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Structural modeling uncertainties and their influence on seismic assessment of existing RC structures

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ABSTRACT

The structural modeling uncertainty may be comparable to that of the ground motion representation in the seismic assessment of existing buildings. This work aims to characterize the uncertainties in material properties and in construction details and to propagate them to estimate the structural performance conditional on code-based seismic demand and capacity definitions. A prior probability distribution for the uncertain parameters is constructed based on the state of knowledge about the building before any in situ test is conducted. The uncertainties in the parameters are then propagated using simulation in order to obtain a probability distribution for structural resistance. The simulation-based methods allow for suitable grouping of uncertain parameters in order to build a simplified model of correlation across structural parameters. The results of tests and inspections are used both to update the prior probability distributions for parameters and also to update the structural reliability in a Bayesian framework. The updated probability distributions take into account the increased level of information eventually gathered as a results of destructive and/or non-destructive tests and inspections on the structure.

1. Introduction

A significant portion of the total seismic risk in Italy comes from the various types of losses induced by inadequate response of existing buildings to ground shaking. This is the case for several other European countries in which the average service life for buildings is larger than that of countries like the United States. Therefore, management of existing building stock is a major concern in such regions. The majority of reinforced concrete (RC) existing structures have been designed mainly for gravity loading only and the seismic provisions considered in the design process are either very poor or non-existent. As a result, more recent European seismic guidelines (e.g., Eurocode 8 [8], OPCM [20–22], NTC[19]) pay particular attention to seismic assessment of existing structures, which is distinguished from that of the new construction by lack of information about both the original features and the current state of building in consideration. In other words, determining the structural modeling parameters such as, material properties and structural detailing parameters is considered a major challenge in the assessment of existing buildings.

These European and Italian seismic guidelines synthesize the effect of structural modeling uncertainties specific to existing buildings due to lack of knowledge about building's characteristics (e.g., uncertainty in the material properties and in construction details) in the so-called *confidence factors* (FC) which have to be applied to mean material properties when performing seismic assessment of the structure. The primary scope for introduction of these confidence factors is to allow for a certain level of conservatism, through the use of material strength values smaller than that of the corresponding mean values determined based on available knowledge, in the seismic assessment of existing structures. It is worthwhile mentioning that after applying the code-based confidence factors, the assessment of an existing building will be performed almost the same as the design of new construction and may be subjected to partial factors employed by the code.[1]

Evaluation of these confidence factors seems to be rather qualitative and based on three increasing levels of knowledge about the structure in question. These levels of knowledge are classified based on a prescribed set of (destructive and non-destructive) verifying tests and inspections. To each level of knowledge a specific value of confidence factor is assigned; for example, the Italian and European codes, recommend confidence factors equal to 1.35, 1.20 and 1.0 for knowledge levels, poor, sufficient and accurate, respectively. Although the confidence factors are applied to

[1] The uncertainties in the component capacity models used for the assessment of existing buildings in both Eurocode 8 and the Italian seismic code are taken into account separately by an overall security factor denoted by γ_{el} in order to take the mean minus standard deviation value for component capacity instead of the mean value. As far as it concerns the uncertainty in the seismic action, the above-mentioned codes do not seem to make a clear distinction between the new construction and the existing buildings.

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the properties of materials, the uncertainties in structural modeling specific to existing buildings are not limited to them and include also other structural detailing parameters entering into the seismic assessment problem (e.g., reinforcement detailing, cover thickness, etc.). However, the extent to which the uncertainties in structural detailing parameters could affect the seismic assessment of existing structures seems not yet to be studied in depth.

This work aims to outline a theoretical basis for quantifying and updating the uncertainties in the material properties and construction details specific to existing buildings which are the basis for the designation of confidence factors in Eurocode 8. This methodology is presented in the context of an existing case-study building where the available in situ test results are used in order to update the modeling uncertainties and the structural reliability, given code-specific seismic actions and limit state capacity threshold, inside a Bayesian probabilistic framework. The focus of the study is on the uncertain parameters that are specific to an existing building as opposed to a building of new construction. Thus, the uncertainties in the seismic action (common between new and existing construction) and the modeling uncertainties in the component capacities such as the modeling uncertainty in determining the ultimate rotation in a section (addressed in the assessment of existing buildings by applying a separate security factor) are not taken into account. This is done particularly in order to single-out those uncertainties specific to seismic assessment of existing construction related to the estimation of code-based confidence factors. The characterization of uncertainties in this framework is preformed in two levels. In the first level, prior probability distributions for the uncertain modeling parameters are constructed based on information available from original design documents and (qualitative) professional judgment. In the second level, the results of in situ tests and inspections are implemented in the Bayesian framework in order to both update the prior distributions for the modeling parameters and also to update the distribution for structural performance variable and structural reliability using simulation-based reliability methods. The Bayesian updating procedure employed allows for updating the probability distributions for both structural modeling parameters and the structural global response within the simulation routine. Moreover, it is general enough to allow for both consideration of various types of inspections ranging from drilled cores tests, ultrasonic tests, and pacometric tests to pseudo-dynamic health-monitoring tests and the consideration of the corresponding measurement errors.

Updating of structural reliability across increasing amount of test results makes it possible to (i) introduce a performance-based probabilistic definition of the confidence factor as the value that, once applied to the mean material properties, leads to a value for structural performance measure with a specified probability of being exceeded (e.g., 5%) and (ii) evaluate the code-based recommendations regarding confidence factors and the corresponding knowledge levels. The methodology presented in this study, in the context of an existing case-study, lays out the fundamentals for a comprehensive work on the characterization of uncertainties specific to existing buildings, implementation of in situ test results, and verification of code-based requirements regarding the confidence factors.

