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Gamma modelling of aftershock reliability of elastic-perfectly-plastic systems

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Abstract. Major earthquakes (i.e., *mainshocks*) typically trigger a sequence of lower-magnitude events clustered both in time and space. Recent advances of seismic hazard analysis stochastically model aftershock occurrence (given the main event) as a non-homogeneous Poisson process with rate that decays in time as a negative power law. Risk management in the post-event emergency phase has to deal with this short-term seismicity. In fact, because the structural systems of interest might have suffered some damage in the mainshock, possibly worsen by damaging aftershocks, the failure risk may be large until the intensity of the sequence reduces or the structure is repaired. At the state-of-the-art, to quantitatively assess the aftershock risk serves to as *building tagging*, that is to regulate occupancy. The study, on the basis of age-dependent stochastic processes, derived closed-form approximations for the aftershock reliability of simple elastic-perfectly-plastic damage-cumulating systems, conditional on different information about the structure. Results show that, in the case hypotheses apply, the developed models may represent a handy tool for risk-informed tagging by stakeholders and decision makers.

Keywords: Performance-based earthquake engineering; Time-variant risk; Cumulative damage process.

1 INTRODUCTION

Short-term risk assessment, that is at the time-scale of weeks/months around a major event, is gathering increasing research attention due to the compelling need of decision makers for quantitative tools enabling to manage such a risk. Of particular interest is the evaluation of the failure probability for mainshock-damaged structures exposed to the following aftershock sequence. This may be referred to as building tagging and allows to monitor the variation of structural risk due to both increased vulnerability, caused by cumulative damage, and time-decaying aftershock hazard, and to decide whether to: prohibit access to anyone (i.e., red tag), allow access only to trained agents for emergency operations (i.e., yellow tag), or to resume from business interruptions allowing normal occupancy (i.e., green tag). Seminal research on the topic is that of Yeo and Cornell (2005, 2009a, 2009b), who developed aftershock probabilistic seismic hazard analysis (APSHA) and then coupled it with state-dependent fragilities in a performance-based approach to aftershock risk.

This study derives closed-form reliability solutions for elastic-perfectly-plastic (EPP) single degree of freedom (SDoF) systems exposed to aftershock hazard, based on a cumulative shock model. Damage increments (i.e., damages in individual seismic shocks) are addressed in the case they are independent and identically distributed (i.i.d.) random variables (RVs), characterized by a gamma distribution. It is shown that such hypotheses apply for simple, yet general, EPP-SDoF systems, in the case of an energy-based collapse criterion.

The following is structured such that essentials of APSHA are reviewed first. Subsequently the structural cumulative degradation stochastic process, based on the hypothesis that damage increments are gamma distributed, is addressed. Then, time-variant reliability formulations, given different knowledge about damage conditions, are derived. Finally, an application, referring to an EPP-SDoF supposed to be located in L'Aquila (central Italy), and exposed to a generic aftershock sequence from a magnitude, *M*, 6.3 event, is developed to illustrate the derived models.

2 APSHA ESSENTIALS

APSHA is expressed in terms of rate of events exceeding a ground motion intensity measure at a site of interest.[1] The main difference with long-term (or mainshock) PSHA (McGuire, 2004) is that such a rate is time-variant. The expected number of events per unit time decreases as the time elapsed since the triggering event increases. In this sense the aftershock process is conditional to the mainshock.

APSHA is based on the model of Reasenberg and Jones (1989) according to which, at time t (assuming that mainshock occurred at $t=0$), the daily rate of the aftershocks' occurrence is that in Eq. (1). Event magnitude is bounded between a minimum value of interest, m_l, and that of the mainshock, m_m. Coefficients *a* and *b* are from a suitable *Gutenberg-Richter relationship*, while *c* and *p* are from the *modified Omori law* for the considered sequence.

