e.g., [6]). To be able to predict the evolution of this kind of wear is especially important in design of maintenance policies (e.g., [7,8]). Often the aging assessment is addressed via *predictable models* (e.g., degradation is assumed to evolve deterministically after a random initiation time). In fact, a stochastic model, which can account for temporal variability of the wear process, can be considered more appropriate [9].

Earthquake shocks potentially cumulate damage on the hit structure during its lifetime, unless partial or total restoration; i.e., within a *cycle*. In general, mainly because earthquake occurrences can be considered instantaneous with respect to structural life, it is advantageous to model the cumulative seismic damage process separately from the progressive aging. Indeed, to describe earthquakes probabilistically, a marked point process, in which each seismic event is represented by its occurrence time and damage it produces, can be conveniently adopted. With respect to this model, engineering interest is in the compound point process, which accounts for the cumulative damage (i.e., the sum of damage increments) produced by all occurring shocks.

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0167-4730/$ - see front matter
http://dx.doi.org/10.1016/j.strusafe.2013.09.001

If both deterioration effects may be measured in terms of the same parameter expressing the structural capacity, for example the residual ductility to collapse, or $\mu(t)$, then the total wear may be susceptible of the representation as a function of time in Fig. 1, where an arbitrary path of the process is depicted, as well as a conventional threshold corresponding to a limit-state of interest.

Formally, the degradation process is that in Eq. (1), where μ_0 is the initial capacity in the cycle (e.g., the *as-new* capacity), and $D(t)$ is the cumulated level of deterioration at the time t. Note that, with respect to Fig. 1, the initial time of the cycle is assumed to be zero, that is $t_0 = 0$.

$$\mu(t) = \mu_0 - D(t) \tag{1}$$

As introduced, $D(t)$ can be seen as the sum of two effects, one due to continuous deterioration and one due to accumulation of seismic damage, as in Eq. (2), where the first term in the right hand side is the continuous loss of capacity at time t due to aging, $\mu_C(t)$, and the second one is the cumulated loss of resistance due to all earthquake events, $N(t)$, occurring until time t. Note that $\mu_C(t)$, $\Delta\mu_i$ (damage in a single seismic event), and $N(t)$ are all random variables (RVs).

$$D(t) = \mu_C(t) + \sum_{i=1}^{N(t)} \Delta\mu_i \tag{2}$$

Given this formulation, the probability the structure fails within t, $P_f(t)$, or the complement to one of the structural reliability $R(t)$, is the probability that the structure reaches or passes a threshold related to a certain limit state, μ_{LS}, at any time before t, Eq. (3).

$$\begin{aligned} P_f(t) &= 1 - R(t) = F_T(t) = P[\mu(t) \le \mu_{LS}] \\ &= P[D(t) \ge \mu_0 - \mu_{LS}] = P[D(t) \ge \overline{\mu}] \end{aligned} \tag{3}$$

In other words, it is the probability that in $(0, t)$ the capacity reduces traveling the distance, $\overline{\mu}$, between the initial value and the threshold. Note that, by definition, Eq. (3) also provides the cumulative probability function (CDF) of structural lifetime, $F_T(t)$. To model such a risk is the objective of the presented study.

The following is structured such that shock-based damage only, that is when continuous deterioration is neglected, is investigated first. The developed compound point process assumes: (1) damage increments, are independent and identically distributed (i.i.d.) and (2) the processes regulating earthquake occurrence and seismic damage are mutually independent.

In particular, it is addressed the case in which damage in an individual earthquake is susceptible of gamma representation (including the special case of exponential distribution). For this case, closed- and/or approximate-form solutions for absolute and conditional reliability problems are derived, if earthquake occurrence follows a HPP. The model also considers that not all earthquakes are necessarily damaging, as not all of them are overloads. Indeed, most of the earthquakes occurring in a region where earthquake magnitude follows a Gutenberg–Richter relationship [10] refer to small magnitude events, with negligible (by structural design) consequences, as also discussed later on.

Subsequently, the *gamma process* [11], of acknowledged suitability for probabilistic representation of wear in engineering systems (e.g., [9]), is considered for continuous deterioration of seismic structural capacity. Then, how to model the total wear when the cumulative earthquake effect and the aging processes may be taken as independent, is approached. Moreover, the special cases, in which total degradation can be described via a single gamma process, are also discussed. The concept of equivalent aging due to earthquake shocks is introduced, which reverting a maintenance concept, is referred to as *forward virtual age*.

Finally, illustrative applications referring to a simple single degree of freedom (SDOF) elastic-perfectly-plastic (EPP) structure, supposed to be located in a comparatively high-seismicity region in central Italy, are developed to address applications of potential earthquake engineering interest, and to shed some light on suitability of the hypotheses at the basis of these simple *age-dependent* reliability models.

2. Cumulative earthquake damage

In the case where only shock-based damage is considered (i.e., $\mu_C(t) = 0, \forall t$), then the deterioration process results as sketched in Fig. 2 and formulated in Eq. (4).

$$\mu(t) = \mu_0 - D(t) = \mu_0 - \sum_{i=1}^{N(t)} \Delta\mu_i \tag{4}$$

In the classical case, where the occurrence of seismic events is described by a HPP, $N(t)$ has a Poisson distribution with constant λ rate. Thus, considering the distribution of cumulative earthquake damage as dependent on the number of occurring earthquakes, and their ground motion intensities measures, $\underline{IM}$, the failure probability may be computed as in Eq. (5), where the integral is of k-th order.

$$\begin{aligned} P_f(t) &= P[D(t) \ge \overline{\mu}] \\ &= \sum_{k=1}^{+\infty} P[D(t) \ge \overline{\mu} | N(t) = k] \cdot P[N(t) = k] \\ &= \sum_{k=1}^{+\infty} P[D(t) \ge \overline{\mu} | N(t) = k] \cdot \frac{(\lambda \cdot t)^k}{k!} \cdot e^{-\lambda \cdot t} \\ &= \sum_{k=1}^{+\infty} \int_{\underline{im}} P\left[\sum_{i=1}^{k} \Delta\mu_i \ge \overline{\mu} \,|\underline{IM} = \underline{im}, N(t) = k\right] \\ &\quad \cdot f_{\underline{IM}}(\underline{im}) \cdot d(\underline{im}) \cdot \frac{(\lambda \cdot t)^k}{k!} \cdot e^{-\lambda \cdot t} \end{aligned} \tag{5}$$

In the classical HPP-based PSHA, *IMs*, for example first mode spectral acceleration, *Sa*, in different earthquakes are i.i.d. RVs, and $f_{\underline{IM}}(\underline{im})$ is simply the product of marginal distributions, $f_{IM_i}(im)$, which are k in number. Therefore, the critical issue to solve the reliability problem is to get the probability of shock-based damage exceeding the threshold conditional to ground motion intensities for a given number of earthquakes, that is $P[\sum_{i=1}^{k} \Delta\mu_i \ge \overline{\mu} | \underline{IM} = \underline{im},\ N(t) = k]$ or, even better, $P[\sum_{i=1}^{k} \Delta\mu_i \ge \overline{\mu} | N(t) = k]$, as per the first line of the equation. The latter may be addressed in a relatively simple manner if three conditions, which are listed below, are met.

(1) Damage in the i-th earthquake, $\Delta\mu_i$, always has the same cumulative distribution in Eq. (6), marginal with respect to IM; i.e., $P[\Delta\mu_i \le \delta\mu] = P[\Delta\mu \le \delta\mu], \forall i$.

$$F_{\Delta\mu}(\delta\mu) = \int_{im} P[\Delta\mu \le \delta\mu | IM = x] \cdot f_{IM}(x) \cdot dx \tag{6}$$

(2) Damages produced in different events are independent RVs. In other words, according to conditions (1) and (2), earthquake's structural effects are i.i.d. (This, in particular, implies that a structure, in an earthquake, suffers damage that is independent of its state.)