2. The methodology

In the presence of structural modeling uncertainty, instead of a unique structural model, a set of plausible structural models can be identified. A *robust* assessment of structural reliability takes into account a whole set of possible structural models that are weighted by their corresponding plausibility. A Bayesian updating framework can be implemented in order to update both the structural modeling properties and the reliability based on test results [1]. Nevertheless, model updating is not an end by itself, and it is normally desirable to also improve the predictions of structural reliability. The Bayesian framework used for updating the structural model and its reliability is described in detail in this section.

2.1. Evaluation of robust reliability

Let the vector $\underline{\theta}$ denote the set of uncertain model parameters.[2] Let D denote some test data and consider that the set of possible structural models can be defined by M to specify (both the structural and the probabilistic) modeling assumptions used in the analysis. The Bayesian framework used herein provides a rigorous method for updating the plausibility of each of the models in representing the structure. The plausibility of a model may be quantified by the probability distribution over the vector of model parameters $\underline{\theta} = [\theta_1, \ldots, \theta_n]$ that define a model within the set of possible models. In other words, each model is uniquely identified by a realization of the vector $\underline{\theta}$. Therefore, the plausibility of the model can be quantified by the probability of such realization among the set of possible values of the vector.

The updated probability distribution can be defined using the Bayes Theorem [2]:

$$p_D(\underline{\theta}) = p(\underline{\theta}|D, M) = \frac{p(D|\underline{\theta}, M)}{p(D|M)} p(\underline{\theta}|M) \tag{1}$$

where $p(\underline{\theta}|M)$ is the prior probability distribution for $\underline{\theta}$ specified by M, $p(D|M)$ is the probability distribution for data D specified by M, and $p(D|\underline{\theta}, M)$ is the (updated) probability distribution for observed data D given the vector of parameters $\underline{\theta}$ specified by M.

Updated response predictions can be made implementing data D through $p_D(\underline{\theta})$ given by Eq. (1). For example, if the probability of a failure event F based on modeling parameters $\underline{\theta}$ is denoted by $P(F|D, M)$, the robust failure probability[3] can be calculated from the following integral defined over the whole domain of $\underline{\theta}$:

$$P(F|D, M) = \int P(F|\underline{\theta}, M) p(\underline{\theta}|D, M)\, d\underline{\theta} \tag{2}$$

where $P(F|\underline{\theta}, M)$ is the failure probability for the structural model defined by $\underline{\theta}$. In particular, given a specific representation of ground motion, $P(F|\underline{\theta}, M)$ reduces to a deterministic index function $I_F(\underline{\theta}, M)$. This index function is equal to one in the event of failure and equal to zero otherwise:

$$P(F|D, M) = \int I_F(\underline{\theta}, M) \frac{p(D|\underline{\theta}, M)}{P(D|M)} p(\underline{\theta}|M)\, d\underline{\theta} \tag{3}$$

This paper utilizes a Markov Chain Monte Carlo (MCMC) simulation method to evaluate the robust reliability in Eq. (3) [2]. The MCMC method employs the Metropolis–Hastings (MH) algorithm [17,13] in order to generate samples as a Markov Chain sequence which are used later to estimate the robust reliability by statistical averaging. The Metropolis–Hastings algorithm is normally used to generate samples according to an arbitrary PDF when the target PDF is known only up to a scaling constant.

[2] It should be noted that the vector $\underline{\theta}$ includes all the uncertain parameters in the problem, such as those representing the uncertainty in the seismic action. However, in this study, we have focused only on structural modeling uncertainties specific to existing buildings for a given level of seismic intensity. Therefore, the robust structural reliability calculated in this work is conditioned on code-based seismic action and limit state capacity assumptions.

[3] This paper is based on a probabilistic framework in which the probability is always conditional on the amount of information available. In this probabilistic framework [14], probability represents the degree of belief in a certain outcome based on the amount of information. We have used the word "robust" instead of the word "predictive" to describe the updated reliability. That is, the plausibilities of all the possible structural models described by M conditional on the amount of information available D are taken into account in order to calculate the updated reliability.

2.2. Generating samples according to target PDF $p(\underline{\theta}|D, M)$

The MH algorithm can be used to generate samples according to the target PDF $p(\underline{\theta}|D,M)$. Using Bayes formula one can derive the PDF as:

$$p(\underline{\theta}|D,M) = \frac{p(D|\underline{\theta},M)p(\underline{\theta}|M)}{p(D|M)} = c^{-1}p(D|\underline{\theta},M)p(\underline{\theta}|M) \quad (4)$$

where $p(\underline{\theta}|M)$ is *the prior probability distribution* for the vector of parameters $\underline{\theta}$ and $p(D|\theta, M)$ known as the *likelihood function* is the probability distribution for the data specified by parameters θ. The MH algorithm can be used to generate samples according to the target updated PDF $f \equiv p(\underline{\theta}|D,M)$ using the product $p^* \equiv p(D|\underline{\theta},M)p(\underline{\theta}|M)$ as the candidate PDF. In order to increase the acceptance rate for the candidate samples during the Markov Chain simulation process, a sequence of intermediate target PDF's denoted by f_i are introduced which vary gradually between the prior PDF $p(\underline{\theta}|M)$ and the updated target PDF $p(D|\underline{\theta}, M)$. The target f_i's can be modeled as updated PDF's according to Bayes theorem based on an increasing amount of data: $f_i \equiv p(\underline{\theta}|D_i,M)$ were $D_1 \subset D_2 \subset \cdots \subset D_n = D$. That is, at the first level with a target PDF equal to f_1, one could use the prior PDF $p_1^* = p(\underline{\theta}|M)$ as the proposal PDF. In order to approximate f_2, the kernel sampling density κ_1 is constructed as a weighted sum of Gaussian PDF's centered about the generated samples [4]. The kernel sampling density generated is then used as the proposal density in the second level: $p_2^* \equiv \kappa_1$.

