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Probabilistic seismic hazard analysis for seismic sequences

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Abstract. Earthquakes are typically clustered both in space and time. Seismic hazard, expressed in terms of rate of exceedance of a ground motion intensity measure, classically refers to mainshocks, which means that such a rate is computed filtering the rate of occurrence of events of largest magnitude within each cluster of earthquakes. This kind of probabilistic seismic hazard analysis (PSHA) is used for structural design or assessment in the long-term. Recently, a similar probabilistic approach has been adopted to perform aftershock probabilistic seismic hazard analysis (APSHA), conditional to mainshock occurrence. It is to be used for short-term risk management. PSHA often refers to a homogeneous Poisson process to describe event occurrence, while APSHA models aftershock occurrence via a non-homogeneous Poisson process, whose rate depends on the magnitude of mainshock that has triggered the considered sequence. However, the whole cluster, comprised of mainshock and aftershocks, occurs with the same rate of the mainshock, and this paper shows how it is possible to analytically combine results of PSHA and APSHA to get a probabilistic seismic hazard analysis for mainshock-aftershocks seismic sequences (SPSHA). Results of the illustrative application presented help to assess the increase in seismic hazard considering the probability of exceeding an acceleration threshold (e.g., that considered for design) also considering aftershocks.

Keywords: earthquake clusters; mainshocks; aftershocks.

1 INTRODUCTION

The probabilistic seismic hazard analysis (PSHA; e.g. McGuire 2004) is a consolidated procedure to assess the seismic threat for a specific site. In the classical case, the homogeneous Poisson process (HPP) is considered to probabilistically model mainshock occurrence at each seismic source. The latter is a *memory-less* model; i.e., the rate of occurrence is time-invariant and independent on the seismic history.

PSHA refers to occurrence of mainshock, that is prominent magnitude earthquakes possibly identified within clusters, which are sequences of events concentrated in both space and time.

Aftershock sequences may be seen as triggered by the mainshock. Aftershock occurrence may be modelled via a non-homogeneous Poisson process (NHPP) with a rate, which depends on the magnitude of the triggering mainshock, according to the model of Reasemberg and Jones (1989).

Recently, Yeo and Cornell (2009) developed aftershock-PSHA (APSHA) to express aftershock hazard similar to the mainshock hazard; i.e., in terms of probability of exceedance of a ground motion intensity measure (IM) threshold. This is useful in the post-mainshock emergency phase and for short-term risk management.

Considering a seismic cluster as the whole of mainshock and following aftershock sequence, it may be argued that the occurrence of clusters is probabilistically described by the same rate of the main event.[1] Foreshocks are neglected, as they are usually very limited in number (Yeo and Cornell 2009).

Following Boyd (2012), it appears possible to extend PSHA, filtering the rate of occurrence of mainshocks by the probability that a ground motion intensity measure threshold is exceeded at least once during the sequence. This leads to sequence-PSHA (SPSHA), for long-term risk analysis, and seismic design, accounting for the aftershock potential.

The study presented in the following shows the analytical formulation of SPSHA. It is built on the hypotheses of HPP occurrence of mainshocks, the NHPP conditional occurrence of aftershocks, and the dependency of the latter process on the mainshock magnitude in terms of: rate of occurrence, magnitude range, and spatial clustering. To this aim PSHA and APSHA essentials are reviewed first. The combination of the two is analytically discussed and, as an illustrative application, a generic seismogenic source is considered, and the SPSHA expressed in terms of annual rate of exceedance of different *IM*-levels is, finally, computed.

2 MAINSHOCK, AFTERSHOCKS AND GROUND MOTION INTENSITY

In this section, stochastic processes used to evaluate mainshock and aftershock hazards, both expressed in terms of rate of exceedance of a ground motion intensity threshold, are briefly reviewed.