Condition (3) is that the distribution of the sum of damages can be expressed in a simple form. A way to satisfy this condition consists of modeling damage via an RV that enjoys the *reproductive* property. A well-known example of this kind of variable is the Gaussian one. However, because in earthquakes it should be $\Delta\mu_i \ge 0\ \forall i$, the deterioration process due to subsequent shocks should also show non-negative increments, rendering the Gaussian representation of damage not perfectly appropriate. Although the lognormal one may appear as a solution (often adopted in the earthquake engineering context), the latter is not reproductive in the addition sense; therefore, it may not be applied if deterioration is seen as the sum of damages.

In the following subsections the gamma RV is considered to derive closed-form solutions for reliability when damage accumulation is due to seismic events only. In fact, the special case of the exponential

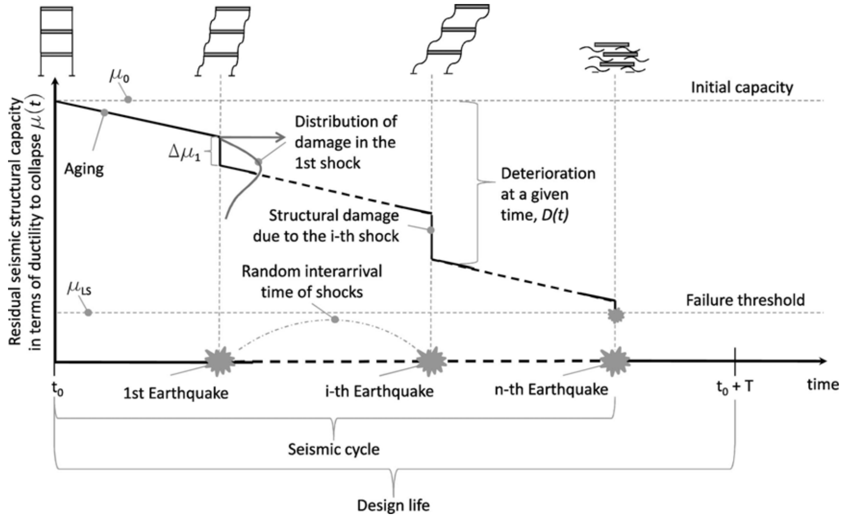

Fig. 1. Seismic cycle representation for a structure subjected to aging and repeated earthquake shocks, when degradation affects residual capacity to failure.

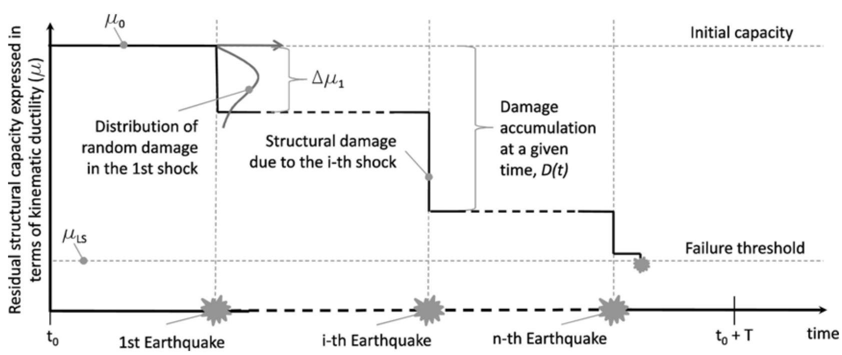

Fig. 2. Seismic cycle representation for a structure subjected to cumulative earthquake damages only.

RV (a gamma distribution with shape parameter equal to one), which the literature has already assumed to model damage, is addressed first.

In the application section it will be shown that the gamma distribution may be a suitable option to model earthquake damage, and that the listed conditions apply for an EPP-SDOF if damage accumulation is based on hysteretic energy dissipation. Conversely, they may not be suitable for those systems in which the structural response in one earthquake depends on the previous shock history. The latter is the case of structures with evolutionary or degrading hysteretic behavior and also EPP-SDOF systems when strain-based damage functionals are considered [12]. In these situations, *state-dependent* approaches (e.g., [13–15]) may be required to describe, from the reliability point of view, performance degradation.

2.1. Damage accumulation in the case of exponential increments

To model earthquake cumulative damage proceeding in one direction only, the simplest option is the exponential distribution, $f_{\Delta\mu}(\delta\mu) = \gamma_D \cdot e^{-\gamma_D \cdot \delta\mu}$ (γ_D is the parameter). The latter was considered as a possibility in [4], in a useful attempt to provide a framework to stochastically model deterioration of earthquake-resistant structures. In fact, in [4] no closed-form solutions were derived for reliability assessment; however, because the sum of i.i.d. exponential RVs is an Erlang-distributed random number, then the failure probability conditional to k_D shocks is that given in Eq. (7), where $\Gamma(\beta) = \int_0^{+\infty} z^{\beta-1} \cdot e^{-z} \cdot dz$ and $\Gamma_U(\beta, y) = \int_y^{+\infty} z^{\beta-1} \cdot e^{-z} \cdot dz$ are the *gamma* and the *upper incomplete gamma* functions, respectively. Indeed, the exponential damage assumption yields the solution of the reliability problem given in Eq. (8).

$$
\begin{aligned}
P\left[D(t) \geq \overline{\mu} \,\middle|\, N_D(t) = k_D\right] &= P\left[\sum_{i=1}^{k_D} \Delta\mu_i \geq \overline{\mu} \,\middle|\, N_D(t) = k_D\right] \\
&= \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{k_D-1}}{\Gamma(k_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx \\
&= \frac{\Gamma_U(k_D, \gamma_D \cdot \overline{\mu})}{\Gamma(k_D)} = \sum_{i=0}^{k_D-1} \frac{(\gamma_D \cdot \overline{\mu})^i}{i!} \cdot e^{-\gamma_D \cdot \overline{\mu}}
\end{aligned}
\tag{7}
$$

$$
\begin{aligned}
P_f(t) &= P\left[D(t) \geq \overline{\mu}\right] \\
&= \sum_{k_D=1}^{+\infty} \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{k_D-1}}{\Gamma(k_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx \cdot \frac{(\lambda_D \cdot t)^{k_D}}{k_D!} \cdot e^{-\lambda_D \cdot t} \\
&= \sum_{k_D=1}^{+\infty} \left[\sum_{i=0}^{k_D-1} \frac{(\gamma_D \cdot \overline{\mu})^i}{i!} \cdot e^{-\gamma_D \cdot \overline{\mu}}\right] \cdot \frac{(\lambda_D \cdot t)^{k_D}}{k_D!} \cdot e^{-\lambda_D \cdot t}
\end{aligned}
\tag{8}
$$

It is to underline that, being continuous and non-negative, the exponential RV is suitable to model only the effect of earthquakes determining loss of capacity, not those events whose intensity is not large enough. In this respect, differently from Eq. (5), k_D in Eq. (7) refers to the *filtered HPP* of parameter $\lambda_D = \lambda \cdot P[\Delta\mu > 0]$, counting damaging events only, $N_D(t)$.

The failure probability in Eq. (8) can be approximated by the conditional probability in Eq. (7), in which $N_D(t)$ is replaced by the expected number of earthquakes until t, Eq. (9).[1]

$$\begin{cases} P_f(t) \approx P\left[D(t) \geq \overline{\mu} \,|N_D(t) = E\left[N_D(t)\right]\right] \\ \qquad = \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot t - 1}}{\Gamma(\lambda_D \cdot t)} \cdot e^{-\gamma_D \cdot x} \cdot dx = \frac{\Gamma_U(\lambda_D \cdot t, \gamma_D \cdot \overline{\mu})}{\Gamma(\lambda_D \cdot t)} \\ E\left[N_D(t)\right] = \lambda_D \cdot t \end{cases} \tag{9}$$

The latter equation, which is obtained via a rough application of the delta method [16], is expected to be a helpful simplification of the reliability problem. In fact, it is to note that this approximation, to provide results similar to the exact case, requires the neglected terms to be comparatively small with respect to those kept. As shown in the illustrative application later on, this approach, even rough, appears suitable in the context of this study.