2.3. Calculating the failure probability using subset simulation

In order to calculate the small failure probabilities encountered in the seismic reliability problem, the failure probability can be calculated using a simulation method known as subset simulation [4], in which the failure region is modeled as the last element in a sequence of embedded failure regions $F = F_m \subset F_{m-1} \subset \cdots \subset F_1$. Therefore, the failure probability can be derived as the following:

$$P(F) = P(F_1)\prod_{i=1}^{m-1} P(F_{i+1}|F_i) \quad (5)$$

where F_1 is the first element in the failure sequence (i.e., largest failure region) and $F = F_m$ is the target failure region and the last element in the failure sequence. The first term in Eq. (5) $P(F_1)$ can be calculated using standard Monte Carlo simulation, generating samples from the original PDF for the modeling parameters:

$$P(F_1) = \int I_{F_1}(\underline{\theta})p(\underline{\theta})\, d(\underline{\theta}) \quad (6)$$

And the intermediate failure probabilities $P(F_{i+1}|F_i)$ is equal to:

$$P(F_{i+1}|F_i) = \int I_{F_{i+1|F_i}}(\underline{\theta})p(\underline{\theta}|F_i)\, d\underline{\theta} \quad (7)$$

Again the MH algorithm can be used to generate samples as the states of a Markov Chain with target distribution equal to the conditional PDF $p(\underline{\theta}|F_i)$ for each intermediate failure region (see [5] for details on how to choose the candidate PDF). The subset simulation is shown to be especially efficient for modeling rare failure events (i.e., when the probability of failure is very small).[4]

[4] It should be mentioned that both the subset simulation algorithm and the structural model updating algorithm exploit the MH algorithm in order to generate samples as states of a Markov Chain. They differ in that the subset simulation moves across embedded failure regions whereas the structural model updating evolves across increasing levels of information.

3. Structural failure

The failure event F can be defined as when structural demand denoted as $D(\underline{\theta})$ exceeds structural capacity $C(\underline{\theta}) : F = \{\underline{\theta} : D(\underline{\theta}) > C(\underline{\theta})\}$. Assuming scalar demand and capacity, the (scalar) demand to capacity ratio can be defined as $Y(\underline{\theta}) = D(\underline{\theta})/C(\underline{\theta})$. Therefore, the failure region F can be defined as $F = \{\underline{\theta} : Y(\underline{\theta}) > 1\}$ and the sequence of embedded intermediate failure regions can be generated as $F_i = \{\underline{\theta} : Y(\underline{\theta}) > y_i\}$ where $0 < y_1 < \cdots < y_m = 1$.

In this study, the structural capacity is obtained using the push-over analysis as the global displacement at which the first element loses its load bearing capacity (i.e., 3/4th of the of ultimate chord rotation in the member).[5] The structural demand is defined as the global displacement corresponding to the intersection of the capacity curve for the equivalent SDOF system and the corresponding code-based seismic response spectra for the seismicity and the soil characteristics at the site of the project (a.k.a, *capacity spectrum method*, [12,9]). It should be mentioned that assumptions involved in the choice of the analysis method, the seismic demand and the structural capacity limit state are considered part of the set of modeling assumptions M. Therefore, the estimated probability of failure will be conditional on M.

4. Modeling of uncertainties

As it is mentioned in the previous section, the vector of parameters $\underline{\theta}$ includes all the uncertain parameters in the problem such as the uncertainty in the seismic action, the uncertainty in the property of the materials and the uncertainty involved in the structural detailing. The present work focuses on the uncertainty in the parameters of structural modeling of resisting elements, since it is characterized differently in existing structures and new construction.[6] Therefore, in order to focus on the modeling uncertainties specific to existing buildings, the seismic assessment is performed conditional on a given level ground motion intensity. The structural modeling uncertainty is directly related to the quantity (and the quality) of information that is available for the structure. In this study, two different sources of uncertainty are considered: (1) uncertainty in the mechanical properties of materials used in construction and (2) the reinforcement details that affect the component capacity in terms of moment-rotation relation; uncertainty in details can be both due to limited information about the design of a specific structure and/or local construction practice and also due to low quality control in construction (also known in the engineering jargon as *structural defects*, not uncommon in structures built after the second world war in Italy). As the uncertainties belonging to the second group mentioned above, those related to the percentage of rebar present in the element, rebar diameter (e.g., different from that specified in the original design notes), the anchorage quality and the cover thickness are considered. The uncertainties in structural detailing are modeled as discrete uncertain variables that can assume a range of possible values with a corresponding plausibility/weight. In the absence of test results and in situ inspections, the plausibility of each possible value is assigned qualitatively based on engineering consensus, judgment and experience. It has to be mentioned that once the

[5] Corresponds to the definition of near-collapse (life-safety) limit state in the Italian code.

[6] It should be noted that the modeling uncertainties in the component capacities are not taken into account in this study, while this study intentionally focuses on the effects of uncertainties related to the actual knowledge of structural features of existing buildings. Moreover, the seismic input representation herein follows a code-based spectral approach not explicitly accounting for variability of the response due to ground motion. Nevertheless, discussions on how to take into account the modeling uncertainties in the component capacities and how it compares to the uncertainty in the representation of the ground motion, one can refer to, for example, [15,16].