2.1 Mainshock probabilistic seismic hazard analysis

Probabilistic seismic hazard analysis usually refers to homogeneous Poisson process (HPP) to probabilistically model earthquake occurrence. The latter is an independent- and stationary-increment (i.e., *memory-less*) model, entirely described by one parameter, the *rate*, ν_E.

According to HPP, the probability of any number of events, N_E, occurring in the time interval of interest, $(t, t+\Delta T)$, is independent of the history of earthquakes occurred in the past and can be expressed via the Poisson probability mass function in Equation (1). It is also consequent to the HPP that the interarrival time distribution of mainshocks is described by an exponential distribution of parameter ν_E (e.g., Reiter 1990).

$$P\left[N_E(t,t+\Delta T)=n\right]=P\left[N_E(\Delta T)=n\right]=\frac{(\nu_E\cdot\Delta T)^n}{n!}\cdot e^{-\nu_E\cdot\Delta T} \quad (1)$$

In PSHA, the exceedance of an *IM* threshold, *im*, at a site of interest is also probabilistically described by a HPP. The rate of the latter, $\lambda_{im,E}$, is obtained from ν_E as in Equation (2), where the term $P\left[IM>im\,|\,x,y\right]$, provided by a ground motion prediction equation (GMPE), represents the probability that intensity threshold is exceeded given an earthquake of magnitude $M_E=x$ on the considered source, from which the site is separated by a distance equal to $R_E=y$. f_{M_E,R_E} is the joint probability density function (PDF) of mainshock magnitude and distance random variables (RVs). In

[1]However, the point-process assumption taken for mainshocks may be less acceptable for seismic sequences, which may last for several months or even years.

the case these two may be considered *s-independent*, f_{M_E} is often based on a *Gutenberg-Richter* (GR) *relationship*, and f_{R_E} depends on the source-site geometrical configuration.

$$\lambda_{im,E} = \nu_E \cdot \int_{r_{E,\min}}^{r_{E,\max}} \int_{m_{E,\min}}^{m_{E,\max}} P\left[IM > im \mid x,y\right] \cdot f_{M_E,R_E}\left(x,y\right) \cdot dx \cdot dy \tag{2}$$

2.2 Aftershocks probabilistic seismic hazard analysis

APSHA is also expressed in terms of rate of events exceeding a ground motion intensity measure threshold at a site of interest. The main difference with PSHA is that such a rate is time-variant. The expected number of events per unit time decreases as the time elapsed since the triggering mainshock event increases. In this sense the aftershock process is conditional to the mainshock occurrence and characteristics.

APSHA is based on a model according to which, at time t (assuming that the mainshock occurred at $t = 0$), the daily rate of occurrence of the aftershocks, $\nu_{A|m_E}\left(t\right)$, is provided in Equation (3). Aftershock magnitude is bounded between a minimum value of interest, $m_{\min}$, and that of the mainshock. Coefficients *a* and *b* are from a suitable GR relationship, while *c* and *p* are from the *modified Omori law* for the considered sequence. From Equation (3) it follows that the expected number of aftershocks in the $\left(t, t+\Delta T_A\right)$ interval, is given by Equation (4), which applies for a NHPP.

$$\nu_{A|m_E}\left(t\right) = \left(10^{a+b\cdot\left(m_E - m_{\min}\right)} - 10^a\right) \Big/ \left(t+c\right)^p \tag{3}$$

$$E\left[N_{A|m_E}\left(t, t+\Delta T_A\right)\right] = \int_t^{t+\Delta T_A} \nu_{A|m_E}\left(\tau\right) \cdot d\tau = \frac{10^{a+b\cdot\left(m_E - m_{\min}\right)} - 10^a}{p-1} \cdot \left[\left(t+c\right)^{1-p} - \left(t+\Delta T_A + c\right)^{1-p}\right] \tag{4}$$

Also APSHA *filters* the rate by the (time-invariant) probability that the *IM* at the site of interest exceeds the threshold. This leads to the rate of the NHPP process, $\lambda_{im,A|m_E}\left(t\right)$, as in Equation (5), where f_{M_A,R_A} is the joint PDF of magnitude and source-to-site distance of aftershocks.

