Selection of the “best” seismic upgrading solution for a RC building by MCDM

Caterino, N., Iervolino, I., Manfredi, G., Cosenza, E.
Department of Structural Analysis and Design, University of Naples Federico II, Via Claudio, 21 – 80125 Naples, Italy

INTRODUCTION

Multi-Criteria Decision Making (MCDM) is made of stepwise procedures useful for complex problems allowing to rank the overall performances of a finite set of alternatives in respect to certain criteria of interest. They help the Decision Maker (DM) to identify the ‘best’ feasible solution which is defined as the one which more closely matches all the relevant goals. MCDM is common in several fields such as resources allocation planning, medical treatment choices, natural resources’ management.
This paper presents an application of this methodological framework to the seismic retrofit of an under-designed RC structure. In fact, several traditional, as well as innovative, upgrade strategies are available for the achievement of the retrofitting goals, each of those scoring different points in respect of different criteria such as installation and maintenance costs, required application time, performances, durability and invasivity. The selection of the most suitable solution depends on the peculiarities of the case under exam and is, often, not straightforward. Due to the several noncommensurable and conflicting criteria, generally there is no solution satisfying all of them simultaneously [1] calling for MCDM. The TOPSIS method [2] is considered being one of the most widely adopted. The eigenvalue approach [3] is used, instead, to express the relative importance (weights) of the criteria.
The application is described in detail; it refers to a three-storeys, irregular, pre-code RC structure built and tested at the European Laboratory for Structural Assessment (ELSA) of the Joint Research Center (JRC) in Ispra, Italy, for the EU project SPEAR [4]. The building is assumed to be located in Pomigliano d’Arco (Naples, Italy) which has been classified as a seismic zone only in 2003 (peak ground acceleration 0.25g) and hence the upgrading needs are actually strongly felt.
For this structure three different retrofitting alternatives, improving the seismic capacity by three different structural performance objectives, were designed: (a) columns’ confinement by Glass Fiber Reinforced Plastics (GFRP); (b) steel bracing; (c) concrete jacketing of columns. The considered criteria are technical and social/economical. The destination of the building is chosen to be residential and the DM is assumed to be the owner.

Keywords: decision, seismic retrofit, alternative strategies, criteria, criteria weight

DESCRIPTION OF THE PROCEDURE

Decision making procedure addressed to the selection of the best retrofit solution should include the following steps:

(1) Design of the alternative interventions;
(2) Selection of the evaluation criteria;
(3) Definition of criteria’s relative importance (criteria weights);
(4) Evaluation of the alternatives according to each criterion;
(5) Conversion of all variables into crisp numbers;
(6) Identification of the best retrofit solution according to the method adopted (TOPSIS herein).

For the sake of brevity the detailed description of the MCDM procedure will be done at the same time with its practical application to the SPEAR building mentioned above. The structure is designed to be representative of old reinforced concrete constructions in Southern Europe designed without earthquake design provisions [4]. The standard floor plan is presented in Fig. 1.

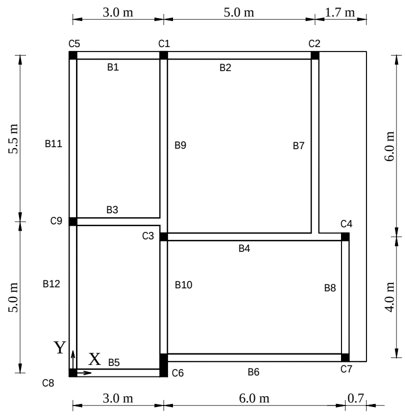


Fig 1. Standard floor plan

The plan irregularity shifts the centre of stiffness away from the centre of mass, causing torsion, while the structure can be considered regular in elevation. The storey height is 3.0 m, from top to top of the slab. All the columns have square cross section (250x250 mm), except for C6 which is 750×250 mm. The beams' depth is 500 mm. The slab thickness is 150 mm.
The frame can be defined as a *weak column-strong beam* system, and is therefore far from the *capacity design* concepts. The reinforcement consists of smooth bars of 12 mm and 20 mm in diameter. Stirrups are smooth 8 mm diameter bars and are spaced by 200 mm in the beams, 250 mm in the columns. They are not continue in the joints. The confinement provided by this arrangement is very low [5].

Assessment of the un-retrofitted structure

Before defining the different alternatives of intervention, the assessment of building is needed in order to identify its major deficiencies. The seismic evaluation of the structure "as-built" was carried out by nonlinear static analysis (pushover) of a lumped plasticity model. In the latter the rotational properties of the plastic hinges are defined according to the last Italian seismic code [6]. The model provided by Mander et al. [7] is adopted (f_c = 25 MPa) for the concrete behaviour. The ultimate strain is assumed to be 0.004. The lateral force pattern corresponds to the first oscillation mode in each direction ($T_{1(x)}$ = 0.52s; $T_{2(y)}$ = 0.46s).
Pushover curves along the four directions are given in Fig. 2, along with the comparison between capacity and required seismic performance. The building does not satisfy the Significant Damage (SD) and barely withstand the Limited Damage (DL) limit state[1].