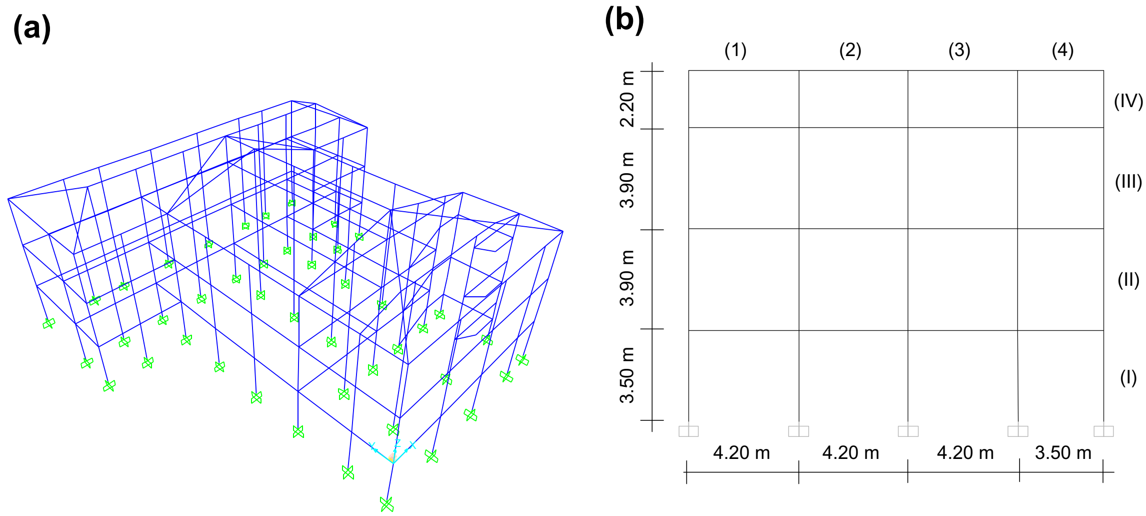

Fig. 1. (a) The tri-dimensional view of the scholastic building. (b) The central frame of the case-study building.

test results are available on the quantity in question, they can be used applying the Bayesian methodology described in the previous sections to update the plausibility of each possible value for the corresponding discrete uncertain variable. As it regards the correlation between different uncertain parameters, a simplified model of correlation is constructed by classifying different sets of correlated uncertain parameters within groups that are not cross-correlated [5].

5. Case-study

As the case-study, an existing school structure located in Avellino (Italy) is considered herein. The structure is situated in seismic Zone II according to the Italian seismic classification of the OPCM 3519 [22]. The structure consists of three stories and a semi-embedded story and its foundation lies on soil type B according to Eurocode 8 [8]. For the structure in question, the original design notes and graphics have been gathered. The building is constructed in the 1960s and it is designed for gravity loads only, as it is frequently encountered in the post second world war construction. In Fig. 1a, the tri-dimensional view of the structure is illustrated; it can be observed that the building is irregular both in plane and elevation. The main central frame in the structure is extracted and used as the structural model (Fig. 1b). The columns have rectangular section with the following dimensions: first storey: $40 \times 55\ \mathrm{cm}^2$, second storey: $40 \times 45\ \mathrm{cm}^2$, third storey: $40 \times 40\ \mathrm{cm}^2$, and forth storey: $30 \times 40\ \mathrm{cm}^2$. The beams, also with rectangular section, have the following dimensions: $40 \times 70\ \mathrm{cm}^2$ at first and second floors, and $30 \times 50\ \mathrm{cm}^2$ for the ultimate two floors. The reinforcement ratios, as defined in the original notes, range from about 0.2% to 0.7% in the columns and 0.7% to 1% in the beams. It can be inferred from the original design notes that the steel rebar is of the type Aq42 (nominal minimum yield resistance $f_y = 2700\ \mathrm{kg/cm}^2$ or 265 MPa) and the concrete has a minimum resistance equal to $180\ \mathrm{kg/cm}^2$ (17.7 MPa, [23]). The finite element model of the frame is constructed assuming that the non-linear behavior in the structure is concentrated in plastic hinges located at the element ends. Each beam or column element is modeled by coupling in series of an elastic element and two rigid–plastic elements (hinges). The stiffness of the rigid–plastic element is defined by its moment-rotation relation which is derived by analyzing the reinforced concrete section at the hinge location. In this study, the section analysis is based on the widely adopted in current practice Mander–Priestly [18] constitutive relation for reinforced concrete, assuming that the concrete is not confined,[7] and that the reinforcing steel behavior is elastic–perfectly-plastic. The behavior of the plastic hinge is characterized by four phases, namely: rigid, cracked, post-yielding, and post-peak. In addition to flexural deformation, the yielding rotation takes into account also the shear deformation and the deformation related to bar-slip based on the code recommendations [21]. Moreover, the shear span used in the calculation of the plastic rotation is based on the code formulas. As it regards the post-peak behavior, it is assumed that the section resistance drops to zero, resulting in a tri-linear curve which is sketched in Fig. 2.[8]

5.1. In situ test methods to quantify material properties

For the case-study building described above, results of both destructive and non-destructive test results are available. As destructive test methods, compression core tests are performed in order to determine the in-place compressive strength of concrete and the tensile test is performed in order to determine the in situ tensile yield and ultimate strength of reinforcing steel. The drilled core test consists of removal of standard cores from primary concrete components followed by the laboratory testing of these samples in order to obtain the core-strength. The core-strength should be converted to the in situ strength following an approved procedure. In this work, the procedure suggested by the British Standard 1881 [3] is used. In the case of destructive tensile tests, reinforcing steel bar samples are removed from the structure and tested in laboratory in order to determine both the tensile yield and ultimate strength [11]. As non-destructive test methods, the ultrasonic test results are performed in order to determine the concrete compressive strength [26]. Ultrasonic testing uses high frequency sound energy to make measurements of the velocity of the sound wave through the reinforced concrete medium. The results of ultrasonic testing are often calibrated against drilled core test results performed at the exact same location in order to convert the sound velocity into concrete in situ strength.

[7] For a building that is not designed according to seismic provisions, it is unlikely the number of stirrups in and outside the section are sufficient to assure that the concrete is confined everywhere inside the stirrups, see also Fardis [10].

[8] It should be mentioned that, in this case-study example, the section resistance is evaluated in terms of the flexural response only and the shear resisting capacity has not been considered.