2.2. Gamma-distributed damage increments

Although the exponential distribution was already considered to model earthquake damage (e.g., in [4]), one may argue about inherent limited flexibility due to its single parameter. In fact, another option, perhaps more attractive, which depends on two of those and includes the exponential RV as a special case, is the gamma distribution. It is shown in Eq. (10), where γ_D and α_D are the scale and shape parameters, respectively. The use of this continuous non-negative RV, due to its reproductive property, yields handy reliability solutions, similar to those of the exponential case; at the same time, depending on its shape parameter, it can take significantly different shapes. For example, α_D equal to one stretches the distribution to the exponential, a large value of the shape parameter let the probability density function (PDF) be similar to that of a Gaussian RV, while for intermediate values of α_D, it is an alternative to the lognormal one to model skewed non-negative RVs.

$$f_{\Delta\mu}(\delta\mu) = \frac{\gamma_D \cdot (\gamma_D \cdot \delta\mu)^{\alpha_D - 1}}{\Gamma(\alpha_D)} \cdot e^{-\gamma_D \cdot \delta\mu} \tag{10}$$

Because the sum of k_D i.i.d. gamma-distributed RVs, with scale and shape parameters γ_D and α_D respectively, is still gamma with parameters γ_D and $k_D{\cdot}\alpha_D$, the probability of cumulative damage exceeding the threshold, conditional to k_D shocks, is given by Eq. (11). Finally, in the gamma case, Eq. (9) becomes Eq. (12), which, again, represents an approximation of $P_f(t)$.

$$\begin{aligned} P\left[D(t) \geq \overline{\mu} \,|N_D(t) = k_D\right] &= \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{k_D \cdot \alpha_D - 1} \cdot e^{-\gamma_D \cdot x}}{\Gamma(k_D \cdot \alpha_D)} dx \\ &= \frac{\Gamma_U(k_D \cdot \alpha_D, \gamma_D \cdot \overline{\mu})}{\Gamma(k_D \cdot \alpha_D)} \end{aligned} \tag{11}$$

$$\begin{aligned} P_f(t) &\approx P\left[D(t) \geq \overline{\mu} \,|N_D(t) = E\left[N_D(t)\right]\right] \\ &= \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot t \cdot \alpha_D - 1}}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx \\ &= \frac{\Gamma_U(\lambda_D \cdot t \cdot \alpha_D, \gamma_D \cdot \overline{\mu})}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)} \end{aligned} \tag{12}$$

2.3. Conditional reliability approximations

Formulations above give absolute (i.e., aprioristic) probability that a new structure fails in $(0, t)$, yet conditional failure probability, which accounts for information possibly available at the epoch of the evaluation, can be obtained. Some of these are given below, and sketched in Fig. 3, for different knowledge levels referring to cases in which:

(i) the current state of the structure is known at the time of the reliability assessment;
(ii) it is only known that the structure is *surviving* at the time the evaluation is performed, yet with unknown residual seismic capacity;
(iii) same of (ii) with the additional information about the number of damaging earthquakes the structure was subjected to.

For the sake of generality, all derivations are given for the case damage is a gamma-distributed RV, yet they also apply to the case it is exponential.

2.3.1. Failure probability when the structural state is known

It may be the case the structure is analyzed at t^*, for example after an earthquake felt in the region where the structure is located, and the current residual capacity is measured, $\mu(t^*)$. The failure probability conditional to observed state has the same expression above, just, replacing $\overline{\mu}$ and t of Eq. (12), with $\overline{\mu}^* = \mu(t^*) - \mu_{LS}$ and t–t^*, respectively (i.e., Eq. (13)).

$$\begin{aligned} P_f(t - t^*) &\approx \int_{\overline{\mu}^*}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot (t - t^*) \cdot \alpha_D - 1}}{\Gamma[\lambda_D \cdot (t - t^*) \cdot \alpha_D]} \cdot e^{-\gamma_D \cdot x} \cdot dx \\ &= \frac{\Gamma_U[\lambda_D \cdot (t - t^*) \cdot \alpha_D, \gamma_D \cdot \overline{\mu}^*]}{\Gamma[\lambda_D \cdot (t - t^*) \cdot \alpha_D]}, \quad t \geq t^* \end{aligned} \tag{13}$$

In fact, the structure has now to undergo a smaller capacity reduction to fail. Equivalent: the same relationship may also be used if the residual capacity $\overline{\mu}^*$ is obtained via a repair at t^*.

2.3.2. Failure probability when survival is known

Also interesting is the case in which one wants to include in the reliability assessment the information that the structure is still surviving at t^*, but with unknown damage condition. It may be computed via Eq. (14), plugging in previous results.

$$\begin{aligned} &P\left[\text{failure within } t > t^* | \text{survival in } t^*\right] \\ &= 1 - P\left[\text{survival in } t > t^* | \text{survival in } t^*\right] \\ &= 1 - \frac{P\left[\text{survival in } t > t^* \cap \text{survival in } t^*\right]}{P\left[\text{survival in } t^*\right]} \\ &= 1 - \frac{R(t)}{R(t^*)} = 1 - \frac{1 - P_f(t)}{1 - P_f(t^*)} \\ &\approx 1 - \frac{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot t \cdot \alpha_D - 1}}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx}{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot t^* \cdot \alpha_D - 1}}{\Gamma(\lambda_D \cdot t^* \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx} \\ &= 1 - \frac{1 - \frac{\Gamma_U(\lambda_D \cdot t \cdot \alpha_D, \gamma_D \cdot \overline{\mu})}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)}}{1 - \frac{\Gamma_U(\lambda_D \cdot t^* \cdot \alpha_D, \gamma_D \cdot \overline{\mu})}{\Gamma(\lambda_D \cdot t^* \cdot \alpha_D)}} \end{aligned} \tag{14}$$

2.3.3. Failure probability when survival and the number of damaging earthquakes are known

Finally, the probability of failure given survival at t^* and the number, $N_D(t^*) = k_D$ (larger than zero), of damaging earthquakes until t^*,

[1] To compute Eq. (9), the following approximation, yielding the same result of Eq. (7) when $\lambda{\cdot}t$ is an integer, can be adopted: $\frac{\Gamma_U(\beta, y)}{\Gamma(\beta)} \approx \sum_{i=0}^{INT(\beta+0.5)-1} \frac{y^i}{i!} \cdot e^{-y}$.

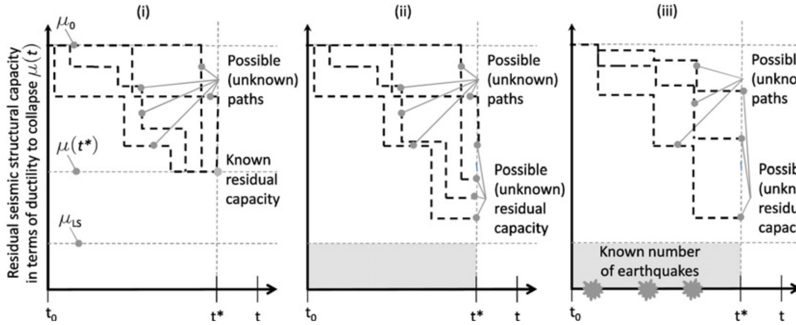

Fig. 3. Conditional probability cases when: (i) current state is known, (ii) non-collapse (survival) is the only available information, and (iii) when survival and the number of damaging earthquakes are known.

yields Eq. (15), where the approximation is analogous to those above.