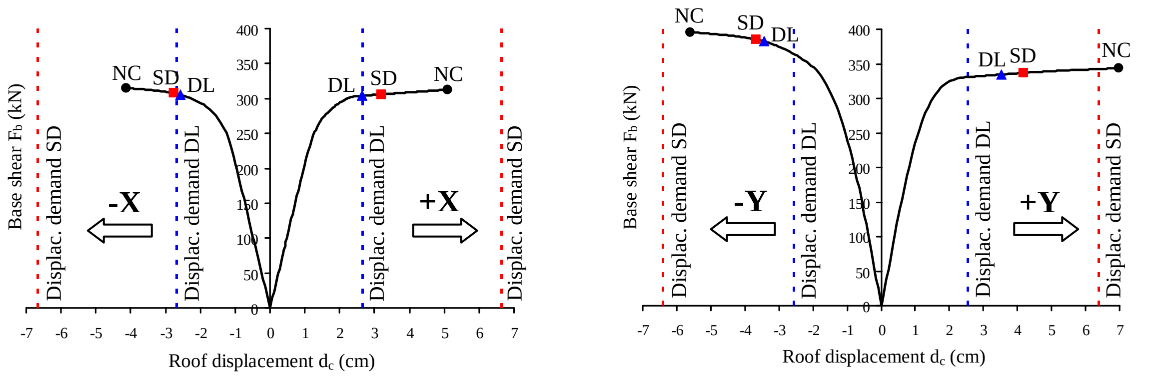


Fig. 2. SPEAR building: pushover curves. Comparison between displacement capacity and code's demand at each limit state, for each of the four directions –X, +X, –Y, +Y.

[1] DL is attained when the Maximum Interstorey Drift Ratio is 0.005; SD corresponds to the attainment of 3/4 of the ultimate rotation of an element.

Alternative retrofit interventions

In first approximation the seismic retrofit strategies may be classified in those reducing the seismic demand (i.e. isolation) and those improving the capacity. These may be ordered further by the global structural feature they improve. In fact, as clearly described in Fig. 3, the capacity upgrade can aim at increasing: ductility only (a), strength only (b) or as combination (c).

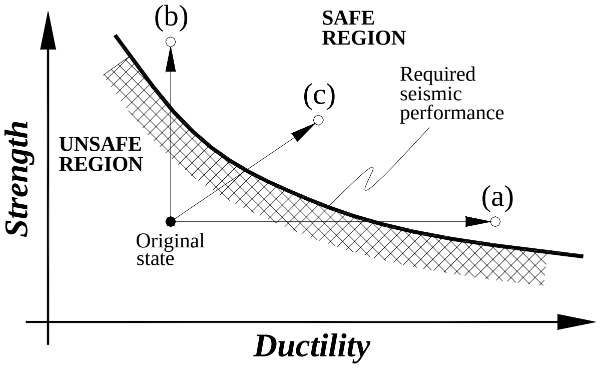
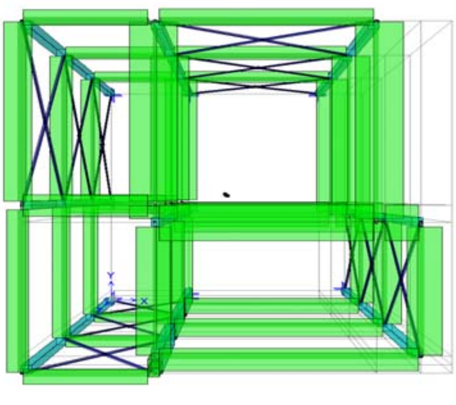
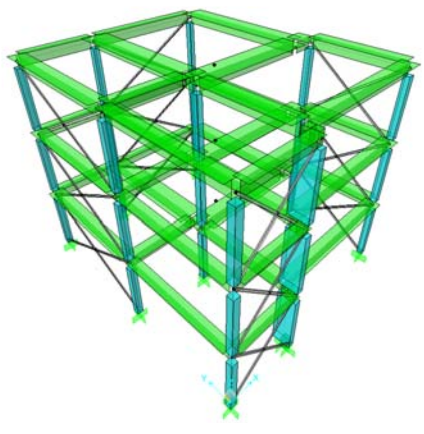


Fig. 3. Retrofit methods: different structural improvement strategies [8].

In this paper one intervention for each direction of the above picture was designed and considered as an alternative solution for the retrofit problem. They will be referred as A_1, A_2, A_3 and correspond to (a), (b) and (c) respectively: A_1 consists in the confinement of columns and joints by Glass Fiber Reinforced Plastics (GFRP); A_2 in the steel bracing of frames and A_3 in the concrete jacketing of selected columns.
The intervention A_1 provides a confinement action to the columns and results in an increasing of the concrete ultimate strain and consequently of the plastic hinge ultimate curvature. Thus, it gives an enhancement of the building's global deformation capacity without changing the strength. The design of the intervention is fully described in Cosenza et al. [10].
The alternative A_2 aims at increasing the global strength of the structure and at centring of plan stiffness by concentric diagonal bracing (Fig. 4). This kind of retrofit does not induce any significant variation in terms of global ductility. The steel considered is Fe430 (f_y = 275 MPa, f_u = 430 MPa). The cross section selected for all the diagonal elements is L-shaped (65x100x7 mm). According to the recommendations of the FIB Bulletin No.24 [11] the diagonal braces are supplemented with a frame of steel members firmly attached to the delimiting concrete members (e.g. columns and beams).
The retrofit option A_3 gives strength centring with consequent reduction of the harmful torsional effects in the non linear response; it consists of the concrete jacketing of selected columns and results in the enhancement of both structural strength and ductility. Columns C1, C3 and C4 (Fig. 1) were strengthened by a concrete jacket 75 mm thick at each storey (concrete R_{cm} = 50 MPa, longitudinal bars 8ϕ16, stirrups ϕ8/100-150 mm). For further details of design and modelling of all solutions see Caterino et al. [9].



(a)



(b)

Fig. 4. Bracing configuration: plan (a) and 3D (b).