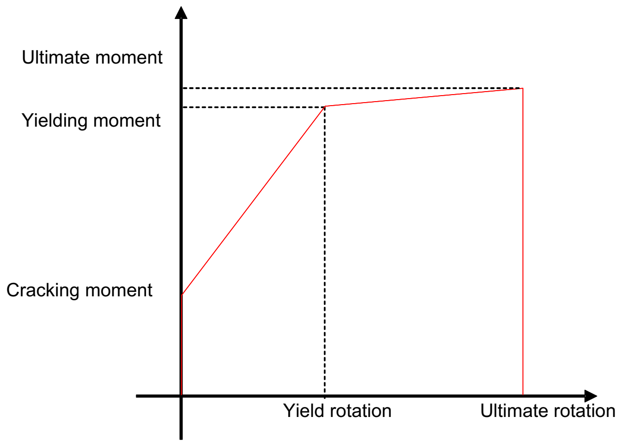

Fig. 2. Schematic diagram of the typical tri-linear behavior characterizing the rigid–plastic hinge.

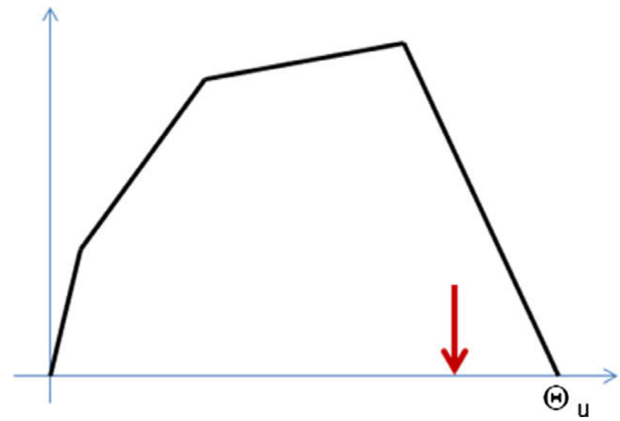

Fig. 3. Schematic diagram of the rotation capacity for limit state of severe damage.

5.2. Evaluation of structural demand and capacity

In accordance with the Eurocode 8 and Italian seismic code [20] the severe damage limit state is reached at the instance when the first element experiences 3/4th of the of its ultimate chord rotation in the member, as shown in Fig. 3.

The structural lateral load resistance curve (a.k.a., *capacity curve*) is represented herein in terms of the base-shear and roof-displacement. The non-linear static analyses are performed using the SAP2000 (Ver. 10) structural analysis commercial software. The information regarding the gravity loading is extracted from the original design notes. The horizontal forces are calculated based on the code procedure using the first fundamental mode shape as the lateral displacement profile along structural height. The same displacement profile is also used in order to calculate the first-mode participation factor for the structure. The capacity curve is transformed into that of an equivalent SDOF system utilizing the modal participation factor and is successively bi-linearized as it is prescribed in capacity spectrum method. The displacement-based structural capacity $C(\underline{\theta})$ corresponds to the point on the capacity curve in which the first element reaches the threshold of severe damage as described above. The ultimate deformation capacity point is followed by a sharp drop in the structural resistance represented by the base-shear.

The seismic action is modeled based on the code-specified elastic spectrum modified to account for the soil condition at the site of the structure (type B). This spectrum is anchored about the peak ground acceleration on rock, a_g, corresponding to the seismic zone in which the site is classified, with a return period equal to 475 years (i.e., 10% probability of exceedance in 50 years). For the site of the case-study structure, this results in a_g equal to 0.25 g. The displacement-based structural demand $D(\underline{\theta})$ is then calculated as the displacement at which the capacity curve for the equivalent SDOF system intersects the spectrum after it is modified based on a factor reflecting the inelastic properties of the structure in question.

As it is mentioned in the section on the methodology applied, the simulation-based methods (e.g., Monte Carlo, Monte Carlo Markov Chain) are used in order to calculate the structural reliability. For each realization within the simulation process, first, the parameters defining the plastic hinge moment-rotation diagram are calculated and implemented in the structural finite element model. In the next step a non-linear static analysis is performed on the (simulated) structural model followed by the application of the capacity spectrum method in order to determine the demand/capacity ratio corresponding to this simulation realization.

5.3. Case 1: structural detailing consistent with design specifications

As the first case, the reliability of the structure is calculated based on the state of knowledge about the building before in situ inspections and tests are conducted. Moreover, in order to evaluate the significance of considering the uncertainty in structural detailing parameters in Case 2 described below, the uncertainty is assumed to be only in material mechanical properties. The prior probability distributions for concrete and steel are constructed using the results of a statistical study done on the characteristics of the steel and concrete used in the constructions of the time [24,25]. As it regards the correlation among material mechanical properties, it is assumed that the material properties for steel and concrete are uncorrelated. Moreover, the concrete resistance for the RC elements on each floor is considered to be uncorrelated with those of the other floors.

Since the probability of failure in this case is predicted to be very small, the subset simulation procedure described in the methodology is used in three levels in order to calculate the probability of failure. In the first level 500 and 200 analyses in the second and third level are performed with intermediate failure probabilities equal to 0.10, 0.20 and 0.005 respectively. The probability of failure is calculated to be equal to 0.0003 with a coefficient of variation equal to 1.57. The coefficient of variation is calculated based on a procedure described in [5] where the failure probability is equal to the expected value for the average of a set of correlated Bernoulli variables generated as members of a Markov Chain sequence.

5.4. Case 2: no test results but considering the uncertainty in the structural detailing

As the second case, the reliability of the case-study frame is calculated based on the state of knowledge about the building before in situ inspections and tests are conducted. The sources of uncertainty are assumed to be the material mechanical properties and the structural detailing. A list of the possible sources of uncertainty in the reinforced concrete section detailing has been constructed by identifying the various possibilities, their relative plausibility, and their correlation with other sources.