$$
\begin{aligned}
&P\left[\textit{failure within } t > t^* \,|\textit{survival in } t^*, N_D(t^*) = k_D\right] \\
&= 1 - \frac{P\left[T > t \cap T > t^* \cap N_D(t^*) = k_D\right]}{P\left[T > t^* \cap N_D(t^*) = k_D\right]} \\
&= 1 - \frac{1 - P\left[\sum_{i=1}^{k_D + N_D(t-t^*)} \Delta\mu_i \geq \overline{\mu}\right]}{1 - P\left[\sum_{i=1}^{k_D} \Delta\mu_i \geq \overline{\mu}\right]} \\
&= 1 - \frac{1 - \sum_{k_D^*=0}^{+\infty} P\left[\sum_{i=1}^{k_D+k_D^*} \Delta\mu_i \geq \overline{\mu} \,|\, N_D(t-t^*) = k_D^*\right] \cdot P\left[N_D(t-t^*) = k_D^*\right]}{1 - P\left[\sum_{i=1}^{k_D} \Delta\mu_i \geq \overline{\mu}\right]} \\
&\approx 1 - \frac{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{k_D \cdot \alpha_D + \lambda_D \cdot (t-t^*) \cdot \alpha_D - 1}}{\Gamma\left[k_D \cdot \alpha_D + \lambda_D \cdot (t-t^*) \cdot \alpha_D\right]} \cdot e^{-\gamma_D \cdot x} \cdot dx}{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{k_D \cdot \alpha_D - 1}}{\Gamma(k_D \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \cdot dx} \\
&= 1 - \frac{1 - \frac{\Gamma_U\left[k_D \cdot \alpha_D + \lambda_D \cdot (t-t^*) \cdot \alpha_D,\, \gamma_D \cdot \overline{\mu}\right]}{\Gamma\left[k_D \cdot \alpha_D + \lambda_D \cdot (t-t^*) \cdot \alpha_D\right]}}{1 - \frac{\Gamma_U(k_D \cdot \alpha_D,\, \gamma_D \cdot \overline{\mu})}{\Gamma(k_D \cdot \alpha_D)}}
\end{aligned}
\tag{15}
$$

Note that in the case $N_D(t^*) = 0$, Eq. (12) applies, in which t is replaced by t–t^*; i.e., in the absence of damaging shocks, the structure is *as new* at t^*.

3. Gamma process for continuous wear modeling

This section refers to the modeling of aging. The key difference with respect to the shock-based damage discussed so far, is that its probabilistic representation is a continuous process, resulting in progressive wear. An attractive option is the gamma process that, if applicable, implies that degradation has independent and stationary gamma-distributed increments, yielding Eq. (16) for the PDF of wear accumulated in $(0, t)$. It may prove suitable to model continuously accumulating degradation, such as wear, fatigue, corrosion, crack growth, creep, and swell; i.e., typical aging-related phenomena in structures [17]; however, the properties of the increment imply that degradation accumulation in any time interval only depends on how wide such interval is, independent of current state and age of the structure.

$$
f_{\mu_C(t)}(\mu) = \frac{\gamma_A \cdot (\gamma_A \cdot \mu)^{s_A \cdot t - 1}}{\Gamma(s_A \cdot t)} \cdot e^{-\gamma_A \cdot \mu} \tag{16}
$$

In Eq. (16) the deteriorating structural parameter is still ductility to collapse, thus it is assumed the phenomenon affects seismic capacity. Therefore, if degradation is due to aging only, the failure probability is given by Eq. (17).

$$
\begin{aligned}
P_f(t) &= P\left[D(t) > \overline{\mu}\right] = P\left[\mu_C(t) > \overline{\mu}\right] \\
&= \int_{\overline{\mu}}^{+\infty} \frac{\gamma_A \cdot (\gamma_A \cdot x)^{s_A \cdot t - 1}}{\Gamma(s_A \cdot t)} \cdot e^{-\gamma_A \cdot x} \cdot dx = \frac{\Gamma_U(s_A \cdot t, \gamma_A \cdot \overline{\mu})}{\Gamma(s_A \cdot t)}
\end{aligned}
\tag{17}
$$

Note that the model of Eq. (16) also implies the mean and the variance of the degradation process vary linearly being equal to: $E[\mu_C(t)] = (s_A/\gamma_A) \cdot t$ and $Var[\mu_C(t)] = (s_A/\gamma_A^2) \cdot t$. Despite this assumption, which helps to keep the illustration simple and is also considered in the structural context (e.g., [9,17,18]), it is to recall that if the shape parameter is defined as a non-linear function of time, the gamma process allows to model degradation trends different from that linear (i.e., non-stationary increments).

4. Degradation due to shock-based damage and aging

In the case the point and continuous degradation processes are independent and modeled as in Sections 2.2 and 3, then the failure probability is that given in Eq. (18), which follows from Eqs. (12) and (17). It may be easily solved numerically; however, it is to mention that analytical-form for the convolution of an arbitrary number of independent gamma random variables, with different parameters, may also be derived; see [19].

$$
\begin{aligned}
P_f(t) &= P\left[D(t) \geq \overline{\mu}\right] = P\left[\mu_C(t) + \sum_{i=1}^{N(t)} \Delta\mu_i \geq \overline{\mu}\right] \\
&= 1 - F_{D(t)}(\overline{\mu}) = 1 - \int_0^{\overline{\mu}} F_{\sum_{i=1}^{N(t)} \Delta\mu_i}(\overline{\mu} - y) \cdot f_{\mu_C}(y) \cdot dy \\
&\approx 1 - \int_0^{\overline{\mu}} \int_0^{\overline{\mu} - y} \frac{\gamma_D \cdot (\gamma_D \cdot x)^{\lambda_D \cdot t \cdot \alpha_D - 1}}{\Gamma(\lambda_D \cdot t \cdot \alpha_D)} \cdot e^{-\gamma_D \cdot x} \\
&\quad \cdot \frac{\gamma_A \cdot (\gamma_A \cdot y)^{s_A \cdot t - 1}}{\Gamma(s_A \cdot t)} \cdot e^{-\gamma_A \cdot y} \cdot dx \cdot dy
\end{aligned}
\tag{18}
$$

The use of Eq. (18) has an important implication. It assumes that the progressive damage accumulation process continues in the same fashion independently of earthquake damage; i.e., occurrence of a seismic damage does not alter future evolution of progressive deterioration. Thus, for example, from the practical point of view, if aging is due to chloride penetration and steel corrosion, it is assumed that evolution of this process is not significantly affected by crack openings due to earthquakes. Specular: damage resulting from an earthquake is independent of both age and amount of deterioration the structure is found when the earthquake occurs.

4.1. Earthquake damage and forward virtual age concept

Due to the properties of the gamma distribution, also invoked in Section 2.2, it follows that in the case continuous aging and seismic damage share the same scale parameter, γ, probability of failure, conditional to the number of shocks, k_D, is that of Eq. (19).

$$P[D(t) \geq \overline{\mu} | N_D(t) = k_D] = \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot t + k_D \cdot \alpha_D - 1}}{\Gamma(s_A \cdot t + k_D \cdot \alpha_D)} \cdot e^{-\gamma \cdot x} \cdot dx = \frac{\Gamma_U(s_A \cdot t + k_D \cdot \alpha_D, \gamma \cdot \overline{\mu})}{\Gamma(s_A \cdot t + k_D \cdot \alpha_D)} \tag{19}$$

The assumption yielding this equation may be restrictive and has to be verified case-by-case. Obviously, when it applies, Eq. (19) gives evident computational advantages. On the other hand, also in the case the hypothesis is not verified, it may be used to quickly obtain a preliminary estimate of the failure probability, to be refined further if needed.

It is also worth noting, as a side result, that Eq. (19) is susceptible of an appealing interpretation, in terms of the virtual age concept, originally developed to account for the effect of maintenance in the reliability assessment [20]. According to virtual age, repair is seen as rejuvenation, such that, from the reliability assessment point of view, the repaired system is equivalent to the original one, but with an age reduced by the number of years canceled by repair (Fig. 4, left).