From Eq. (1) it follows that the expected number of aftershocks in the $(t, t+\Delta t)$ interval, is given by Eq. (2), which applies for a non-homogenous Poisson process (NHPP).

$$\lambda(t) = \left(10^{a+b\cdot(m_m - m_l)} - 10^a\right) \Big/ (t+c)^p \tag{1}$$

$$E\left[N(t,t+\Delta t)\right] = \int_t^{t+\Delta t} \lambda(\tau)\cdot d\tau = \frac{10^{a+b\cdot(m_m-m_l)} - 10^a}{p-1} \cdot \left[(t+c)^{1-p} - (t+\Delta t + c)^{1-p}\right] \tag{2}$$

APSHA *filters* the rate by the (time-invariant) probability that the ground motion intensity measure, *IM*, at the site of interest exceeds a threshold, $P[IM > im]$. This leads to the rate of the NHPP process, $\lambda_{im}(t)$, as in Eq. (3), where $P[IM > im \mid m, r_s]$ is provided by a ground motion prediction equation, and $f_{M,R_s}(m, r_s)$ is the joint probability density function (PDF) of magnitude and source-to-site distance, R_s, of aftershocks, which is sequence-specific (see also section 4).

$$\lambda_{im}(t) = \lambda(t)\cdot P[IM > im] = \lambda(t)\cdot \iint_{m,r_s} P[IM > im \mid m, r_s]\cdot f_{M,R_s}(m, r_s)\cdot dm\cdot dr \tag{3}$$

If only aftershocks above a certain intensity threshold, im^*, are damaging (e.g., above the intensity corresponding to the elastic limit of the structure), Eq. (3) may also serve to compute the rate of the NHPP characterizing their occurrence: $\lambda_D(t) = \lambda(t)\cdot P\left[IM > im^*\right]$, to follow.

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[1] While the reader should refer to (Yeo and Cornell, 2009a) for details, this section only recalls those essential results of aftershock probabilistic seismic hazard analysis, which are required by the models derived in section 3.

3 GAMMA MODEL FOR THE CUMULATIVE DAMAGE PROCESS

Given a mainshock-damaged structure, aftershocks may potentially further increase damage, unless partial or total restoration. Cumulative damage, or degradation, measured for example by means the residual ductility to collapse, or $\mu(t)$, may be susceptible of the graphical representation in Figure 1, which refers to a failure threshold corresponding to a limit-state of interest.

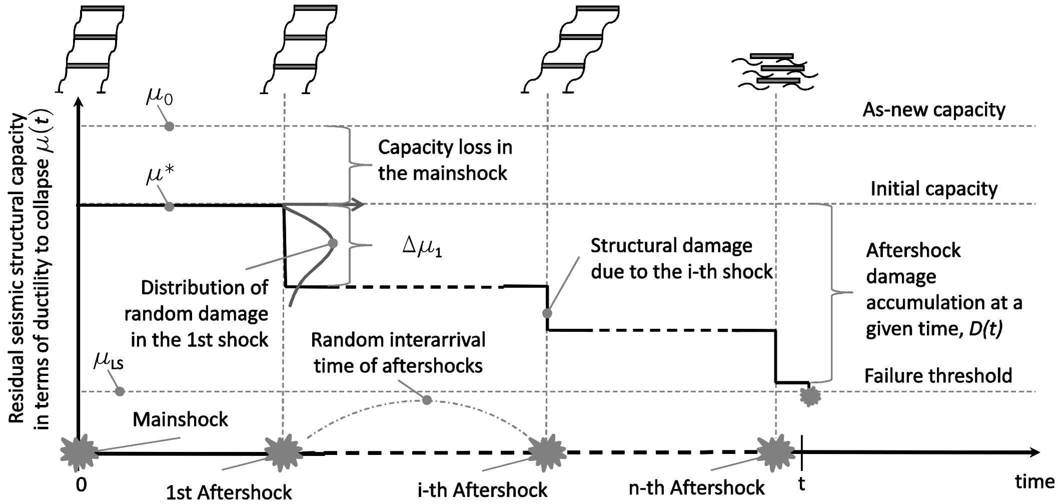

Figure 1. Degradation process for a mainshock-damaged structure exposed to aftershocks.