Evaluation criteria

Judgement criteria can be defined as different points of view from which each alternative can be evaluated. They allow to compare and rank the available retrofit solutions and, generally, can be sort in *social/economical* (e.g. costs, duration of works, disruption to occupants, functional and aesthetic compatibility, reversibility, historical significance etc.) and *technical* criteria (e.g. structural compatibility, regularity of stiffness, strength and ductility, non-structural components' protection, foundation system, repair materials, technology available etc.) [12]. Obviously not all the mentioned criteria have a significant rule in all cases. It depends on the peculiarities of the building under exam and on its destination; it may depends also on the decision maker's profile. Moreover in some cases other criteria could be included in the list to take into account other special issues. Since the building is supposed to be residential and the DM is assumed to be the owner, the following criteria have been selected as those having a significant influence on the decision:

From the *social/economical* point of view

C_1) *Installation cost*: obtained by listing the stages necessary to completely realize each interventions (except for the foundation, as will be discussed below) and by adding the respective costs including materials and labour;

C_2) *Maintenance cost*: ordinary maintenance is considered;

C_3) *Duration of works/disruption of use*: calculated by analyzing the time required for each stage and considering a team of four workmen (two of those are specialized workers). A rational sequence of operations was considered by Gantt diagrams;

C_4) *Functional compatibility*: aims to measure the compatibility of each intervention with the destination of the building.

From the *technical* point of view

C_5) *Skilled labour requirement/needed technology level*: it is important to discriminate the alternatives also in respect to this aspect since, for example, a more specialized team is generally more difficult to find available on the market;

C_6) *Size of the needed intervention at foundation*: defined by means of a "global" parameter that is the maximum ratio, measured for each column at first storey, between axial load due to the seismic action (plus gravity loads) and that due to the gravity loads only;

C_7) *Significant Damage risk*: defined as the probability of exceeding SD limit state in 50 years;

C_8) *Limited Damage risk*: calculated as the probability of sustain repair cost in 50 years.

More comments about each criteria will be given in the following along with the definition of their relative importance and the evaluation of alternatives in respect of them.

Weighting the criteria

As discussed in the following, the alternatives will be numerically evaluated in terms of each criterion. However, since not all the criteria have the same importance to the final decision, the definition of the "weights" w_i of the criteria C_i (i=1,2, …,8) is needed. The weights amplify or de-amplify the evaluations of the solutions in terms of C_i considering its relative importance. To compute the weights Saaty [3] proposed a simple approach based on the eigenvalue theory. It consists in making pairwise comparisons of criteria: a number (a_{ij}) determined according to the instructions in Tab. 1 estimates the relative importance of criteria C_i when it is compared with C_j.

Intensity of Importance	Definition	Explanation
1	Equal importance.	Two activities contribute equally to the objective.
3	Moderate importance of one to another	Experience and judgment slightly favour one activity over another
5	Essential or strong importance	Experience or judgment strongly favours one activity over another
7	Demonstrated importance	An activity is strongly favoured and its dominance is demonstrated in practice
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
2, 4, 6, 8	Intermediate values between the two adjacent judgments	When compromise is needed
Reciprocal of above	If criterion *i* compared to *j* gives one of the above then *j*, when compared to *i* gives its reciprocal	

Tab. 1. Scale of relative importance [3]

The a_{ij} numbers are collected in a matrix (A). (Obviously it results a_{ij}=1/a_{ji} and a_{ij}=1). So, if n is the total number of criteria (which is the order of the matrix A), the DM has to assign only $n(n-1)/2$ independent numbers. In the case in exam (n=8) it results $n(n-1)/2=28$; the A matrix is in Eq. (1) (where the 28 independent numbers are included in the dashed line).

$$\boldsymbol{A} = \left[\boldsymbol{a}_{ij}\right]_{8x8} = \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \dots & \boldsymbol{a}_{18} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} & \dots & \boldsymbol{a}_{28} \\ \dots & \dots & \dots & \dots \\ \boldsymbol{a}_{81} & \boldsymbol{a}_{82} & \dots & \boldsymbol{a}_{88} \end{bmatrix} = \begin{bmatrix} 1 & 1/3 & 1 & 1/5 & 4 & 1/3 & 4 & 1/3 \\ 3 & 1 & 3 & 1/2 & 6 & 1 & 6 & 1 \\ 1 & 1/3 & 1 & 1/5 & 4 & 1/3 & 4 & 1/3 \\ 5 & 2 & 5 & 1 & 6 & 2 & 5 & 2 \\ 1/4 & 1/6 & 1/4 & 1/6 & 1 & 1/6 & 1/2 & 1/5 \\ 3 & 1 & 3 & 1/2 & 6 & 1 & 5 & 3 \\ 1/4 & 1/6 & 1/4 & 1/5 & 2 & 1/5 & 1 & 1/3 \\ 3 & 1 & 3 & 1/2 & 5 & 1/3 & 3 & 1 \end{bmatrix} \tag{1}$$

As discussed, these numbers are based on the DM (owner) judgment and goals. To better understand how the matrix A was retrieved some clarifications are given here. For example, the owner considers the reduction of the maintenance costs (C_2) moderately more important than installation cost (C_1) because the former implies additional undesirable disruption of use, then a_{12}=1/3.
Installation cost (C_1) is considered to be as important as the duration of works (C_3) since the latter results in a monetary loss (e.g. rent) for the owner, therefore a_{13}=1.
The functional compatibility (C_4) is judged to be very important in comparison to the other criteria ($a_{4j} \geq 1, j=1, \dots, 8$) due to the residential destination of the structure. In other words, given its small size, the building is very sensitive in terms of even small architectonical impact that an intervention may have on the normal use of the space.
The criteria concerning the significance of the needed intervention at foundation (C_6) is also assumed to be generally important ($a_{6j} \geq 1$, j =1, 2, 3, 5, …, 8) since the corresponding variable implicitly accounts for the additional time, cost and disruption to be sustained. For example, a reduced intervention at foundation, is considered highly desirable if compared to criterion C_5 (required workers' specialization) and therefore a_{65}=6. On the other hand, C_6 is only moderately more important than the installation cost, the duration of works and the limited damage risk ($a_{61}=a_{63}=a_{68}=3$), and it is assumed as important as maintenance costs (a_{62}=1).
The criteria C_5 regarding the skilled labour requirement and the needed technology level is considered less important than the others ($a_{5j} \leq 1$, j =1, …, 8) since the owner prefers to have better performances in terms of compatibility, costs, duration, increment of demand at foundation even if he has to engage a more specialized team that is generally more difficult to find on the market.
The criteria C_7 (Significant Damage risk) is judged to be less relevant than C_8 (Limited Damage risk) because, since the design target is SD and it is satisfied by all the alternatives (design target), the owner is more interested to reduce the expected loss related to the repair in case of DL limit state occurrence; consequently a_{78}=1/3.
If the comparison among criteria are carried out in a perfectly consistent manner, should be $a_{ij} = w_i / w_j$, where w_i and w_j denote the weights of importance of criteria C_i and C_j respectively. In this ideal case the matrix A has rank 1 and $\lambda=n$ is its nonzero principal right eigenvalue; moreover it is easy to show that the vector W of relative weights $w_1, w_2, \dots, w_8$ is the principal right eigenvector of matrix A. In the non-consistent case, which is not uncommon at all, a_{ij} may deviate from the ratio w_i/w_j [13] and then the eigenvalues change consequently. In particular the maximum eigenvalue λ_{max} results to be greater than n (but close to it) while the other eigenvalues are close to zero. In the non-consistent case is, thus, reasonable to assume the vector W of weights w_i equal to eigenvector that corresponds to λ_{max} that is the vector satisfying the equation $A\ W=\lambda_{max}\ W$. In the case under exam it results λ_{max} = 8.447 (> n=8) and the vector W results to be the following:

