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Models and issues in history-dependent mainshock hazard

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ABSTRACT: The purpose of this paper is to critically review some history-dependent probabilistic mainshock occurrence models with respect to probabilistic seismic hazard analysis (PSHA). In PSHA, earthquake occurrence is usually referred to homogeneous Poisson process (HPP), which is memory-less; however, when a single fault is of concern and/or the time scale is different from that of the long term, history-dependent models, may be appropriate. In the study, two categories of models are reviewed: (i) *renewal processes*, which are used to model the sequence of interarrival times of characteristic earthquakes, and (ii) simple *Markov renewal processes*, in which beyond earthquake occurrence, modeling of correlation between magnitude and interarrival time is also explicitly addressed. As an illustrative application, the Paganica fault (in central Italy) is considered to compute the seismic hazard, in terms of exceedance probability of a ground motion intensity measure value, according to each of the models rendered comparable by means of working hypotheses. Results show that as larger is the time since the last earthquake the more different hazard is implied by the different models, rendering critical their selection for sources where the time since the last event is large.

1 INTRODUCTION

Probabilistic seismic hazard analysis (PSHA; e.g., McGuire, 2004) usually refers to homogeneous Poisson process (HPP) to probabilistically model earthquake occurrence. The latter is an independent- and stationary-increment (i.e., *memory-less*) model, which may prove suitable when several (independent) sources contribute to the seismic threat for a site. However, when a single fault is of concern and/or the time scale is different from that of the long term, other models may be more appropriate to probabilistically describe the earthquakes occurrence process.

The long-term mainshock occurrence is considered in this paper, neglecting other cases as the short-term aftershock sequence modeling (e.g., Yeo and Cornell, 2005) or the multi-scale operational forecasting (e.g., Jordan et al., 2011). In fact, the study focuses on two types of history-dependent models. The first category is that of renewal processes, which applies when *characteristic* earthquakes are of concern, that herein is when the considered source may produce a specific magnitude. The second type, which can be formalized on the basis of the theory of *Markov renewal processes*, enables, as an additional feature, modeling of correlation between magnitude and interarrival time; e.g., Anagnos and Kiremidijan (1988); Cornell and Winterstein (1988).

The study is structured such that assumptions common to all the considered models are presented first. Then, modeling of the random variables (RVs) involved in each of them, is reviewed. Moreover, an illustrative application is set-up with respect to evaluate the conditional probability of exceedance of a ground motion intensity measure (IM) value for a site of interest, and in a given time-frame.

To this aim, the Paganica fault (in central Italy; believed to be the source of the 2009 L'Aquila earthquake) and a site close to it are considered. This allows to compute, for each history-dependent model, the probability of observing *one event* in the time interval of interest, and the probability of exceedance of an IM-level, as a function of the time elapsed since the last earthquake.

2 RENEWAL PROCESSES FOR EARTHQUAKE OCCURRENCE

A renewal process (RP) is, by definition, a sequence of independent and identically distributed (i.i.d.) non negative RVs (whose distribution completely characterizes the model). In the considered application the RV of interest is the time between successive occurrences of earthquakes (i.e., interarrival time, *T*).

In the seismic context, RPs appear suitable to describe a sequence of similar and large magnitude events on a specific seismic source in the context of the *elastic-rebound theory* (Reid, 1910), which suggests that large tectonic earthquakes may recur at the onset of large elastic strain in the crust. Strain will then re-accumulate slowly by steady tectonic forcing until the next event.

In fact, in all RPs it is assumed that the system (i.e., the earthquake source) restarts *as-new* after the occurrence of each earthquake. In this sense, they appear suitable to model occurrence of characteristic earthquakes, that is sources that tend to produce specific-magnitude events.

The renewal processes considered are: (1) an inverse Gaussian RP, related to the *Brownian relaxation oscillator* model, (2) an Erlang RP, featuring an analytically tractable counting process; (3) and finally an inverse gamma RP, related to a model in which load increases deterministically over time with random loading rate.

2.1 *Inverse Gaussian RP*

This RP relates to the *Brownian relaxation oscillator* model. According to this model, load state, $X(t)$, increases gradually over time until it reaches an earthquake-triggering threshold. The model assumes that earthquake occurrence instantaneously *relaxes* back the system to some ground level.

Load state process is modeled through a process with independent and stationary Gaussian-distributed increments, as in Equation (1) and sketched in Figure 1. In the equation $\dot{u}$ is the rate, $W(t)$ is the standard Brownian motion, which has stationary and independent Gaussian increments, and σ is a scaling factor that models process variance (Matthews et al., 2002). The deterministic (linearly increasing) part of the process, takes into account the constant-rate average loading, the random part represents contributions of all other factors affecting the eventual rupture of the considered source.

$$X(t) = \dot{u} \cdot t + \sigma \cdot W(t) \tag{1}$$

It is possible to show that, according to the above assumptions, the probability density function (PDF) of interarrival time, $f_T(t)$, follows an *inverse Gaussian* distribution, Equation (2). This PDF, which is also called the *Brownian passage time* (BPT) distribution, is entirely described by two parameters: the mean recurrence time (μ, the *mean interarrival time*, also referred to as the *return period*, *Tr*) and the coefficient of variation, or aperiodicity, of interarrival time (α). The return period is in relation with the load rate $(\dot{u})$ and the threshold $(\bar{u})$.

$$\begin{cases} f_T(t) = \sqrt{\dfrac{\mu}{2 \cdot \pi \cdot \alpha^2 \cdot t^3}} \cdot e^{-\frac{(t-\mu)^2}{2 \cdot \mu \cdot \alpha^2 \cdot t}} \\ \mu = \dfrac{\bar{u}}{\dot{u}} \end{cases} \tag{2}$$

The mean interarrival time or its reciprocal, the mean rate of occurrence, is the parameter of first order interest, that is the best-estimate of frequency at which events occur. The aperiodicity is a measure of irregularity in the event sequence, that is, a deterministic sequence features $\alpha = 0$.