In fact, in the case under study, the effect of the shock may be seen as equivalent to aging. Indeed, defining the time warp $\tau = \alpha_D / s_A$, Eq. (19) may be rewritten as Eq. (20), which shows that failure probability of a structure of age t and subject to k_D earthquakes, may be computed as that of a structure with age $t + k_D \cdot \tau$, and no shocks. This model may be referred to as *forward* virtual age (as opposite to that *backward* of [20]).

$$\begin{aligned} &P[D(t) \geq \overline{\mu} | N_D(t) = k_D] \\ &= P[D(t + k_D \cdot \tau) \geq \overline{\mu} \,|\, N_D(t + k_D \cdot \tau) = 0] \\ &= \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot (t + k_D \cdot \tau) - 1}}{\Gamma[s_A \cdot (t + k_D \cdot \tau)]} \cdot e^{-\gamma \cdot x} \cdot dx = \frac{\Gamma_U[s_A \cdot (t + k_D \cdot \tau), \gamma \cdot \overline{\mu}]}{\Gamma[s_A \cdot (t + k_D \cdot \tau)]} \end{aligned} \tag{20}$$

4.2. Life-cycle and conditional reliability approximations

Eq. (19) allows the derivation of handy approximations of reliability considering both degradation phenomena; the failure probability may be approximated as in Eq. (21), with the use of the same approximation as in Eq. (12). In fact, if the *equivalent* shape parameter, $s = s_A + \lambda_D \cdot \alpha_D$, is introduced, the term at the second line of the Eq. (21) coincides with that of Eq. (17).

$$\begin{aligned} P_f(t) &\approx \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot t + E[N_D(t)] \cdot \alpha_D - 1}}{\Gamma(s_A \cdot t + E[N_D(t)] \cdot \alpha_D)} \cdot e^{-\gamma \cdot x} \cdot dx \\ &= \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot \left(1 + \lambda_D \cdot \frac{\alpha_D}{s_A}\right) \cdot t - 1}}{\Gamma\left[s_A \cdot \left(1 + \lambda_D \cdot \frac{\alpha_D}{s_A}\right) \cdot t\right]} \cdot e^{-\gamma \cdot x} \cdot dx \\ &= \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s \cdot t - 1}}{\Gamma(s \cdot t)} \cdot e^{-\gamma \cdot x} \cdot dx = \frac{\Gamma_U(s \cdot t, \gamma \cdot \overline{\mu})}{\Gamma(s \cdot t)} \end{aligned} \tag{21}$$

Conditional probabilities, given the same pieces of information discussed in Section 2.3, can be also retrieved for this model. In particular, if the current residual seismic capacity, $\mu(t^*)$, is measured at a certain time t^*, the failure probability is that of Eq. (22).

$$\begin{aligned} P_f(t - t^*) &\approx \int_{\mu(t^*) - \mu_{LS}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s \cdot (t - t^*) - 1}}{\Gamma[s \cdot (t - t^*)]} \cdot e^{-\gamma \cdot x} \cdot dx \\ &= \frac{\Gamma_U[s \cdot (t - t^*), \gamma \cdot (\mu(t^*) - \mu_{LS})]}{\Gamma[s \cdot (t - t^*)]}, \quad t \geq t^* \end{aligned} \tag{22}$$

Recalling Eqs. (21) and (14), if the information is only that the structure is still surviving at t^*, the failure probability is approximated by Eq. (23). If the number of damaging earthquakes is also known, Eq. (24), applies.

$$\begin{aligned} &P[\text{failure within } t > t^* | \text{survival in } t^*] \\ &\approx 1 - \frac{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s \cdot t - 1}}{\Gamma(s \cdot t)} \cdot e^{-\gamma \cdot x} \cdot dx}{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s \cdot t^* - 1}}{\Gamma(s \cdot t^*)} \cdot e^{-\gamma \cdot x} \cdot dx} \\ &= 1 - \frac{1 - \frac{\Gamma_U(s \cdot t, \gamma \cdot \overline{\mu})}{\Gamma(s \cdot t)}}{1 - \frac{\Gamma_U(s \cdot t^*, \gamma \cdot \overline{\mu})}{\Gamma(s \cdot t^*)}} \end{aligned} \tag{23}$$

$$\begin{aligned} &P[\text{failuire within } t > t^* \,|\, \text{survival in } t^*, N_D(t^*) = k_D] \\ &\approx 1 - \frac{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot t + k_D \cdot \alpha_D + \lambda_D (t - t^*) \cdot \alpha_D - 1}}{\Gamma[s_A \cdot t + k_D \cdot \alpha_D + \lambda_D (t - t^*) \cdot \alpha_D]} \cdot e^{-\gamma \cdot x} \cdot dx}{1 - \int_{\overline{\mu}}^{+\infty} \frac{\gamma \cdot (\gamma \cdot x)^{s_A \cdot t^* + k_D \cdot \alpha_D - 1}}{\Gamma(s_A \cdot t^* + k_D \cdot \alpha_D)} \cdot e^{-\gamma \cdot x} \cdot dx} \\ &= 1 - \frac{1 - \frac{\Gamma_U[s_A \cdot t + k_D \cdot \alpha_D + \lambda_D (t - t^*) \cdot \alpha_D, \gamma \cdot \overline{\mu}]}{\Gamma[s_A \cdot t + k_D \cdot \alpha_D + \lambda_D (t - t^*) \cdot \alpha_D]}}{1 - \frac{\Gamma_U(s_A \cdot t^* + k_D \cdot \alpha_D, \gamma \cdot \overline{\mu})}{\Gamma(s_A \cdot t^* + k_D \cdot \alpha_D)}} \end{aligned} \tag{24}$$

5. Illustrative application

In this section structural modeling is addressed with reference to a simple EPP-SDOF system with unloading/reloading stiffness, which is the same as the initial one. The reason to choose this model is threefold: (i) it is at the basis of earthquake engineering and the results developed for it are expected to be of significant generality; (ii) earthquake-resistant structures, especially those reflecting modern codes, may be often rendered equivalent to this kind of system; (iii) it shows stable hysteretic cycles that repeat themselves despite of the sequence of excitation it undergoes. This latter property is especially important with respect to the age-dependent reliability models discussed herein, which are based on independent and identically distributed damage increments. In the following, earthquake-based cumulative damage is addressed first, subsequently, continuous deterioration, and finally the sum of the two.

5.1. Structural response and ductility-based collapse criterion

The elastic period of the EPP-SDOF is equal to 0.5 s; weight is 100 kN and the yielding force is equal to 19.6 kN, which corresponds to a strength reduction factor equal to 2.5 given a 0.49 g spectral acceleration.

Chosen engineering demand parameter (EDP) is the kinematic ductility, μ; i.e., the maximum displacement, when the yielding displacement is the unit. In fact, such an EDP is chosen as the simplest proxy for the dissipated hysteretic energy during one earthquake event. Note that this implies, as in all energy-based damage measures, structural damage in all seismic events with intensity larger than that required to yield the structure; see also [21] for a discussion.

The collapse is assumed to occur when kinematic ductility, conservatively accumulated independently on the sign of maximum displacement, reaches some capacity value.

If the considered limit state (LS) is *collapse prevention* (CP) derived from [22], which assumes conventional collapse of concrete structures at a maximum drift ratio equal to 0.04, the system has an (initial) ductility capacity $\mu_0 = 3.3$, and each damaging shock drains some of this ductility supply (Fig. 5).

Once structural system and collapse criterion are defined, it is possible to address the i.i.d. hypotheses of damage increments. Due to its force–displacement relationship, the considered SDOF has a

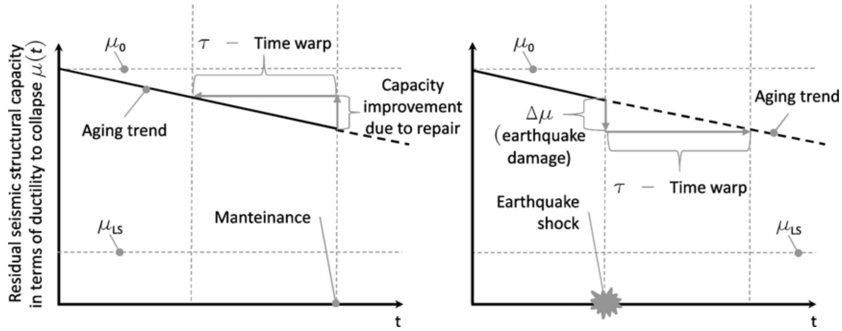

Fig. 4. Backward virtual age from repairable systems' maintenance theory (left); forward virtual age, that is, continuous deterioration equivalent to earthquake damage (right).