$$\boldsymbol{W} = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \dots, \boldsymbol{w}_8\} = \{0.073, 0.172, 0.073, 0.280, 0.026, 0.201, 0.035, 0.141\} \tag{2}$$

The weights' values w_i can be used to rank the criteria with reference to their relative importance, as shown in the following Tab. 2. The pie chart in Fig. 5 represents in an effective way the shares of importance levels that the DM has implicitly defined by making the 28 pairwise comparisons among criteria.

Ranking order	Weights w_i	Criteria	Description
I	0.280	C_4	Functional compatibility
II	0.201	C_6	Significance of the needed intervention at foundation
III	0.172	C_2	Maintenance cost
IV	0.141	C_8	Limited Damage risk
V-VI	0.073	C_1, C_3	Installation cost, Duration of works
VII	0.035	C_7	Significant Damage risk
VIII	0.026	C_5	Skilled labour requirement/needed technology level

Tab. 2. Scale of relative importances [3]

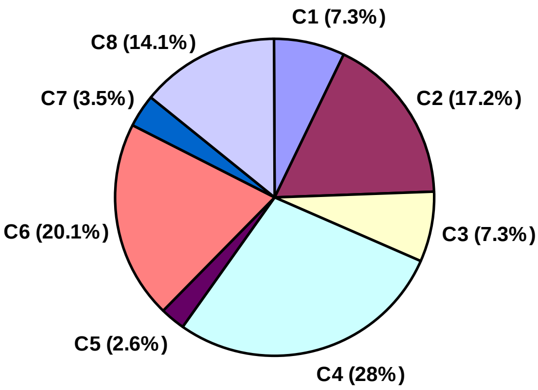


Fig. 5. Shares of importance levels of criteria

It is worth noting that different approximated ways are available to compute the weights w_i from the matrix A rather than solving the equation $A\ W=\lambda_{max}\ W$. Saaty suggested to evaluate the terms w_i of the vector W simply normalizing (the weights' values should add up to one) the rows of A by the geometric mean of the corresponding row. Fishburn [14] suggested instead to calculate w_i values by normalizing each matrix' element dividing it by the sum of all the elements located in the same column and then by taking the arithmetic mean of each row. In Eq. (3) and (4) the weights vectors obtained by these two quick methods are shown respectively. The weights retrieved in these ways are very close to those of the eigenvector of Eq. (2).

$$\{0.072, 0.178, 0.072, 0.280, 0.026, 0.200, 0.034, 0.139\} \quad (3)$$

$$\{0.075, 0.172, 0.075, 0.277, 0.026, 0.198, 0.036, 0.140\} \quad (4)$$

Moreover Saaty proposed to quick estimate the value of λ_{max} by adding the elements of the same column of matrix A and then multiplying the resulting row vector with the column vector W. The so derived value of λ_{max}=8.408 is, again, very close to that (8.447) evaluated as the maximum eigenvalue of A.

Consistency of weights assignments

The "Perfect consistency" of pairwise comparisons means that if C_i is defined to be more important than C_j by a factor a_{ij} and C_j is assumed to be more important than C_k by a factor a_{jk}, then C_i should be judged to be more important than C_k by a factor $a_{ik}= a_{ij}\ a_{jk}$. In this ideal case in fact it results $a_{ij}= w_i / w_j$, $a_{jk}= w_j / w_k$ and then $a_{ik}= w_i / w_k = (w_i/w_j)\ (w_j/w_k)= a_{ij}\ a_{jk}$. Since the DM arbitrarily assign a_{ij}, without any mathematical constraint, then the perfect consistency is generally not achieved and a_{ij} may deviate from the ratio w_i/w_j.
A way to measure the degree of consistency and then to check if it is tolerable or not is provided by the *Consistency Index (CI)* [3] in Eq. (5).

$$CI = \frac{\lambda_{\max} - n}{n-1} \quad (5)$$

CI has to be normalized by the *Random Consistency Index RCI* that is an average random consistency measure depending on n (0, 0, 0.58, 0.90, 1.12, 1.24, 1.32, 1.41, 1.45 for n=1, 2, …, 9 respectively) and obtained by numerous empirical studies. Then, the *Consistency Ratio CR* is obtained.
The pairwise comparisons in the judgement matrix A can be considered sufficiently consistent if *CR* is less than 5% if n=3, 9% if n=4, 10% if n>4 [17], being n the order of the judgement matrix. Otherwise it would be desirable to re-examine the pairwise judgments until acceptable consistency is achieved.
In the case in exam it results *CI*=0.058 and, since for n=8 it is *RCI*=1.41, *CR*=4.1%. The judgement matrix A can be thus considered sufficiently consistent (*CR*<10%).
It is important to underline that an acceptable *CR* ensures that no intolerable conflicts exist, and that the final decision is logically sound and not a result of random prioritization [18].