$$\lambda_{im,A|m_E}\left(t\right) = \nu_{A|m_E}\left(t\right) \cdot \int_{r_{A,\min}}^{r_{A,\max}} \int_{m_{\min}}^{m_E} P\left[IM > im \mid w,z\right] \cdot f_{M_A,R_A}\left(w,z\right) \cdot dw \cdot dz \tag{5}$$

3 COMBINING MAINSHOCKS AND AFTERSHOCKS: SPSHA

In this section the probabilistic seismic hazard analysis accounting for both mainshock and aftershocks effects is formulated. The whole sequence is described by a HPP (point) process and, within a sequence, aftershocks are described by a NHPP whose rate function is conditional to the mainshock magnitude. The aim is, again, to evaluate the rate of exceedance of a ground motion intensity measure λ_{im} . Such a rate corresponds to the event defined as *the exceedance of an IM threshold at least once during the sequence*, Equation (6). In the equation, *IM* is the maximum in the cluster, IM_E is the mainshock intensity measure, and $IM_{\cup A}$ is the maximum intensity measure in the aftershock sequence.

$$\begin{aligned}\lambda_{im} &= \nu_E \cdot P[IM > im] = \nu_E \cdot P\left[IM_E > im \cup IM_{\cup A} > im\right] = \\ &= \nu_E \cdot \left\{1 - P\left[IM_E \le im \cap IM_{\cup A} \le im\right]\right\}\end{aligned} \tag{6}$$

In fact, according to APSHA, the aftershock sequence entirely depends on the mainshock magnitude and location. Indeed, the number of events, their magnitude, and their location, are function of the size of the sequence-triggering earthquake and source position. Therefore, conditional to M_E, the non-exceedance in the mainshock and in the aftershocks are independent events, Equation (7).

$$\begin{aligned}\lambda_{im} &= \nu_E \cdot \left\{1 - \iint_{M_E,R_E} P\left[IM_E \le im \cap IM_{\cup A} \le im \mid x,y\right] \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\right\} = \\ &= \nu_E \cdot \left\{1 - \iint_{M_E,R_E} P\left[IM_E \le im \mid x,y\right] \cdot P\left[IM_{\cup A} \le im \mid x,y\right] \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\right\}\end{aligned} \tag{7}$$

The probability of not exceeding the threshold during the aftershock sequence, $P\left[IM_{\cup A} \le im \mid x,y\right]$, is formulated accounting for the fact that such a sequence is comprised of a random number of events, N_A. According to the NHPP assumption, such a random variable is described by a Poisson distribution, as Equation (1), of rate given in Equation (3). Therefore, applying the total probability theorem, Equation (8) results.

$$\begin{aligned}\lambda_{im} &= \\ &= \nu_E \cdot \left\{1 - \iint_{M_E,R_E} P\left[IM_E \le im \mid x,y\right] \cdot \sum_{i=0}^{+\infty} \left(P\left[IM_A \le im \mid x,y\right]\right)^i \cdot P\left[N_A = i \mid x\right] \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\right\} = \\ &= \nu_E \cdot \left\{1 - \iint_{M_E,R_E} P\left[IM_E \le im \mid x,y\right] \cdot \sum_{i=0}^{+\infty} \left(P\left[IM_A \le im \mid x,y\right]\right)^i \cdot \frac{\left(\int_0^{\Delta T_A} \nu_{A|x}(\tau) \cdot d\tau\right)^i}{i!} \cdot e^{-\int_0^{\Delta T_A} \nu_{A|x}(\tau) \cdot d\tau} \times \right. \\ &\left. \times f_{M_E,R_E}(x,y) \cdot dx \cdot dy\right\}\end{aligned} \tag{8}$$