Formally, the degradation process is that in Eq. (4), where μ^* is the capacity at $t=0$, immediately after the mainshock of interest, and $D(t)$ is the cumulated damage due to all aftershocks, $N(t)$, occurring within t. Both $\Delta\mu_i$ (damage in one aftershock) and $N(t)$ are RVs.

$$\mu(t) = \mu^* - D(t) = \mu^* - \sum_{i=1}^{N(t)} \Delta\mu_i \tag{4}$$

Given this formulation, the probability the structure fails within *t*, $P_f(t)$, is the probability that the structure passes the limit-state threshold, μ_{LS}, or the complement to one of reliability, $R(t)$, Eq. (5). In fact, it is the probability that the capacity reduces travelling the distance, $\bar{\mu} = \mu^* - \mu_{LS}$, between the initial value and the threshold.

$$P_f(t) = 1 - R(t) = P\left[\mu(t) \le \mu_{LS}\right] = P\left[D(t) \ge \mu^* - \mu_{LS}\right] = P\left[D(t) \ge \bar{\mu}\right] \tag{5}$$

In the case the occurrence of seismic shock is described by a NHPP, and considering the cumulated damage as dependent on the vector of the ground motion intensities measures of occurring aftershocks, $P_f(t)$ may be computed from Eq. (6), where the integral is of *k-th* order.

$$\begin{aligned} P_f(t) &= \sum_{k=1}^{+\infty} P\left[D(t) \ge \bar{\mu} \middle| N(t) = k\right] \cdot P\left[N(t) = k\right] = \\ &= \sum_{k=1}^{+\infty} \int_{\underline{im}} P\left[\sum_{i=1}^{k} \Delta\mu_i \ge \bar{\mu} \middle| \underline{IM} = \underline{im}, N(t) = k\right] \cdot f_{\underline{IM}}(\underline{im}) \cdot d(\underline{im}) \cdot \frac{\left(E\left[N(t)\right]\right)^k}{k!} \cdot e^{-E\left[N(t)\right]} \end{aligned} \tag{6}$$

Because in APSHA, IMs, for example first mode spectral acceleration, or *Sa*, in different earthquakes are i.i.d. RVs, then $f_{\underline{IM}}\left(\underline{im}\right)$ is simply the product of $f_{IM_i}\left(im\right)$ distributions, which are k in number. Therefore, the critical issue to solve the reliability problem is to obtain $P\left[D(t)\geq\bar{\mu}\middle|\underline{IM}=\underline{im},N(t)=k\right]$, the probability of cumulative damage exceeding the threshold conditional to intensities and number of shocks, or, even better, $P\left[D(t)\geq\bar{\mu}\middle|N(t)=k\right]$.

3.1 Damage increment characterization and reliability approximations

Reliability may be easily addressed if three conditions are met. (1) Damage in the *i-th* earthquake, $\Delta\mu_i$, has always the same distribution, $f_{\Delta\mu_i}\left(\delta\mu\right)$, marginal with respect to *IM*; i.e., $f_{\Delta\mu_i}\left(\bullet\right)=f_{\Delta\mu}\left(\bullet\right),\forall i$. (2) Damages produced in different events are independent RVs. In other words, according to these two conditions, earthquake's structural effects are i.i.d. (Iervolino et al., 2012).[2]

Condition (3) is that distribution of sums of damages can be expressed in a simple form. A way to satisfy this condition consists of modeling damage via a RV that enjoys the *reproductive* property.[3] Indeed, an attractive option to probabilistically model damage increment is the gamma distribution, Eq. (7), which depends on γ_D and α_D; the scale and shape parameters, respectively.[4] The *D* subscript emphasizes that, being continuous and non-negative, such RV is suitable to model only the effect of earthquakes factually determining loss of capacity, not those events whose intensity is not large enough (see section 4).

$$f_{\Delta\mu}\left(\delta\mu\right)=\int_{im} f_{\Delta\mu|IM}\left(\delta\mu\middle|x\right)\cdot f_{IM}\left(x\right)\cdot dx\triangleq\frac{\gamma_D\cdot\left(\gamma_D\cdot\delta\mu\right)^{\alpha_D-1}}{\Gamma\left(\alpha_D\right)}\cdot e^{-\gamma_D\cdot\delta\mu} \qquad (7)$$

Because the sum of k_D i.i.d. gamma-distributed RVs, with scale and shape parameters γ_D and α_D respectively, is still gamma with parameters γ_D and $k_D\cdot\alpha_D$, the probability of cumulative damage exceeding the threshold, conditional to k_D shocks, is given by Eq. (8). $\Gamma\left(\beta\right)=\int_0^{+\infty}z^{\beta-1}\cdot e^{-z}\cdot dz$ and $\Gamma_U\left(\beta,y\right)=\int_y^{+\infty}z^{\beta-1}\cdot e^{-z}\cdot dz$ are the *gamma* and the *upper incomplete gamma* functions, respectively, and $N_D\left(t\right)$ is the NHPP counting function for the damaging events.