Evaluation of the solutions

Each retrofit alternative A_i (i =1, 2, 3) has to be evaluated according to each criterion C_j (j =1, 2, …, 8) defined above. Given the different nature of the criteria, the corresponding evaluations are expressed in different units and have to be therefore normalized. Furthermore some variables are not crisp numbers (e.g. qualitative) and have to be converted. The (3x8=24) evaluations should be collected in the *Decision Matrix (D);* in Tab. 3 the *D* matrix with the quantitative scores is given.

	C_1 (€)	C_2	C_3 (days)	C_4	C_5	C_6	C_7	C_8
A_1	23,096	*qualitative*	21.38	*qualitative*	*qualitative*	2.90	0.022	0.311
A_2	53,112		41.25			15.18	0.024	0.002
A_3	10,574		17.13			2.97	0.040	0.172

Tab. 3. Decision matrix

Evaluations in the matrix have been computed as follows:

Criteria C_1 - the values in the first column of the Decision Matrix are obtained by listing the stages necessary to completely realize each interventions (except for the foundation, as discussed above) and by adding the respective costs (including materials and labour).

Criteria C_2 (qualitative) - a crisp value (expressed in €/year and representing the mean annual maintenance charges) should be computed for each alternative. However, since the application of composite materials to structures is relatively recent, the durability and consequently the maintenance needs are still open issues [19]. Specific codes [20] recommend periodical monitoring of the status of the fibres and aggressiveness of the environment (exposition to UVA rays, humidity, temperature, etc.) in order to promptly take non-ordinary maintenance actions if required. Herein authors did not get into quantification of costs and how often monitoring should be carried out; therefore, a qualitative comparison of the durability for the different materials (solutions) seemed to be a practical proxy for the maintenance costs.

Criteria C_3 - the duration of each intervention is calculated by analyzing the time required for each stage and considering a team of four workmen (two of those are specialized workers). A rational sequence of operations (also including some stops, when necessary, and some simultaneous activities, when possible, allowing to optimize the time) was considered.

Criteria C_4 and *C_5* (qualitative) - the qualitative evaluation of A_1, A_2 and A_3 with reference to these two criteria and the consequent conversion into crisp values is described in the next section.

Criteria C_6 measures the size of the intervention at foundation for each alternative. The evaluation according to C_6 consisted of computing a “global” parameter that is the maximum ratio, measured for each column at first storey, between axial load due to the seismic action plus gravity loads and that due to the gravity loads only. Each column is assumed to have its own independent plinth of foundation. However, authors are aware that a more rigorous assessment of the substructure should be performed in order to evaluate its present capacity and then to compare it with the demand due to the earthquake.

Criteria C_7 and *C_8* are related to the seismic capacity of the building being defined as the earthquake intensity (measured by the peak ground acceleration, PGA) at which a certain limit state is attained [15]. Nonlinear static (pushover) analysis was performed for all the 3 alternatives and in all the 4 directions (±X, ±Y) [9]. The comparison among the corresponding pushover curves is shown in Fig. 6.

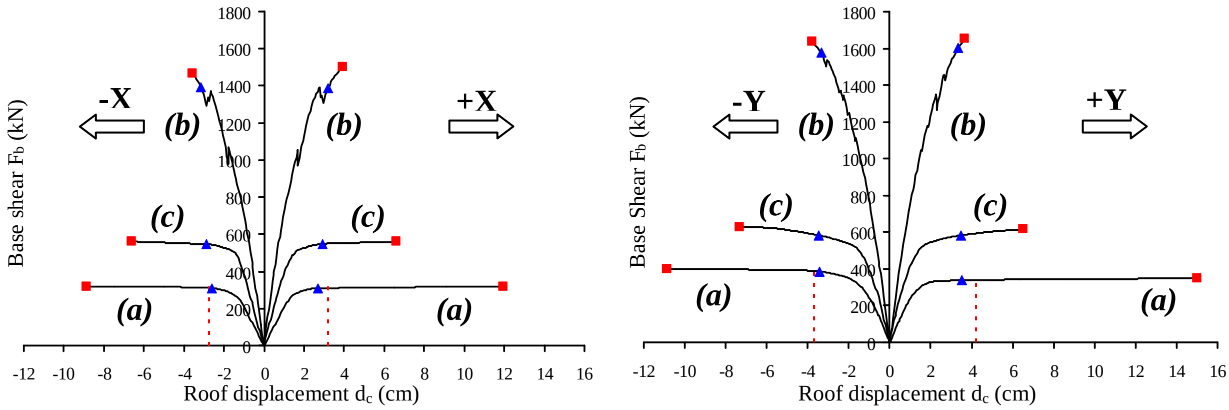


Fig. 6. Pushover curves for the interventions A_1 (a), A_2 (b), A_3 (c) for each direction (+X, -X, +Y, -Y). Triangles and squares indicate the DL and SD limit states attainment respectively.

The un-retrofitted building's curves coincide in strength with those relative to the building retrofitted by GFRP up to the vertical dashed lines indicating the attainment of the SD limit state (as far as the DL limit state is concerned, the original and retrofitted by GFRP building have almost the same capacity). The corresponding values of PGA of "failure" at SD and DL limit states are then obtained by applying the N2 method [16] to the capacity curves (Tab. 4). The probability of exceeding the PGA capacity in 50 years is calculated by the hazard curve of Pomigliano d'Arco shown in Fig. 7 [21]. The PGA values are given in Tab. 5.