In the equation $\nu_{A|x}$ reflects the fact that such a rate is conditional to the mainshock magnitude, and ΔT_A is the reference duration for the aftershock sequence (e.g., 90 days). $P\left[IM_A \le im \mid x,y\right]$ is the non-exceedance probability in one aftershock marginal with respect to its possible magnitude and location, yet given magnitude and location of the mainshock. Acknowledging that, known magnitude and location of the aftershock, the non-exceedance probability is conditionally independent of the mainshock, and applying the total probability theorem, Equation (9) results.

$$\begin{aligned}\lambda_{im} &= \nu_E \cdot \left\{1 + \right. \\ &- \iint_{M_E,R_E} P\left[IM_E \le im \mid x,y\right] \cdot \sum_{i=0}^{+\infty} \left(\iint_{M_A,R_A} P\left[IM_A \le im \mid w,z\right] \cdot f_{M_A,R_A|M_E,R_E}(w,z \mid x,y) \cdot dw \cdot dz\right)^i \times \\ &\left. \times \frac{\left(E\left[N_{A|x}(0,\Delta T_A)\right]\right)^i}{i!} \cdot e^{-E\left[N_{A|x}(0,\Delta T_A)\right]} \cdot f_{M_E,R_E}(x,y) \cdot dx \cdot dy\right\}\end{aligned} \tag{9}$$

In the equation $f_{M_A,R_A|M_E,R_E}$ is the joint PDF of aftershock magnitude and distance conditional to the mainshock features, and $P\left[IM_A \le im \,|\, w,z\right]$ term is the non-exceedance probability in one aftershock of known magnitude and location.

Applying the *first moment approximation* replacing the summation with only the term corresponding to the expected number of aftershocks in the time interval of interest, that is a further simplification of the *delta method* (e.g., Oehlert, 1992), Equation (10) may be obtained for SPSHA.

$$\lambda_{im} \approx \nu_E \cdot \left\{ 1 - \iint_{M_E,R_E} P\left[IM_E \le im \,|\, x,y\right] \times \right.$$
$$\left. \times \left(\iint_{M_A,R_A} P\left[IM_A \le im \,|\, w,z\right] \cdot f_{M_A,R_A|M_E,R_E}\left(w,z \,|\, x,y\right) \cdot dw \cdot dz \right)^{E\left[N_{A|x}\left(0,\Delta T_A\right)\right]} \cdot f_{M_E,R_E}\left(x,y\right) \cdot dx \cdot dy \right\} = \tag{10}$$

4 ILLUSTRATIVE APPLICATION

As an illustrative application of SPSHA, hazard was computed for a site enclosed in a generic seismogenic areal source, the size of which is $30 \times 100\, km^2$; Figure 1. Mainshock epicentres were assumed as uniformly distributed in the source zone. Their rate was, arbitrarily, assumed to be $\nu_E = 0.054\, event/yr$. The magnitude distribution of mainshock was taken as a *truncated exponential* defined in the $\left[4.3, 5.8\right]$ range, as in Figure 2 (left). The *b*-value of the GR relationship is 1.056. In the application magnitude and source-to-site distance were considered to be independent RVs.

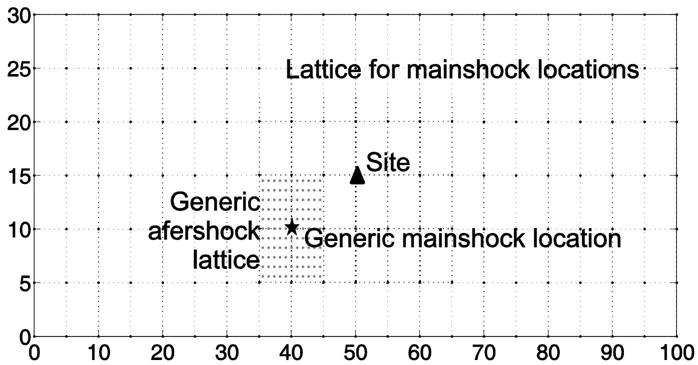

Figure 1. Seismogenic source lattice for mainshocks, generic aftershock lattice around a mainshock epicentre, and site.