$$P\left[D(t)\geq\bar{\mu}\middle|N_D\left(t\right)=k_D\right]=\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{k_D\cdot\alpha_D-1}}{\Gamma\left(k_D\cdot\alpha_D\right)}\cdot e^{-\gamma_D\cdot x}\cdot dx=\frac{\Gamma_U\left(k_D\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right)}{\Gamma\left(k_D\cdot\alpha_D\right)} \qquad (8)$$

If $P_f\left(t\right)$ in Eq. (6) can be approximated, via the *moment approximation* (Oehlert, 1992), that is by its value conditional to the expected number of damaging earthquakes until *t*, Eq. (9) results.

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[2] Which, in particular, implies increment of damage in an earthquake is independent of structural state. Otherwise, for example in the case of systems with evolutionary hysteretic behaviour, *state-dependent* approaches may be required; e.g., (Luco et al, 2002, Giorgio et al., 2010, Giorgio et al., 2011).

[3] A well-known example of reproductive RV is the Gaussian one. However, because it should be $\Delta\mu_i\geq 0\;\forall i$, the cumulative degradation process due to subsequent aftershocks should also show non-negative increments, disqualifying the Gaussian representation of damage. Although the lognormal PDF may appear as a solution, it is not reproductive in the addition sense.

[4] This PDF is quite flexible: a shape parameter equal to one, stretches the distribution to the exponential; a large value of α_D, say larger than four, let the PDF be similar to that Gaussian; an intermediate value, around two, makes it similar to the lognormal shape.

$$
\begin{aligned}
&P_f\left(t\right)\approx P\left[D\left(t\right)\geq\bar{\mu}\middle|N_D\left(t\right)=E\left[N_D\left(t\right)\right]\right]= \\
&=\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{E\left[N_D(t)\right]\cdot\alpha_D-1}}{\Gamma\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D\right)}\cdot e^{-\gamma_D\cdot x}\cdot dx=\frac{\Gamma_U\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right)}{\Gamma\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D\right)}
\end{aligned}
\tag{9}
$$

Also interesting is the case in which one wants to include in the reliability assessment the information that the structure is still surviving at a certain time after the mainshock, t_1, but with unknown damage condition. It may be computed via, Eq. (10). Finally, the probability of failure given survival in t_1, and a given number of aftershocks, k_D, yields Eq. (11).

$$
\begin{aligned}
&P_{f|D(t_1)<\bar{\mu}}\left(t\right)=1-\frac{R\left(t\right)}{R\left(t_1\right)}\approx \\
&\approx 1-\frac{1-\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{E\left[N_D(t)\right]\cdot\alpha_D-1}}{\Gamma\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D\right)}\cdot e^{-\gamma_D\cdot x}\cdot dx}{1-\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{E\left[N_D(t_1)\right]\cdot\alpha_D-1}}{\Gamma\left(E\left[N_D\left(t_1\right)\right]\cdot\alpha_D\right)}\cdot e^{-\gamma_D\cdot x}\cdot dx}=1-\frac{1-\frac{\Gamma_U\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right)}{\Gamma\left(E\left[N_D\left(t\right)\right]\cdot\alpha_D\right)}}{1-\frac{\Gamma_U\left(E\left[N_D\left(t_1\right)\right]\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right)}{\Gamma\left(E\left[N_D\left(t_1\right)\right]\cdot\alpha_D\right)}}
\end{aligned}
\tag{10}
$$