Table 1 demonstrates a list of possible sources for structural modeling uncertainty represented by discrete probability mass functions and the corresponding correlation structure. The variables representing uncertainty in the quality of anchorage (in this case 180° hooks) are modeled by considering two possibilities, either the hooks are done according to the specifications or not done at all. In the case where they are done well, it is assumed that the full area of the present rebar at the ends is effective; otherwise, in the absence of the hooks, only half of the rebar area is considered to be effective at member ends. It is further assumed that in 90% of the cases the hook is done according to the specifications. Moreover, anchorage quality is supposed to be uniform throughout each floor; that is the corresponding variables are fully correlated. In the next category, the error in diameter is considered. This type of error usually occurs when rebar diameters are close-enough to be mistaken visually. This has been modeled only for columns of the ground floor where both diameter 14 and 16 rebar were specified in the design notes. It is supposed that in 95% of the cases the correct diameter is being placed. The next category addresses the possible errors in superposition of columns across floors. It is assumed that, if the overlapping splice length is sufficient, the whole area of column rebar extending from the story beneath is effective. Otherwise, as a possible variation, a case is considered in which 75% of the rebar area is effective. This type of error is supposed to occur uniformly across each floor. Another type of error in detailing can take place when the rebar in column section is positioned without particular attention to whether it is oriented towards the weak direction or otherwise. In such case, two possibilities can be considered, one (the more plausible one) in which the rebar is positioned so that the section has more flexural resistance in the weak direction and its opposite case (less plausible case) where the section has more flexural resistance in the strong direction. This kind of error is supposed to be uniform across the same type of section throughout the same floor. The kind of human error related to missing rebar is taken into account only in beams where a large quantity of short-length diagonal rebar are specified. This could cause accidental loss of a rebar. It is assumed that any specified rebar is present as specified with 90% probability. This kind of error is also modeled as uniform across section type and the floor to which it belongs. The last category addresses the errors in concrete cover which is discretized by considering only three possible values. The most likely value is taken to be 3 cm with a lower limit of 2 cm and higher limit of 4 cm. This kind of error is assumed to be systematic across each floor.

It should however be emphasized that probabilities/weights assigned to each possibility are herein determined qualitatively and based on judgment and experience. Table 2 demonstrates the parameters for constructing a prior probability distribution for the steel yielding strength and concrete strength in compression as material properties. The prior probability distributions for concrete and steel are modeled the same as in Case 1.

Since in this case, there are additional sources of uncertainty compared to in Case 1, the failure probability is predicted to be large enough to be estimated using standard Monte Carlo simulation. Hence, the reliability of the frame is calculated using the Monte Carlo Simulation with 200 simulations (the pushover curves corresponding to a few simulation realizations are shown in Fig. 4). The probability of failure is calculated to be equal to 0.005 with a coefficient of variation equal to 1.0. The coefficient of variation is calculated based on a procedure described in [5] where the failure probability is equal to the expected value for the average of a set of independent Bernoulli variables (binary outcome 1 with probability equal to P_f and 0 with probability $1 - P_f$).

Table 2
Probabilistic characterization of the mechanical property of RC.

Var.	Dist.	Mean (kg/cm^2)	COV
f_c	LN	165 (16 Mpa)	0.15
f_y	LN	3200 (314 Mpa)	0.08

5.5. Case 3: using test results to update predictions

Table 3 demonstrates the actual test results available for the case-study structure which consist of (non-destructive) ultrasonic results (6 data per floor), (destructive) drilled core tests (2 data per floor) for determining the concrete resistance and the tension test for reinforcing steel (1 data). It should be noted that the standard error assigned to the ultrasonic tests is larger than that assigned to the drilled core tests to take into account the fact that the ultrasonic results are calibrated (using regression analysis) with respect to the drilled core tests.

The results of the tests are used in two levels in order to update the probability distribution for concrete and reinforcing steel resistance at different floors in the structure and to calculate the robust reliability. In the first level the destructive test results are implemented and in the second level the non-destructive test results are used. The test results are implemented using the MH algorithm with 200 simulations at each level in order to update the probability distribution for concrete and reinforcing steel resistance and also to update the structural reliability. However, in the first level before the data are employed, the same 200 samples generated employing standard Monte Carlo simulation in Case 2 are used.

6. The results

Fig. 5a and b demonstrates the histograms for structural performance variable (the demand to capacity ratio) corresponding to the Case 1 (uncertainty in only material properties) and Case 2 (uncertainty in both material properties and defects) described above, respectively. The lognormal PDF's fit to the two histograms are illustrated in Fig. 5c. It can be observed that taking into account the uncertainty in structural defects leads to a significant increase

Table 1
Probabilistic characterization of the structural detailing parameters.

Defects	Possibilities	Prob.	Type
Insufficient anchorage (beams)	Sufficient (100% effective)	0.900	Systematic over floor
	Absent (50% effective)	0.100	
Error in diameter (columns)	$\phi 16$	0.950	Systematic over floor and section type
	$\phi 14$	0.050	
Superposition (columns)	100% of the area effective	0.950	Systematic over floor
	75% of the area effective	0.050	
Errors in configuration (columns)	More plausible configuration	0.950	Systematic over floor and section type
	Less plausible configuration	0.050	
Absence of a bar (beams)	Absence of a bar	0.100	Systematic over floor and section type
	Presence of a bar	0.900	
Conceret cover	2 cm	0.125	Systematic over floor
	3 cm	0.750	
	4 cm	0.125	

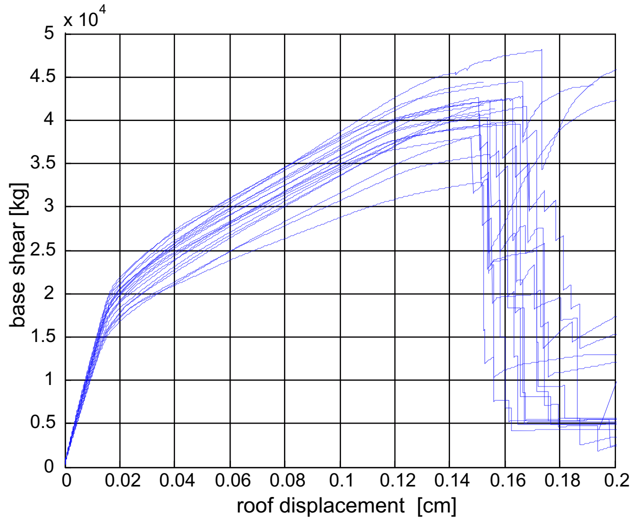

Fig. 4. Pushover curves generated for different realizations of the simulation procedure (Case 1).