It was assumed that each mainshock has its aftershocks constrained in an area around its epicentre. The size of the aftershock seismogenic zone in squared kilometres, S_A, depends on the main event's magnitude according to Equation (11) (Utsu 1970); Figure 2 (right). Within this area, arbitrarily assumed to be a square, epicentres are uniformly distributed.

$$S_A = 10^{\left(m_E - 4.1\right)} \tag{11}$$

The parameters used in the modified Omori law and in the Gutenberg-Richter relationship for aftershocks, that is the parameters of Equation (3), were: $a = -1.66$, $b = 0.96$, $c = 0.03$, $p = 0.93$, and $m_{\min} = 4.2$ (Lolli and Gasperini 2003). The sequence time in Equation (8), and those following, is $\Delta T_A = 90$ days since the time of occurrence of the mainshock. To evaluate the $P\left[IM_E \le im | m,r\right]$ and the $P\left[IM_A \le im | m,r\right]$ terms, the Ambraseys et al. (1996) GMPE was considered; in the latter the R_{jb} distance is used (Joyner and Boore 1981).

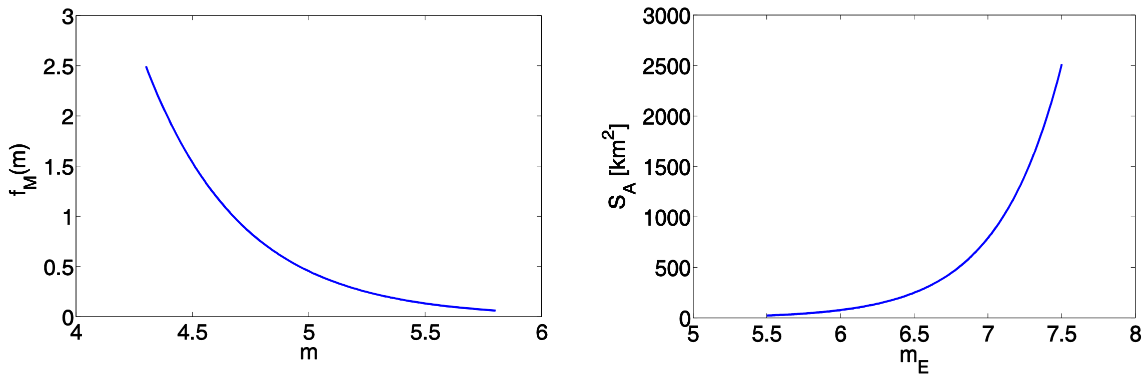

Figure 2. Magnitude distribution for mainshocks (left); mainshock magnitude versus aftershock source area size (right).

Figure 3 (left) shows hazard in terms of annual rate of exceedance of different thresholds when PGA (peak ground acceleration on rock) is the *IM*. The figure refers to the hazard considered only in terms of mainshocks, that is Equation (2), and considering also aftershocks according to Equation (10).

In Figure 3 (right) the relative difference between the two cases is also depicted; it can be noted that differences up to around 30% in rate are found.