$$
\begin{aligned}
&P_{f|D(t_1)<\bar{\mu},N_D(t_1)=k_D}\left(t\right)=1-\frac{1-P\left[\sum_{i=1}^{k_D+N_D(t_1,t)}\Delta\mu_i\geq\bar{\mu}\right]}{1-P\left[\sum_{i=1}^{k_D}\Delta\mu_i\geq\bar{\mu}\right]}\approx \\
&\approx 1-\frac{1-\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{\left(k_D+E\left[N_D(t_1,t)\right]\right)\cdot\alpha_D-1}}{\Gamma\left[\left(k_D+E\left[N_D\left(t_1,t\right)\right]\right)\cdot\alpha_D\right]}\cdot e^{-\gamma_D\cdot x}\cdot dx}{1-\int_{\bar{\mu}}^{+\infty}\frac{\gamma_D\cdot\left(\gamma_D\cdot x\right)^{k_D\cdot\alpha_D-1}}{\Gamma\left(k_D\cdot\alpha_D\right)}\cdot e^{-\gamma_D\cdot x}\cdot dx}=1-\frac{1-\frac{\Gamma_U\left[\left(k_D+E\left[N_D\left(t_1,t\right)\right]\right)\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right]}{\Gamma\left[\left(k_D+E\left[N_D\left(t_1,t\right)\right]\right)\cdot\alpha_D\right]}}{1-\frac{\Gamma_U\left(k_D\cdot\alpha_D,\gamma_D\cdot\bar{\mu}\right)}{\Gamma\left(k_D\cdot\alpha_D\right)}}
\end{aligned}
\tag{11}
$$

Even if ugly looking, as shown in the next section, these equations allow an easy reliability assessment, because solutions are readily available for the gamma functions.

4 ILLUSTRATIVE APPLICATION

The application refers to a simple EPP-SDoF system with reloading/unloading stiffness, which is the same as the initial one. The elastic period is equal to 0.5 s; weight is 100 kN and the yielding force is equal to 19.6 kN. Chosen engineering demand parameter (EDP) is the kinematic ductility, μ; i.e., the maximum displacement demand, when the yielding displacement is the unit. To focus on the reliability analysis more than on structural modeling, this EDP is considered as the simplest proxy for the dissipated hysteretic energy during one earthquake event; i.e., the amount of dissipated energy is proportional to the maximum plastic excursion (Cosenza and Manfredi, 2000). Note that this implies, as in all energy-based damage measures, an amount of structural damage in all seismic events with intensity larger than that required to yield the structure.

The collapse is assumed to occur when kinematic ductility, conservatively accumulated independently on the sign of maximum displacement, reaches the limit state (LS) threshold. Quantification of such

LS is derived from FEMA 356 (2000), which identifies the conventional *collapse prevention* (CP) for concrete structures at a maximum drift ratio equal to 0.04.[5] Appling to the considered system, (initial) ductility capacity is $\mu_0 = 3.3$, and each damaging shock drains some of this ductility supply (Figure 2).

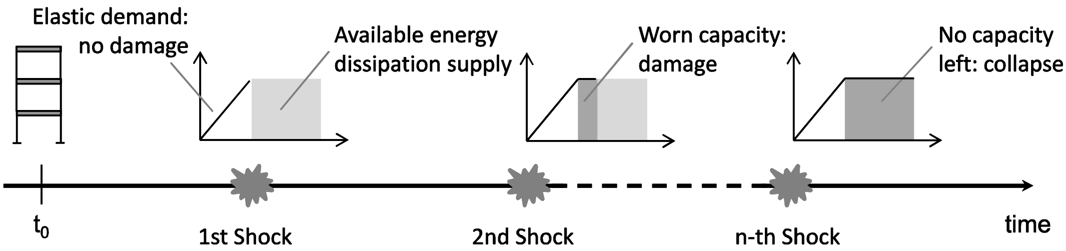

Figure 2. Accumulation of damage with respect to kinematic ductility for EPP-SDoF systems.

This system shows stable hysteretic cycles that repeat themselves despite of the sequence of excitation it undergoes to, which is important with respect to the reliability models based on i.i.d. damage increments. In fact, the force-deformation response, is such that the demand of kinematic ductility in the *i-th* earthquake of a sequence is just the same as if the damaging event hit the structure in initial conditions. In other words, *variation* of drained capacity in the *i-th* earthquake is independent of the state the shock finds the structure in and ductility is computed as if the residual displacement at beginning of each ground motion is zero. Thus, damage increments are i.i.d. In the following subsection a gamma distribution is calibrated for the damage increment of the SDoF considered.