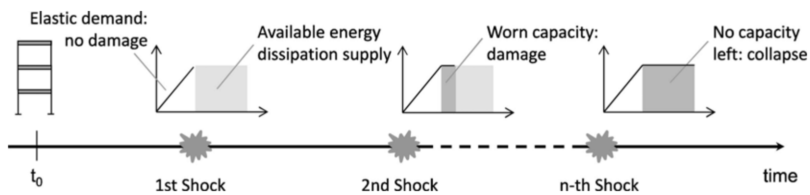

Fig. 5. Accumulation of damage in shock sequence with respect to kinematic ductility for EPP-SDOF systems.

response, which is stable with respect to subsequent earthquakes. It is easy to recognize that this means the maximum displacement reached in the *i-th* earthquake of a sequence (required to compute the ductility assumed to be related to the dissipated energy) is just the same as if the damaging event hit the new structure, plus the residual displacement from the preceding shaking. In other words, according to the assumed damage criterion, variation of drained capacity in the *i-th* earthquake is independent both of the age and the state the shock finds the structure in. Thus, different earthquakes produce i.i.d. effects, independent of the structural conditions.

5.2. Evaluating the distribution of damage increments

The marginal CDF of the damage increment in a shock, $\Delta\mu$, may be computed via Eq. (6), where $f_{IM}(im)$ is derived from the HPP hazard curve for the site where the structure is supposed to be located. The probabilistic seismic demand term, $f_{\Delta\mu|IM}(\delta\mu|IM=im)$, may be computed via *incremental dynamic analysis* (IDA) assuming the spectral acceleration at the elastic period of the SDOF, as an *IM*.[2] IDA is developed in terms of structural ductility normalized by μ_0, in a way that the demand is equal to 1 when the CP-LS is attained. Thus, $\Delta\mu$ may be defined as in Eq. (25), where μ_{before} and μ_{after} refer to residual capacity before and after the generic earthquake shock.

$$\Delta\mu = \frac{\mu_{before} - \mu_{after}}{\mu_0} \tag{25}$$

Fig. 6 (left) shows IDAs[3] output for the considered EPP system; in the same figure, recalling the normalization of ductility demand, collapse limit corresponding to 1, is also reported. Fig. 6 (right) shows $f_{\Delta\mu|IM}(\delta\mu|IM=im)$ for some ground motion intensities, under the assumption they are lognormal RVs (a well-established hypothesis in PBEE context).

Marginalization of the distribution of damage increments by $f_{IM}(im)$, as per Eq. (6), is site-specific. Considered site is (arbitrarily) Sulmona (13.96 Lon.; 42.05 Lat.), close to L'Aquila in central Italy. PSHA for the site was carried out by software specifically developed and described in [26], to which the reader should refer for details. $f_{IM}(im)$ for the spectral ordinate corresponding to the SDOF's elastic period, is reported in Fig. 7 (left). Note that, it is not exactly the hazard curve for the site, while it is the distribution of ground motion intensity *given the occurrence of an earthquake*. In fact, this is required to obtain the marginal distribution of capacity reduction in one shock, and it was obtained from the hazard curve divided by the annual rate of occurrence of events in Sulmona, which is equal to 1.95 (between magnitude 4.3 and 7.3).

In Fig. 7 (right), the result of the marginalization in Eq. (6) is reported. To comment on the plot it has to be recalled that, given the structure, not all earthquakes are strong enough to yield the structure, and $\Delta\mu = 0$ for such shocks. In particular, $\Delta\mu$ is larger than zero only for spectral accelerations larger than about 1.96 m/s^2, which is, in fact, the yielding acceleration of the considered EPP. Thus, damage increment is not a continuous RV and its CDF has the expression in Eq. (26).

$$F_{\Delta\mu}(\delta\mu) = P[\Delta\mu \le \delta\mu] = \begin{cases} P_0 & \delta\mu = 0 \\ P_0 + \int_0^{\delta\mu} \tilde{f}_{\Delta\mu}(x)\cdot dx & \delta\mu > 0 \end{cases} \tag{26}$$

In other words, the distribution of $\Delta\mu$ is defined by means of a probability density for $\Delta\mu > 0$, and a *probability mass* for $\Delta\mu = 0$. In fact, $P_0 = P[\Delta\mu = 0]$ accounts for the probability that earthquakes are not strong enough to damage the structure. Also $P[\Delta\mu > 1]$ has an interesting meaning; it is the marginal (i.e., with respect to earthquakes magnitude and location) probability that the new structure fails in just one event. In this application $P[\Delta\mu = 0]$ and $P[\Delta\mu > 1]$ are equal to 0.9924 and 0.0006, respectively. This means that only 0.76% of earthquakes is expected to be damaging, while 0.06% is expected to be *catastrophic*; i.e., directly causing collapse.

The expected value of $\Delta\mu$, $E[\Delta\mu] = 0.0026$, is also reported, yet barely visible, in Fig. 7 (right). It means that, for the considered structure at the considered site, a generic earthquake produces a capacity reduction of about 0.26%, on average with respect to both damaging and undamaging events. Thus, referring to the seismic hazard of Sulmona, given that average number of earthquakes in 1 yr is equal to 1.95, the considered SDOF is expected to undergo an average capacity reduction equal to $0.0026 \cdot 1.95 = 0.0051$ or 0.5% per year. Therefore,

[2] Due to the mentioned repetitive features of the EPP response, it is also easy to show that a single set of IDAs is required to estimate the distribution of damage increments given *IM* (see [23] for details).

[3] To develop IDAs thirty records were selected via REXEL [24], with moment magnitude between 5 and 7, epicentral distances lower than 30 km, and site class A according to Eurocode 8 [25].

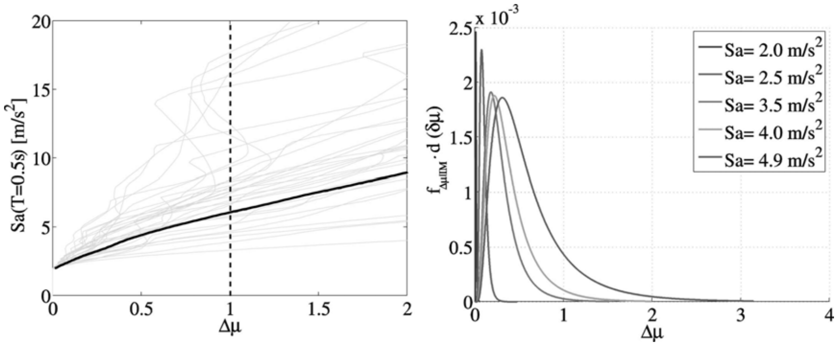

Fig. 6. Ductility demand from IDA analyses (left) and some distributions of structural damage conditional to ground motion intensity (right).

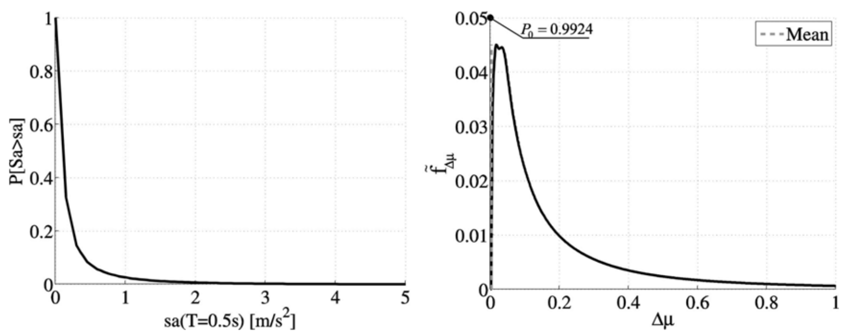

Fig. 7. Complementary CDF of ground motion intensity given earthquake occurrence at the site of interest (left); marginal distribution of $\Delta\mu$ for the structure at the site of interest (right).

according to the considered criterion, the structure fails after about 200 yr on average.