PGA capacity at SD LS (g)					PGA capacity at DL LS (g)				
Dir.	As built structure	GFRP A_1	Steel braces A_2	RC jackets A_3	Dir.	As built structure	GFRP A_1	Steel braces A_2	RC jackets A_3
+X	0.12	0.43	0.32	0.25	+X	0.10	0.10	0.31	0.12
-X	0.10	0.33	0.32	0.25	-X	0.10	0.10	0.31	0.12
+Y	0.16	0.52	0.34	0.25	+Y	0.14	0.12	0.34	0.14
-Y	0.15	0.40	0.35	0.29	-Y	0.14	0.13	0.35	0.14
min	0.10	0.33	0.32	0.25	min	0.10	0.10	0.31	0.12

Tab. 4. Capacity in terms of PGA at Significant Damage and Limited Damage limit states

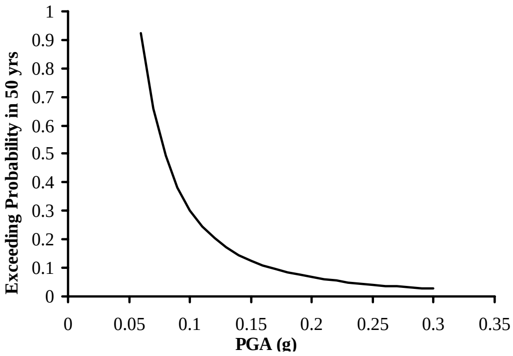


Fig. 7. Hazard curve of Pomigliano d'Arco (Naples, Italy).

Significant Damage Limit State					Limited Damage Limit State				
Dir.	As built structure	GFRP A_1	Steel braces A_2	RC jackets A_3	Dir.	As built structure	GFRP A_1	Steel braces A_2	RC jackets A_3
+X	0.20	0.01	0.02	0.04	+X	0.31	0.32	0.03	0.20
-X	0.28	0.02	0.02	0.04	-X	0.32	0.32	0.03	0.21
+Y	0.10	0.01	0.02	0.04	+Y	0.15	0.20	0.02	0.14
-Y	0.13	0.02	0.02	0.03	-Y	0.16	0.18	0.02	0.14
max	0.28	0.02	0.02	0.04	max	0.32	0.32	0.03	0.21

Tab. 5. Probability of exceeding in 50 years the PGA capacities at SD and DL limit states

Note that the performance of the solutions according to the criteria C_7 is measured by the maximum probability (among all the 4 directions) of exceeding in 50 years the capacity at Significant Damage limit state (last row of the Tab. 5, left). The evaluation in respect of the criteria C_8 is computed by the probability that in 50 years the seismic capacity at the DL limit state is exceeded while SD is not, otherwise the building is likely to be uneconomic to repair. Therefore these values are calculated as the maximum difference (again, among all the 4 directions) between the probability of exceeding the DL and SD limit states respectively, see the column C_8 of Tab. 7.

Conversion of criteria C_2, C_4, C_5 evaluations into crisp numbers

Criteria C_4 and C_5, due to their own nature, do not allow the evaluation of alternatives in a quantitative way. The performances of A_1, A_2 and A_3 according to C_4 and C_5, in fact, can be done only by adopting qualitative or linguistic variables. The criteria C_2 regarding the maintenance's cost is also treated here qualitatively for the reasons described above.

In order to apply any MCDM procedure (TOPSIS herein) the conversion of such variables into crisp numbers is needed. Many are the methods in literature that allow this kind of operation. In this paper pairwise comparisons and the eigenvalue approach is adopted, exactly in the same way as already done for the evaluation of criteria weights. Therefore the procedure consists of making linguistic comparisons among the performance of each alternative with reference to the criteria C_i (i = 2, 4, 5) and then quantifying these statements by using the already mentioned linear scale. After composing these values into a 3x3 matrix, the eigenvalue's method is adopted to calculate the 3 numbers representing the numerical score (priority) of each retrofit option according to the criteria C_i. In the Tab. 6 the judgement matrices and the priorities calculated with reference to criteria C_2, C_4 and C_5 are shown.

C_2	A_1	A_2	A_3	Priority
A_1	1	1/7	1/2	0.094
A_2	7	1	5	0.740
A_3	2	1/5	1	0.167
	λ_{max}=3.014; CR=1.23%			

C_4	A_1	A_2	A_3	Priority
A_1	1	7	3	0.669
A_2	1/7	1	1/3	0.088
A_3	1/3	3	1	0.243
	λ_{max}=3.007; CR=0.61%			

C_5	A_1	A_2	A_3	Priority
A_1	1	4	7	0.705
A_2	1/4	1	3	0.211
A_3	1/7	1/3	1	0.084
	λ_{max}=3.032; CR=2.80%			

Tab. 6. Quantitative evaluation of alternatives according to criteria C_2, C_4 and C_5.

As done for the criteria weights, a consistency check is needed also for the three judgement matrices. The Consistency Index *CI* has the already known expression in Eq. (5). In this case is n=3 and the *RCI* is equal to 0.58 so the *CR* has to be calculated as in Eq. (6). Its value with reference to the criteria C_2, C_4 and C_5 is given in the last row of the Tab. 6.

$$CR = \frac{CI}{RCI} = \frac{\lambda_{\max} - n}{n-1} \cdot \frac{1}{RCI} = \frac{\lambda_{\max} - 3}{2} \cdot \frac{1}{0.58} \tag{6}$$

The pairwise comparisons in the focused judgement matrices can be considered sufficiently consistent since *CR* is always less than 5% (limit value for n=3 [17]). Then, the computed priorities have to be introduced in the correspondent column of the Decision Matrix (Tab. 3) obtaining its final and complete form (Tab. 7).