Table 3
Test results available for the structure.

Test	# data	Type	Standard error
Drilled core test Basement	2	Destructive	0.15
Drilled core test Ground floor	2	Destructive	0.15
Drilled core Fisrt floor	2	Destructive	0.15
Ultrasonic test Basement	6	Non-destructive	0.335
Ultrasonic test Ground floor	6	Non-destructive	0.335
Ultrasonic test First floor	6	Non-destructive	0.335
Tension test reinforcing steel	1	Destructive	0.08

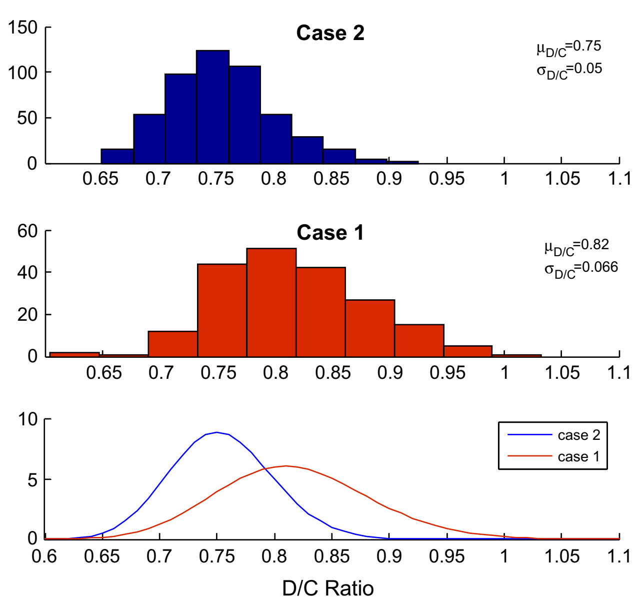

Fig. 5. (a) The histogram for the demand to capacity ratio when only the uncertainty in the material properties are considered (Case 2, based on 500 samples). (b) The histogram for the demand to capacity ratio when both the uncertainty in material properties and structural defects are considered (Case 1, based on 200 samples). (c) The lognormal probability distribution functions fit to the demand to capacity ratio data presented in Case 2 (a) and Case 1 (b).

both in the mean value of the demand to capacity ratio and also in the standard deviation for the demand to capacity ratio.

Fig. 6 demonstrates the data for concrete strength at the basement of the structure for the case where the uncertainty in both material properties and structural defects are taken into account. The first row of the figure represents the prior distribution for the concrete strength before the test and inspection results are known or are taken into consideration. The rightmost column in the figure represents the lognormal curves fit to the histograms. It can be observed that Bayesian updating of the concrete strength based on the destructive test results (most accurate) leads to an increase in the mean value for concrete strength and to a decrease in its standard deviation. The addition of the non-destructive tests in the third level leads to further increase in the mean value and reduction the standard deviation.

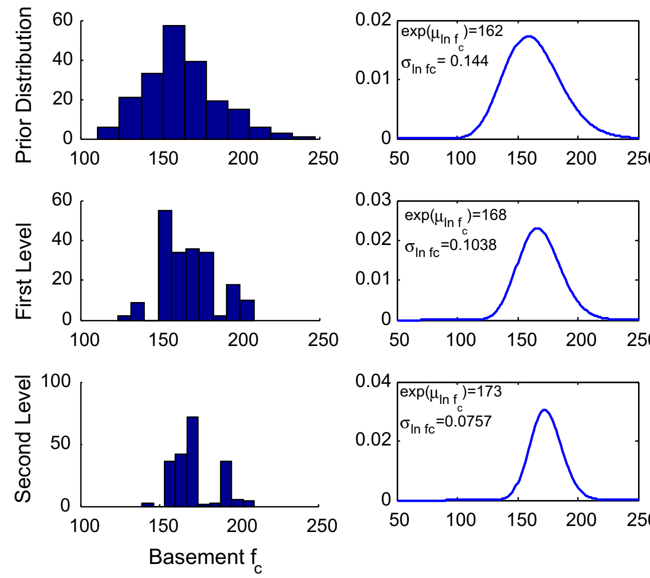

Fig. 6. The distribution of the concrete strength in the basement: (a) the first level corresponds to the prior distribution for the concrete strength before test results are taken into account (uncertainty in both material properties and structural defects, Case 1). The histogram is based on 200 samples and the rightmost diagram demonstrates the lognormal PDF fit to the data, (b) the second level corresponds to the updated distribution of the basement concrete strength after the destructive test results for both concrete and steel are implemented, and (c) the third level corresponds to the updated distribution for basement concrete strength after the non-destructive test results are also considered.

Fig. 7 demonstrates the data for the steel yielding strength for the case where the uncertainty in both material properties and structural defects are taken into account. The second row represents the updated data for steel yield strength after the destructive test results are implemented in the Bayesian framework. It can be observed that the addition of test results (in this case only one data point for steel strength) leads to an increase in the mean value and to a decrease in the dispersion.

Fig. 8 demonstrates the histograms and the lognormal curves fitted to the demand to capacity ratio for the three increasing levels of data. The first level corresponds to the prior lognormal probability distribution for the demand to capacity ratio before taking into consideration the test results. The second level corresponds to the updated distribution after considering the drilled core test results for concrete and the tension test result for reinforcing steel. The last level illustrates the updated distribution for structural performance variable after considering also the ultrasonic test results for concrete.