5.3. Absolute and conditional reliability for the cumulative earthquake damage case

The gamma distribution is adopted to model the PDF of shock effect in the case of damage larger than zero, $f_{\Delta\mu}(\delta\mu) = \tilde{f}_{\Delta\mu}(\delta\mu)/(1-P_0)$; see Section 2.2. Scale, γ_D, and shape, α_D, parameters of the model are set equal to 0.5539 and 0.1916, respectively.

The criterion to calibrate the gamma distribution was to set its mean and variance equal to the conditional mean and variance of damage (in the case it is larger than zero) computed by means of the structural analysis described in Section 5.2. Therefore, the scale and shape parameters were obtained solving the equations $\alpha_D/\gamma_D = E\,[\Delta\mu|\Delta\mu > 0] = 0.3459$ and $\alpha_D/\gamma_D^2 = Var[\Delta\mu|\Delta\mu > 0] = 0.6245$; where 0.3459 and 0.6245 are the mean and variance of the curve in Fig. 7 (right), when its area is normalized to one.

Failure probabilities are computed in the following illustrative cases: (1) failure probability within 50 yr, Eq. (12); (2) failure probability within 50 yr given that after the first 25 yr a reduction of 30% of the as-new capacity has been measured, Eq. (13); (3) failure probability within 50 yr given that no collapse was recorded in the first 25 yr, Eq. (14); and (4) failure probability within 50 yr given that one damaging earthquake hit the structure without causing collapse in the first 25 yr, Eq. (15). Results are given in Eq. (27), where it also recalled that the expected number of damaging earthquakes is computed filtering the *all earthquakes* HPP.

$$\begin{cases} (1) & P_f(50) \approx 0.076 \\ (2) & P_{f|D(25)=0.3}(50) \approx 0.0524 \\ (3) & P_{f|D(25)<1}(50) \approx 0.0407 \\ (4) & P_{f|D(25)<1,k_D=1}(50) \approx 0.047 \\ \{1,2,3\} & k_D = \lambda_D \cdot t = (1-P_0)\cdot\lambda\cdot t = 0.0076\cdot 1.95\cdot t \end{cases} \quad (27)$$

At this point, it is appropriate to check tolerability of the gamma-assumption for the damage increments. Moreover, it is the occasion also to verify the implications of using the approximation, based on the delta method, in Eq. (12). To this aim, in Fig. 8 (left) the CDF of the lifetime of the structure, $F_T(t)$, according to the model in Eq. (12), is reported.

In the same figure, $P_f(t)$ is also computed: (i) under the assumption damage increments are gamma distributed and explicitly considering the probability associated to any number of shocks as in Eq. (5); and (ii) adopting for $\Delta\mu$ the empirical distribution obtained from structural simulation (i.e., without fitting a gamma PDF to it), and explicitly considering the probability associated to any number of shocks. The figure shows that, at least up to 300 yr, where failure probability is 0.6 (hardly tolerable for a civil construction), the gamma assumption, even in case of the delta-method-based approximation, gives results in agreement with those of the empirical model (on the safe side). This is quantitatively shown by the ratios in Fig. 8 (right), which are computed taking as a reference $P_f(t)$ from the gamma-based model.

5.4. Total degradation and virtual age

This section starts considering the case of a structure subject (only) to continuous deterioration of seismic capacity that can be described via a gamma process with mean and variance function $s_A/\gamma_A \cdot t = 10^{-3}$ and $(s_A/\gamma_A^2)\cdot t = 10^{-4}\cdot t$, respectively. The corresponding process, and the CDF of lifetime, are reported in Fig. 9; it emerges that it was assumed continuous deterioration has a mild effect (in accordance to the literature; e.g., [18]), and the structure has a median life of

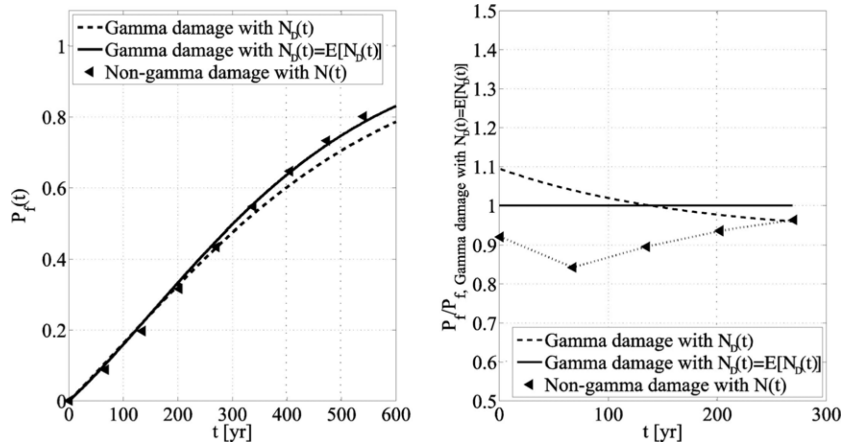

Fig. 8. Structural lifetime distribution in the case of earthquake damage according to the approximated model of Eq. (12) and removing the approximation of the gamma distribution for the damage increments and the expect number of shocks in lieu of any number of earthquakes.

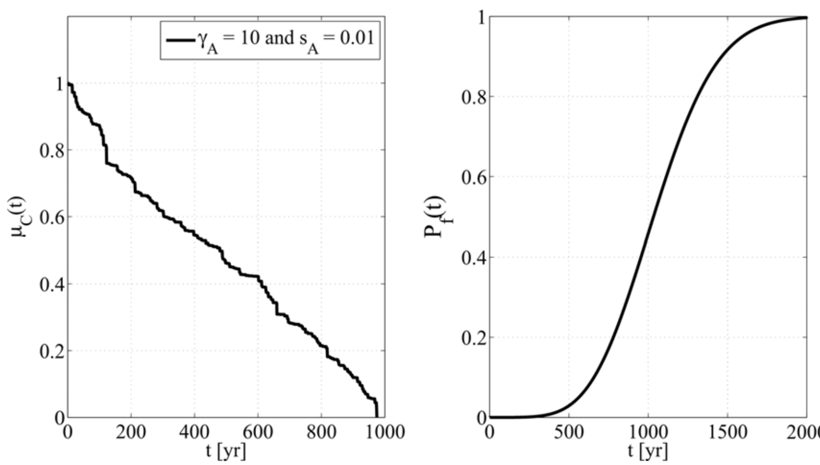

Fig. 9. Realization of continuous deterioration process (left); corresponding lifetime CDF (right).

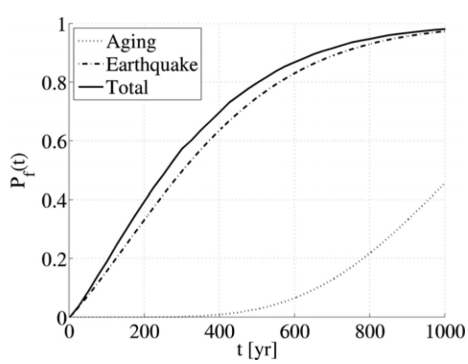

Fig. 10. Lifetime cumulative distribution functions for individual and combined processes.

about 1000 yr, while P_f is close to one after about 2000 yr, if this is the sole source of degradation. This arbitrary choice was to simulate aging mildly affecting seismic capacity if compared to shock-based damage.

In the case effects of earthquake and aging are independent, the failure probability due to both can be computed numerically solving Eq. (18), in which $\{\gamma_D, \alpha_D\}$ and $\{\gamma_A, s_A\}$ are equal to $\{0.5539, 0.1916\}$ and $\{10, 0.01\}$, respectively. Resulting lifetime distribution is reported in Fig. 10, along with analogous results for the two individual processes.