4.1 Calibrating the gamma distribution for damage increments

The PDF of damage in a shock may be obtained via the marginalization in Eq. (7). The probabilistic seismic demand term, $f_{\Delta\mu|IM}$, may be computed via *incremental dynamic analysis* (IDA) assuming *Sa* at the elastic period of the SDoF, as an *IM*.[6] IDA is developed herein[7] in terms of structural ductility response normalized by μ_0, so that ductility demand is equal to 1 at onset of CP-LS. Indeed, damage increment is defined as $\Delta\mu = \left(\mu_{before} - \mu_{after}\right) / \mu_0$, where μ_{before} and μ_{after} refer to capacity before and after the generic aftershock. Figure 3 (left) shows $f_{\Delta\mu|IM}$ for some ground motion intensities under the, formally tested, lognormal assumption.

Considered site for risk assessment is (arbitrarily) L'Aquila (13.40 lon., 42.35 lat.). Figure 3 (center) shows the site-specific f_{IM} which is also required by Eq. (7).[8] This is the distribution of ground motion intensity *given the occurrence of an aftershock*; therefore it was computed via the integral term at the right hand side of Eq. (3), assuming M and R_s, in one aftershocks, to be stochastically independent RVs. Following section 2, $f_M(m)$ is considered to be an exponential PDF with the upper bound of magnitude equal to 6.3, the 2009 L'Aquila earthquake magnitude. The minimum aftershock magnitude value is arbitrarily taken equal to 4.3.

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[5] This is to obtain a purely conventional measure of energy dissipation structural supply, which may be not perfectly appropriate if such a threshold refers to a maximum displacement collapse criterion.

[6] In fact, due to the mentioned repetitive features of the EPP response, it is also easy to show that a single set of IDAs can be used to estimate the distribution of damage increment given *IM* (see Chioccarelli and Iervolino, 2012, for details).

[7] To develop IDAs thirty records were selected via REXEL (Iervolino et al., 2010), with moment magnitude between 5 and 7, epicentral distances lower than 30 km, and stiff site class.

[8] The hazard software and ground motion prediction equation used for the calculations are the same of Iervolino et al. (2011).

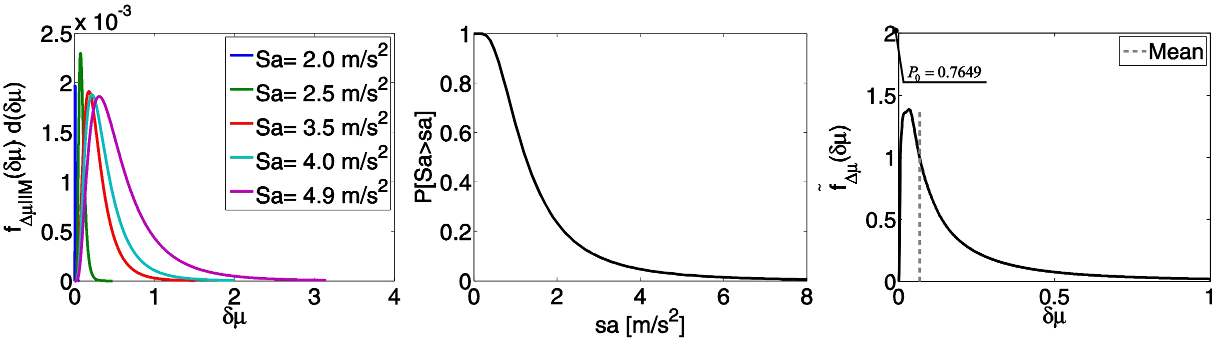

Figure 3. Examples of distribution of structural damage conditional to ground motion intensity (left), distribution of Sa(0.5s) give the earthquake occurrence (center), and damage increment distribution (right).

The *b*-value of the GR relationship was taken equal to 0.96 (Lolli and Gasperini, 2003), as justified in the following section. To constrain the epicenters of aftershocks with respect to *M* 6.3, a 400 km^2 aftershock source area around the considered site was considered. Therefore, f_{R_s} only accounts for such a seismogenic zone, the size of which was obtained by the aftershock area versus mainshock magnitude relationship in Utsu (1970).

It is to note that, although all the assumptions of the application are generic and reference to 2009 earthquake is only in terms of mainshock magnitude, aftershock area considered is not much larger than the fault supposed to have originated the 2009 earthquake (Chioccarelli and Iervolino, 2010).