	C_1 (€)	C_2	C_3 (days)	C_4	C_5	C_6	C_7	C_8
A_1	23,096	*0.094*	21.38	*0.669*	*0.705*	2.90	0.022	0.311
A_2	53,112	*0.740*	41.25	*0.088*	*0.211*	15.18	0.024	0.002
A_3	10,574	*0.167*	17.13	*0.243*	*0.084*	2.97	0.040	0.172

Tab. 7. Completely defined Decision Matrix (evaluation of solutions in respect of qualitative criteria are given in italic)

Ranking the alternatives: selection of the best solution

The adopted *TOPSIS* method (*Technique for Order Preference by Similarity to Ideal Solution*) was developed by Hwang and Yoon [2] and consists in identifying the best alternative among those in exam as the one having the shortest distance from a so called *ideal solution* A^* and the farthest distance from a so called *negative-ideal solution* A^- which will be better defined in the following.
The generic element of the Decision Matrix (Tab. 7) which is the performance measure of the i-th alternative (i =1, 2, 3) in terms of the j-th criteria (j =1, 2, …, 8) will be referred as x_{ij}. The first step of the TOPSIS procedure consists in converting all the x_{ij} values (each of those has a different dimension) into dimensionless r_{ij} numbers by normalizing them according to Eq. (7). The *Normalized Decision Matrix* R=[r_{ij}] is thus obtained (Eq. (8)).

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{k=1}^{3} x_{kj}^2}} \tag{7}$$

$$R = \left[r_{ij}\right] = \begin{bmatrix} 0.392 & 0.123 & 0.432 & 0.932 & 0.951 & 0.184 & 0.408 & 0.876 \\ 0.902 & 0.968 & 0.833 & 0.122 & 0.284 & 0.964 & 0.408 & 0.005 \\ 0.179 & 0.218 & 0.346 & 0.338 & 0.113 & 0.188 & 0.816 & 0.480 \end{bmatrix} \tag{8}$$

The next step is weighting the matrix R by multiplying each value of the i-th column by the weight w_i of the i-th criterion. The so evaluated *Weighted Normalized Decision Matrix* V=[v_{ij}] is reported in Eq. (9).

$$V=\left[v_{ij}\right]=\begin{bmatrix} w_1r_{11} & \dots & w_8r_{18} \\ w_1r_{21} & \dots & w_8r_{28} \\ w_1r_{31} & \dots & w_8r_{38} \end{bmatrix}=\begin{bmatrix} 0.028 & 0.021 & 0.032 & 0.261 & 0.024 & 0.037 & 0.014 & 0.123 \\ 0.065 & 0.166 & 0.061 & 0.034 & 0.007 & 0.193 & 0.014 & 0.0008 \\ 0.013 & 0.037 & 0.025 & 0.094 & 0.002 & 0.037 & 0.028 & 0.067 \end{bmatrix} \tag{9}$$

A graphical comparison of the alternatives in terms of normalized performances r_{ij} is shown by the bar diagram in Fig. 8(a) where it is also indicated, for each criteria, if the Decision Maker's goal is maximizing or minimizing the correspondent performance values. The bar chart in Fig. 8(b) shows the comparison done with reference to the weighted normalized values $v_{ij} = w_j\, r_{ij}$ and the relative amplification and reduction of the variables corresponding to more important (i.e. C_4, C_6) or less important (i.e. C_5, C_7) criteria. The transformation of the first diagram into the second one, depending on the criteria weights, reflects the Decision Maker profile and the building's destination.

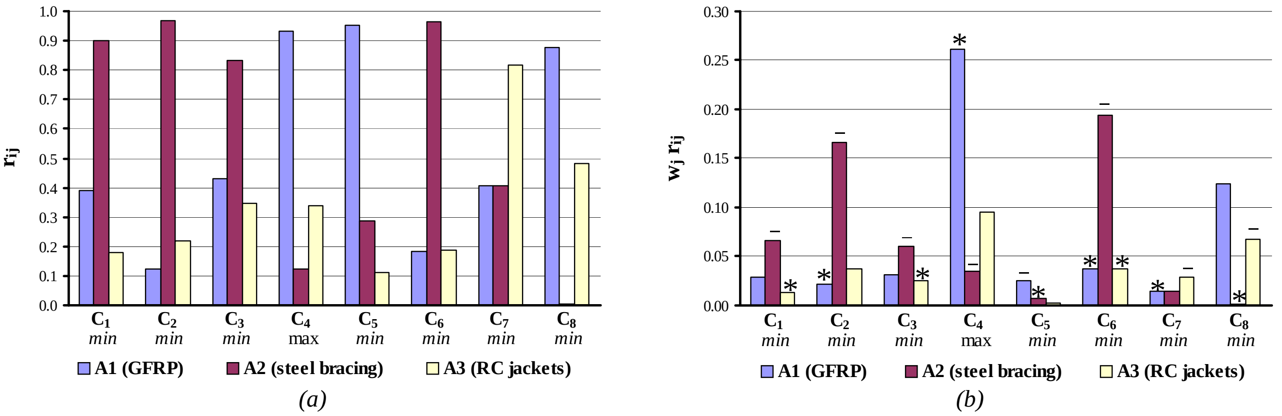


Fig. 8. Graphical comparison among alternatives in terms of normalized performances r_{ij} (a) and weighted normalized performances v_{ij} (b)

The two opposite fictitious solutions A^* and A^- mentioned above are completely defined by 8 values, each of those representing the (weighted normalized) performance measured according to each criterion. In particular, the ideal solution A^* is obtained by taking for each criterion the *best* performance value among A_1, A_2 and A_3 (indicated by an asterisk in Fig. 8(b)). In the opposite way, the negative-ideal solution A^- is composed by considering for each criterion the *worst* performance measure among the alternatives (indicated by a minus). It's important to remark that the *best* value among the alternatives' performances has to be interpreted has the *maximum* value if the criterion is a *benefit* criteria (to be maximized); it has instead to be interpreted has the *minimum* value if the considered criterion is a *cost* criteria (to be minimized). In the case in exam all the criteria are cost criteria, except for the criteria C_4 (functional compatibility). So the ideal and negative-ideal solutions are the following:

$$A^*=\left\{\left(\min_i v_{ij}\mid j\in 1,2,3,5,6,7,8\right),\left(\max_i v_{ij}\mid j=4\right), i=1,2,3\right\}=\{0.013, 0.021, 0.025, 0.261, 0.002, 0.037, 0.014, 0.0008\}$$

$$A^-=\left\{\left(\max_i v_{ij}\mid j\in 1,2,3,5,6,7,8\right),\left(\min_i v_{ij}\mid j=4\right), i=1,2,3\right\}=\{0.065, 0.166, 0.061, 0.034, 0.024, 0.193, 0.028, 0.123\}$$