For all three values of confidence level suggested by the code (i.e., FC = 1, 1.2, 1.35) the corresponding demand to capacity ratios for the structure is calculated. The resulting three values for demand to capacity ratio are marked on the curves illustrated in Fig. 8. Note that the failure threshold is also marked at the value $\ln Y = \ln 1.0 = 0.0$. The confidence factors can be estimated, for

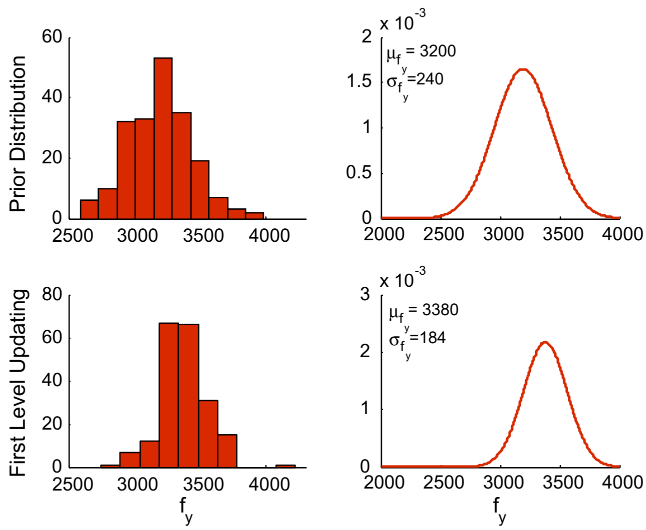

Fig. 7. The distribution of the steel yielding strength: (a) the first level corresponds to the prior distribution for the steel yield strength before test results are taken into account (uncertainty in both material properties and structural defects, Case 1). The histogram is based on 200 samples and the rightmost diagram demonstrates the normal PDF fit to the data and (b) the second level corresponds to the updated distribution of the steel yield strength after the destructive test results for both concrete and steel are implemented.

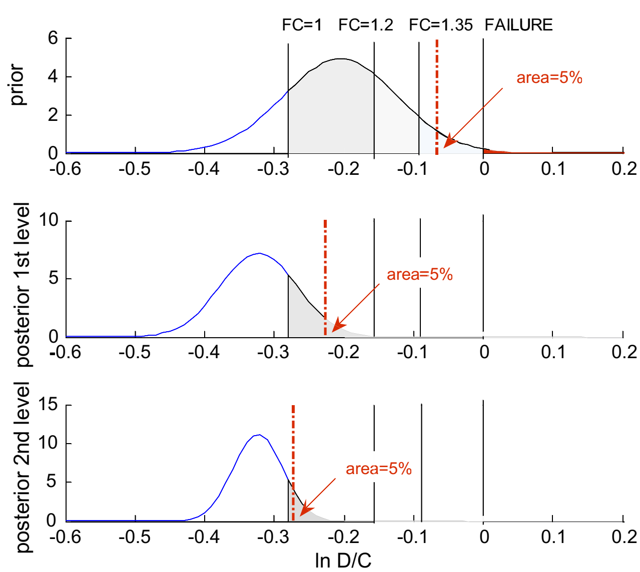

Fig. 8. Distribution of the demand to capacity ratio when both the uncertainty in the material property and in the structural defects are taken into account: (a) the prior lognormal PDF fit to the demand to capacity ratio before test results are being considered, (b) the updated lognormal PDF fit to the demand to capacity ratio after φ implementing the destructive test results, (c) the updated lognormal PDF fit to the data after the non-destructive test results are also implemented. The probability of failure is indicated by the area under the curve for demand to capacity ratios greater than unity (0 on the logarithmic scale). The demand to capacity ratios corresponding to the code-specified values for the confidence factor are also marked on the figure.

example, as the value of FC that leads to a demand to capacity ratio with say 5% probability of exceedance. In the prior stage (Fig. 8a), the confidence factor corresponding to a value of demand to capacity ration with 5% probability of exceedance is larger than (but close to) FC = 1.35. In Fig. 8b, where the distribution for demand to capacity ratio is updated after the destructive test results are considered, the demand to capacity ratio with 5% probability of exceedance corresponds to a confidence factor between FC = 1.0 and FC = 1.20. In Fig. 8c, after the non-destructive test results are also considered, the demand to capacity ratio with 5% probability of exceedance corresponds to a confidence factor slightly greater than 1.0 which corresponds to the code-recommended value for the most complete level of knowledge.

7. Conclusions

This study aims to characterize, to quantify and to update the uncertain modeling parameters, namely, mechanical properties of materials and the structural detailing specific to existing RC buildings, as a function of the amount of information available. The motivation behind this research effort is to create a benchmark against which the *confidence factors* (FC's) recommended by international codes for seismic assessment of existing buildings can be evaluated. A Monte Carlo Markov Chain (MCMC) algorithm is employed inside the Bayesian updating framework in order to both update the structural modeling parameters and reliability as a result of in situ tests and inspections. The Bayesian updating algorithm implemented is general with respect to the type of in situ test performed and the standard error associated with it. The procedure is applied to the seismic assessment of an existing school structure in Italy which serves as case-study, given the code-based seismic action and capacity models.

- Considering the uncertainty in structural detailing parameters increases both the mean and standard deviation of the demand to capacity ratio for the structure.
- The quality and quantity of the acquired in situ information affects significantly the structural reliability assessment of the existing building.
- With regard to benchmarking of the European Code FC's, an *optimal* FC is defined as the value which leads to a demand to capacity ratio with a specified probability of being exceeded (e.g., 5%). For the case-study analyzed, a certain degree of consistency was found between these optimal FC's, with respect to the code-specified FC values defined as a function of increasing knowledge levels.

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