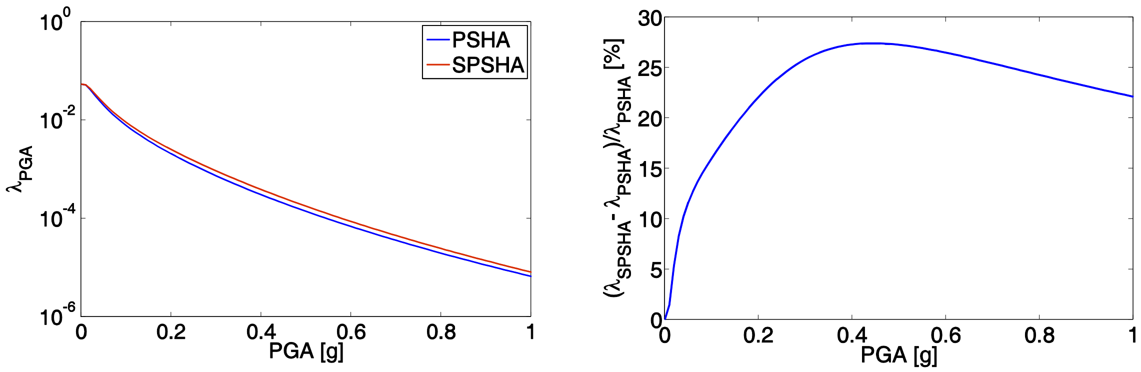

Figure 3. PSHA and SPSHA results in terms of PGA for the illustrative application (left); relative differences in the two cases (right).

As a further analysis, SPSHA was also computed taking the 5% damped pseudo-spectral-acceleration, $Sa(T)$, in the 0s-2s range of periods, as an *IM*. In Figure 4 (left) the resulting 475 yr return period uniform hazard spectrum (UHS) is compared with its PSHA counterpart.

Figure 4 (right) shows the relative difference between the two cases of Figure 4 (left). Differently from the previous figure, in this case the relative difference between the two spectra in terms of spectral acceleration is given. Differences appear to be in the order of about 10%.

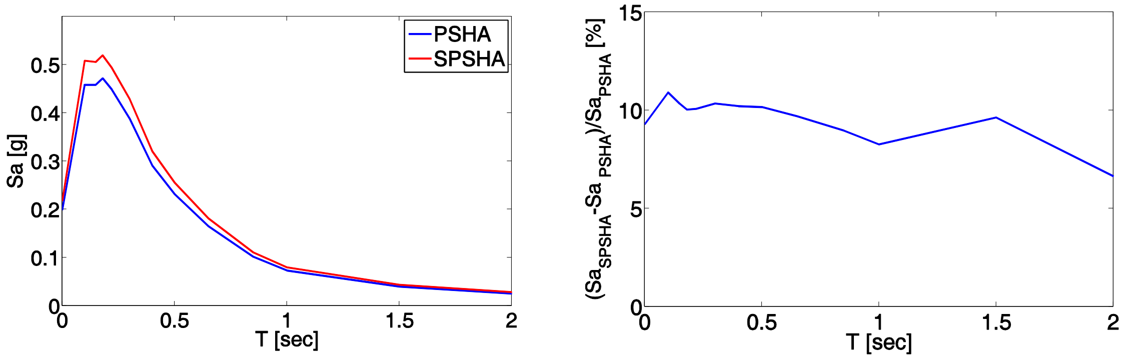

Figure 4. PSHA and SPSHA results in terms of 475 yr UHS for the illustrative application (left); relative differences in the two cases (right).

5 CONCLUSIONS

The study aimed at including aftershocks to a main earthquake event in probabilistic seismic hazard analysis expressed in terms of probability of exceedance of a ground motion intensity measure.

Probabilistic seismic hazard analysis for mainshock-aftershocks seismic sequences was built on the homogeneous Poisson process assumption for mainshock occurrence, and on the non-homogeneous Poisson process for the aftershock occurrence. The latter depends on the features of the mainshock via the modified Omori law and via an empirical relationship between the mainshock magnitude and the aftershock source area.

SPSHA was formulated analytically considering that the aftershock process is conditionally independent on the mainshock, given the magnitude and location of the latter. Finally, SPSHA was approximated by means of the first moment approximation applied to the expected number of aftershocks following a major event.

The illustrative application refers to a generic source zone for mainshocks and to generic aftershock sequences. The SPSHA was compared to the PSHA results both in terms of hazard curves as well as uniform hazard spectrum. Results show that, at least in the considered example, an increase up to about 30% in the exceedance rate of PGA threshold can be found. The comparison in terms of 475 yr uniform hazard spectrum highlights, in the specific application, an increase in spectra acceleration in the order of 10%.

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