The marginalization as per Eq. (7) leads to the distribution of damage in Figure 3 (right). To comment such a plot it has to be recalled that, given the structure, not all aftershocks are strong enough to yield the structure, and $\Delta\mu = 0$ in the case of *weak* motions. In particular, $\Delta\mu$ is larger than zero only for spectral accelerations larger than 1.96 m/s^2; i.e., the yielding acceleration of the considered EPP-SDoF. Thus, damage increment is not a continuous RV and its cumulative distribution function has the expression in Eq. (12). In other words, the distribution of $\Delta\mu$ is defined by means of a probability density for $\Delta\mu > 0$, and a *probability mass* in zero. In fact, $P_0 = P\left[\Delta\mu = 0\right]$ accounts for the probability that shocks are not strong enough to damage the structure. In this case, P_0 is equal to 0.7649, that is only 23.5% of aftershocks is expected to be damaging. In fact, in Figure 3 the mean of the damage increment (dashed vertical line), accounting for both damaging and undamaging aftershocks, is also given.

$$P\left[\Delta\mu \le \delta\mu\right] = \begin{cases} P_0 & \delta\mu = 0 \\ P_0 + \int\limits_0^{\delta\mu} \tilde{f}_{\Delta\mu}(x) \cdot dx & \delta\mu > 0 \end{cases} \tag{12}$$

Average degradation increment due to a damaging aftershock, $E\left[\Delta\mu \mid \Delta\mu > 0\right]$, is 0.2884, while the variance is $Var\left[\Delta\mu \mid \Delta\mu > 0\right] = 0.3522$. To retain such moments, the gamma PDF adopted to model the shock effect in the case of damage larger than zero, $f_{\Delta\mu}(\delta\mu) = \tilde{f}_{\Delta\mu}(\delta\mu)/(1-P_0)$, must have scale and shape parameters equal to 0.8187 and 0.2361, respectively. This is because the mean and the variance in the gamma distribution are equal to α_D/γ_D and α_D/γ_D^2, respectively.

4.2 Reliability in the aftershock sequence of a M 6.3 mainshock

Risk assessment as per the developed model, first requires the rate, $\lambda(t)$, of aftershock from Eq. (1). To this aim, coefficients of the *generic sequence* $a = -1.66$, $b = 0.96$, $c = 0.03$, $p = 0.93$, from (Lolli and Gasperini, 2003) were used, considering 4.3 and 6.3 as m_l and m_m, respectively. Then the rate of damaging aftershocks is $\lambda_D(t) = \lambda(t) \cdot P[\Delta\mu > 0] = \lambda(t) \cdot (1 - 0.7649)$. Thus, the expected number of damaging events, in any time interval, is given by Eq. (2), plugging in $\lambda_D(t)$. Figure 4a plots the total and damaging-only aftershock rates until three months since the mainshock.