Assume now the same scale parameter, for example that of cumulative earthquake damage, can be attributed to both degradation processes. For example, imposing that $\gamma_A = \gamma_D$, the shape parameter of continuous deterioration process may be reshaped such that the same linear trend is preserved, that is $s_A = \gamma_D \cdot E[\mu_C(t)]/t$; however, this implies to force the variance of the process to be $(s_A/\gamma_D^2)\cdot t$. In this case, if $s_A = \gamma_D \cdot 10^{-3} = 0.5539 \cdot 10^{-3}$, the same mean of aging in Fig. 9 is kept, yet the variance results to be $Var[\mu_C(t)] = 0.0018 \cdot t$. This allows to apply Eq. (21) with $\{\gamma = \gamma_D, s = s_A + \lambda_D \cdot \alpha_D\}$ parameters, Eq. (28).

$$
\begin{aligned}
s &= s_A \left(1 + \lambda_D \cdot \frac{\gamma}{s_A} \cdot E[\Delta\mu | \Delta\mu > 0]\right) \\
&= 0.5539 \cdot 10^{-3} \cdot \left(1 + 0.01482 \cdot \frac{0.5539}{0.5539 \cdot 10^{-3}} \cdot 0.3459\right) \\
&= 0.0034
\end{aligned} \tag{28}
$$

The resulting lifetime CDF, compared with individual and total degradations, is given in Fig. 11 (left), where also suitability of the approximation is depicted (Fig. 11, right). It may be deduced that, in this particular case, the approximation provided by the simple model, appears acceptable.

Along the same line, it is also possible to get results analogous to Eq. (27) using Eqs. (21)–(24). Results are given in Eq. (29).

$$
\begin{cases}
(1)\ P_f(50) \approx 0.0920 \\
(2)\ P_{f|D(25)=0.3}(50) \approx 0.0629 \\
(3)\ P_{f|D(25)<1}(50) \approx 0.0499 \\
(4)\ P_{f|D(25)<1, k_D=1}(50) \approx 0.0572
\end{cases} \tag{29}
$$

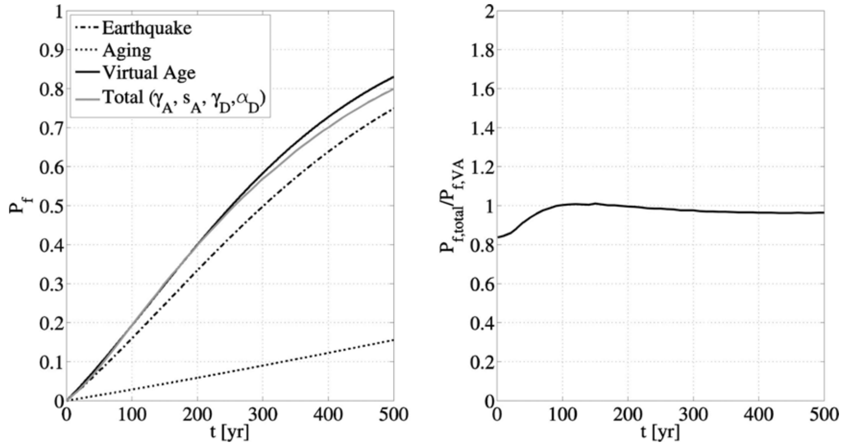

Fig. 11. Lifetime CDFs (left); failure probability ratio of superimposed degradations and virtual age, or VA (right).

It is interesting to note that, according to the parameters of the application, $\tau = \alpha_D/s_A = 0.1916/(0.5539{\cdot}10^{-3}) = 346$ yr, meaning that a generic damaging earthquake is computationally equivalent to an aging of the structure of more than 300 yr. This illustrates how the forward virtual age concept, if applicable, is attractive: it provides, at a glance, vulnerability of a structure subject to the considered sources of deterioration.

6. Conclusions

Life-cycle reliability analysis of deteriorating structures was discussed. The addressed approach potentially accounts for both progressive aging and damage accumulation due to earthquake shocks. The structural performance measure considered is the ductility capacity to collapse as a simplistic proxy for an energy-based damage criterion. This has the advantage to enable treating aging and earthquake damage effects altogether.

First, models for reliability analysis of structures cumulating seismic damage were discussed in the case of exponential and gamma distributions. Closed- and/or approximate-form reliability solutions were formulated; these also enable accounting for information possibly available at the epoch of the evaluation. Second, the gamma-process, especially suitable to represent continuous wear due to its non-negative, independent, and stationary increments characteristics, was adopted to model structural progressive damage accumulation. Then, reliability of structures, subject to both degradation phenomena, was formulated in the case of independent gamma-based processes. Finally, the computationally attractive forward virtual age option was also introduced.

The suitability of the discussed reliability model in the performance-based earthquake engineering context was also illustrated via a simple application, which refers to a bilinear SDOF system located in a relatively high seismicity site in central Italy. Conventional collapse prevention limit-state was considered and the gamma distribution's parameters were calibrated based on structural analysis. The results of the models were also discussed with respect to invoked assumptions and approximations.

Results support the conclusion that gamma-process-based stochastic modeling of degrading structures, may be useful in the performance-based earthquake engineering context.

Acknowledgements

The study was developed in the framework of *AMRA – Analisi e Monitoraggio dei Rischi Ambientali scarl* (http://www.amracenter.com). The results refer to the *Strategies and tools for Real-Time Earthquake Risk Reduction* (REAKT; http://www.reaktproject.eu) and to the *New Multi-Hazard and Multi-Risk Assessment Methods for Europe* (MATRIX; http://matrix.gpi.kit.edu/), projects. REAKT and MATRIX are funded by the European Community via the *Seventh Framework Program for Research* (FP7), with Contract Nos. 282862 and 265138, respectively. Authors want also to thank Ms. Racquel K. Hagen, of Stanford University, for proofreading the paper.

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Erratum

Erratum to "Gamma degradation models for earthquake-resistant structures" [Struct Saf 45 (2013) 48–58]

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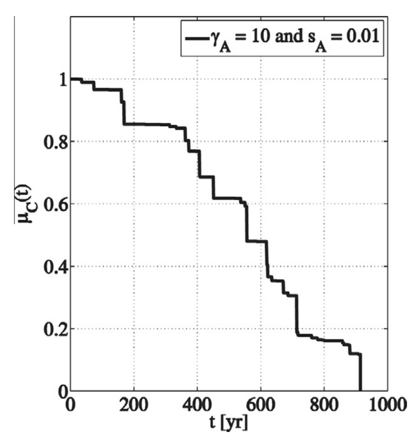
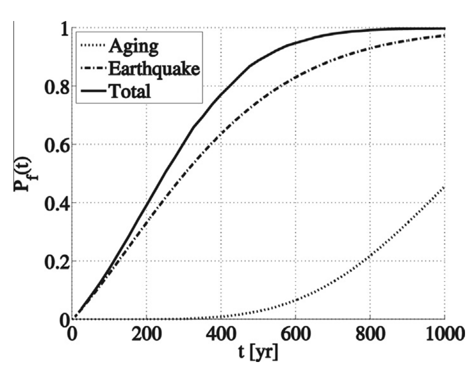
Because of a minor error found in the software generating gamma-distributed random numbers used to simulate aging, a few curves in the paper are slightly incorrect. Even if this error does not alter neither the comments about the plots nor the conclusions of the study, the correct curves are given here to enable the interested reader to exactly reproduce the results of the illustrative application in the paper.

1. The left panel of Fig. 9 should be replaced by this one:



2. The "Total" curve in Fig. 10 should be replaced by that in the following figure (the others are correct):



DOI of original article: http://dx.doi.org/10.1016/j.strusafe.2013.09.001
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http://dx.doi.org/10.1016/j.strusafe.2013.12.002

3. The "Total" curve in the left panel Fig. 11 and the right panel of Fig. 11 should be replaced by those in the following figure (the others are correct):