The Euclidean distances S_{i*} and S_{i-} of the i-th alternative A_i from the ideal and negative-ideal solutions A^* and A^- are defined in Eqq. (10) and (11) respectively. The relative closeness C_{i*} ($0\le C_{i*}\le 1$) of A_i with respect to A^* is defined as in Eq. (12)[2].

$$S_{i*}=\sqrt{\sum_{j=1}^{8}\left(v_{ij}-v_{j*}\right)^2}\quad ;\quad i=1,2,3 \tag{10}$$

$$S_{i-}=\sqrt{\sum_{j=1}^{8}\left(v_{ij}-v_{j-}\right)^2}\quad ;\quad i=1,2,3 \tag{11}$$

[2] $C_{i*}=1$ if $A_i=A^*$; $C_{i*}=0$ if $A_i=A^-$. Therefore the best alternative (with the shortest distance to the ideal solution) is the one with the highest C_{i*} value.

$$C_{i*} = \frac{S_{i-}}{S_{i*} + S_{i-}} \quad ; \quad i = 1,2,3 \tag{12}$$

Since a graphical representation in the 8-dimensional space is not possible, the location of each alternative (actual and fictitious) and the meaning of the distances (with reference only to A_1, for simplicity) defined in Eq. (10) and (11) are shown by their projection onto two of the eight planes (Fig. 9); the plane v_{i3}, v_{i8} (in which the weighted normalized values in respect to the criteria C_3 and C_8 are reported) and the plane v_{i4}, v_{i5} (related to the criteria C_4 and C_5).

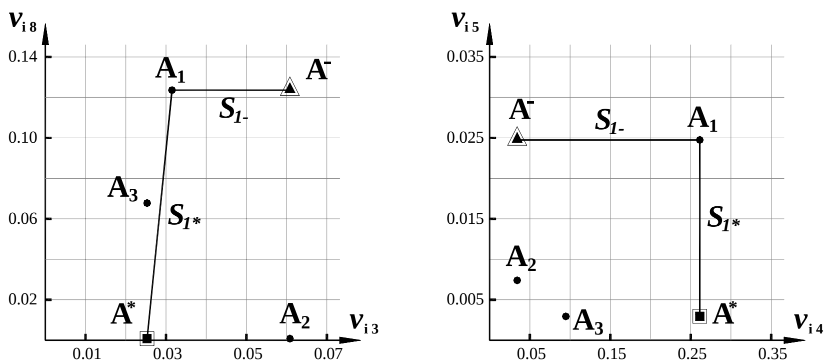


Fig. 9. Graphical comparison among alternatives in terms of normalized performances r_{ij} (a) and weighted normalized performances (b)

C_{i*} of Eq. (12) calculated for each alternative is shown in Tab. 8. It allows to rank the solutions and choose the optimal one. The best solution results to be A_1 (GFRP), followed by A_3 (concrete jackets) and A_2 (steel bracing).

		S_{i*}	S_{i-}	C_{i*}
GFRP	A_1	0.126	0.315	0.715
Steel bracing	A_2	0.318	0.125	0.282
Concrete jackets	A_3	0.181	0.229	0.559

Tab. 8. Distances S_{i*}, S_{i-} and C_{i*} values for each alternative (weighted criteria).

It is worth noting that considering the un-weighted criteria, which corresponds to give the same importance to all criteria (w_i =1, for i =1, 2, …,8), leads to a different best solution. It means to use the normalized r_{ij} values in Eq. (8) rather than the weighted v_{ij} values in Eq. (9) and corresponds to neglecting the influence of the DM profile and of the building's destination of use on the final decision. In this way the values in Tab. 9 are obtained and the best solution would be A_3 (jacketing of columns), followed by A_1 and A_2. This conclusion is justified by observing that the assigned weights give less importance to the criteria (i.e. C_1, C_3, C_5) in respect of which the alternative A_3 shows better performances than A_1.

		S_{i*}	S_{i-}	C_{i*}
GFRP	A_1	1.231	1.602	0.566
Steel bracing	A_2	1.664	1.171	0.413
Concrete jackets	A_3	0.869	1.682	0.659

Tab. 9. Distances S_{i*}, S_{i-} and C_{i*} values for each alternative (non-weighted criteria).

Conclusions

This paper briefly discuss Multi-Criteria Decision Making for the choice of the best seismic retrofit solution for a under-designed RC structure. The procedure consisted of structural design of alternatives and evaluations of them according to criteria of interest to the stake holder. Criteria are both technical and non-technical and of the quantitative or qualitative kind. They are weighted considering non-uniform relevance to the final decision.

The best solution resulted to be the GFRP retrofit. This solution increases the displacement capacity of the

structure by a low architectonic impact and requires a comparatively moderate intervention at the foundation and maintenance. Conversely steel bracing is more demanding for foundations and has a higher impact and disruption being less suitable for residential constructions even if gives better performances in terms of operational limit states improving the global strength and stiffness.
The influence of the DM's judgments on the final choice, formally included into the weights' value, was clearly highlighted by showing that the best solution changes (A_3 rather than A_1) if un-weighted criteria are used. Neglecting the influence of the DM profile and building's destination of use is considered to be not realistic, but a sensitivity analysis should be performed in order to identify if slight modifications of user preferences might lead to the selection of a different alternative.

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