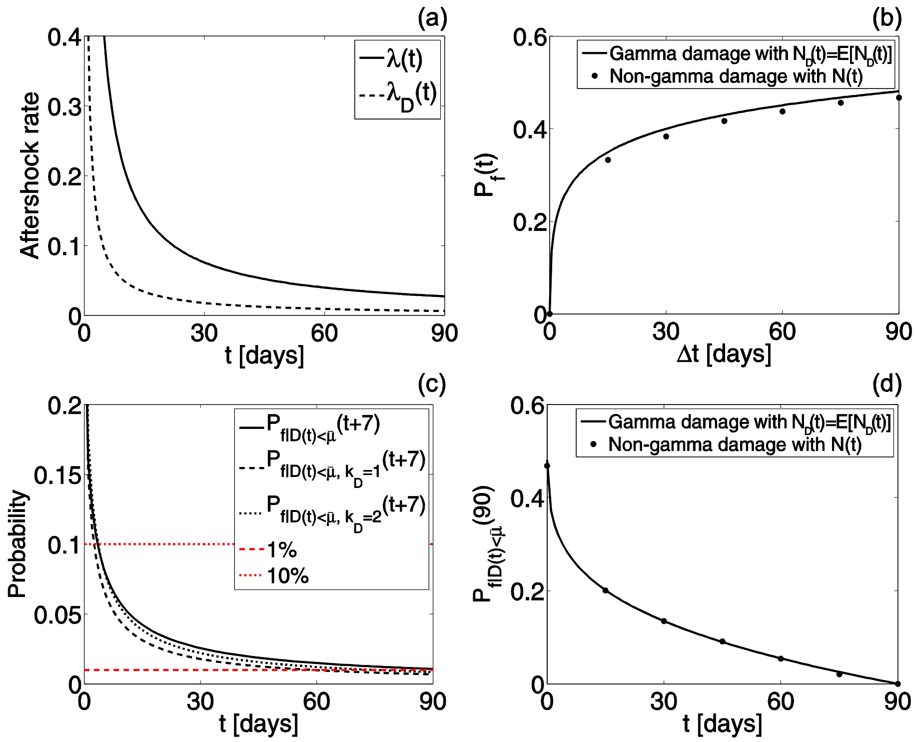

Figure 4. Aftershock rates for the considered sequence (a), failure probability for the structure as time since the mainshock passes (b), failure probability in the next seven days given survival at *t* and/or major aftershocks (c), and failure probability in what remains of three months since the mainshock, given survival at *t* (d).

It is arbitrarily assumed that the structure has survived to the mainshock event with a residual capacity $\mu^* = 0.7$ (i.e., 30% capacity reduction). At this point, the failure probabilities according to the models of section 3.1 can be easily computed via, for example, the *gammainc* function implemented in MATHWORKS-MATLAB®. Figure 4, panels b-d, reports about the cases listed below, where *t* is always the time elapsed since the mainshock.

- Figure 4b: absolute failure probability in the $(0,t)$ interval as per Eq. (9); the trend is increasing because the larger the interval, the larger the expected number of shocks, see Eq. (2).
- Figure 4c: this panel reports three curves: $P_{f|D(t)<\bar{\mu}}(t+7)$ from Eq. (10), it is the failure probability the week following *t*, given survival in *t*; $P_{f|D(t)<\bar{\mu},k_D=1}(t+7)$ from Eq. (11), that is the same as the previous case, except that it is known that one damaging aftershocks has occurred before the time of the reliability evaluation; and $P_{f|D(t)<\bar{\mu},k_D=2}(t+7)$, that is, it is known that two damaging aftershocks have occurred before the time of the reliability evaluation.
- Figure 4d: $P_{f|D(t)<\bar{\mu}}(90)$ from Eq. (10), that is, assuming three months as reference duration of the sequence, the failure probability in the remaining exposure time, given survival at *t*.

Dotted graphs in Figure 4b and Figure 4d, represent the same results computed, via Monte Carlo simulation, removing the hypothesis that increments are gamma-distributed and without using the first moment approximation; i.e., directly employing Eq. (6). Errors introduced by the approximated models appear tolerable, and on the safe side.

As an illustration of use of these results, Figure 4c also reports two (time-invariant) probabilities. The lower one is 1%, the larger one is 10%. Following the approach in Yeo and Cornell (2005), and arbitrarily assuming the probability 1% as a tolerable collapse risk in one week during an aftershock sequence, and ten times its value as a tolerable risk for emergency operations, it may be said that: before the decaying risk intersects the largest probability, the structures is red tagged (i.e., cannot be accessed) in the next week, is green tagged after the aftershock risk gets smaller than the lower value, and can be entered only by trained agents in between.

5 CONCLUSIONS

A model for the reliability assessment of damage-cumulating elastic-perfectly-plastic systems in aftershock environment was discussed. It is based on a cumulative shock model, which makes use of the gamma distribution to describe damage in a shock and non-homogeneous Poisson occurrence for shocks. The resulting degradation process, considering the ductility capacity to collapse as the structural damage measure, is an independent increments one, enabling approximate-form solutions for reliability, given different information about the structure.

The applicability of the model was illustrated via an example referring to a mainshock-damaged system, supposed to be subjected to a generic aftershock sequence following a M 6.3 event. Different kinds of failure risks during the sequence were evaluated. Some of them were also compared with arbitrary values of tolerable risk, which were used as tagging criteria in a virtual application of short-term seismic risk management.

The results from model were also discussed with respect to approximations invoked, which appear tolerable, if compared with the near-real-time risk assessment the closed-forms allow.

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