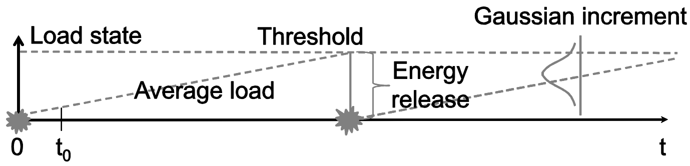

Figure 1. Sketch of source load modeling in the BPT model.

2.2 *Erlang-distributed interarrival time RP*

To define this process, an Erlang distribution (i.e., a gamma PDF with, k, as the integer shape parameter and λ as the scale parameter) for the interarrival time is considered. The interarrival time distribution is given in Equation (3), where Γ is the gamma function. This PDF has a flexible shape that can easily characterize any data-derived distribution (Takahashi et al., 2004).

Note that the mean and the coefficient of variation (CoV) in this case are given by k/λ and $1/\sqrt{k}$, respectively. These may be put in relation with the return period and the aperiodicity of the BPT model.

$$\begin{cases} f_T(t) = \dfrac{\lambda \cdot (\lambda \cdot t)^{k-1}}{\Gamma(k)} \cdot e^{-\lambda \cdot t} \\ E[T] = \dfrac{k}{\lambda} \\ CoV[T] = \dfrac{1}{\sqrt{k}} \end{cases} \tag{3}$$

One of the main advantages of this process is that it allows a closed-form solution for the probability of occurrence of at least one event in time interval (t_0, t), given that the last earthquake occurred at

$t=0$, Equation (4). In the equation, t_0 is the time of the probabilistic assessment, and $N(t)$ is the function counting events in $(0,t)$.

$$
\begin{aligned}
&P\left[N(t)\geq 1 \mid N(t_0)=0\right]= \\
&=1-P\left[N(t)=0 \mid N(t_0)=0\right]= \\
&=1-\frac{\sum_{i=0}^{k-1}\frac{(\lambda\cdot t)^i}{i!}\cdot e^{-\lambda\cdot t}}{\sum_{i=0}^{k-1}\frac{(\lambda\cdot t_0)^i}{i!}\cdot e^{-\lambda\cdot t_0}}
\end{aligned}
\tag{4}
$$

If the probability of having exactly one event is computed as in Equation (5), it is possible to evaluate how likely is that more than one earthquake occurs in the time-frame of interest as a function of the time elapsed since the last event, Figure 2.

This allows to understand that if the interval of interest is small with respect to the average recurrence time, as it usually happens for seismic risk analysis of engineering interest, the probability of having more than one event is very close to the probability of one event. In other words, it is unlikely that more than one earthquake occurs in a small time interval. This result will be helpful in probabilistic seismic hazard analysis discussed in Section 4.

$$
\begin{aligned}
&P\left[N(t)=1 \mid N(t_0)=0\right]= \\
&=\frac{1}{\sum_{i=0}^{k-1}\frac{(\lambda\cdot t_0)^i}{i!}\cdot e^{-\lambda\cdot t_0}}\cdot\sum_{i=0}^{k-1}\left[\frac{(\lambda\cdot t_0)^i}{i!}\cdot e^{-\lambda\cdot t_0}\times\right. \\
&\left.\times\sum_{j=k-i}^{2\cdot k-i-1}\frac{\left[\lambda\cdot(t-t_0)\right]^j}{j!}\cdot e^{-\lambda\cdot(t-t_0)}\right]
\end{aligned}
\tag{5}
$$

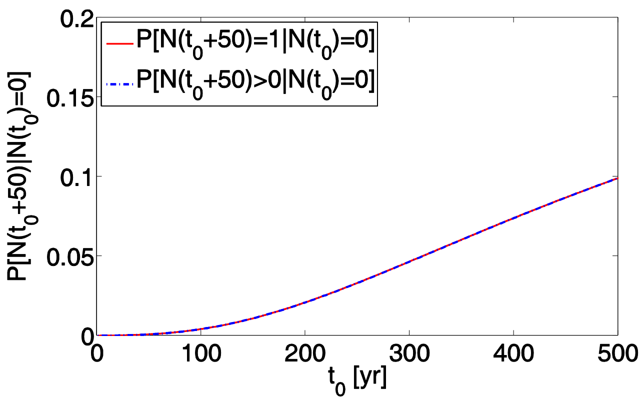

Figure 2. Comparison between the probability of observing exactly one event, and at least one event, for the renewal process with gamma interarrival time distribution in a 50 yr time frame, as a function of the time elapsed since the last earthquake.

2.3 *Inverse-Gamma-distributed interarrival time RP*

This RP relates to a (simple) model, which assumes that the load on the fault increases linearly and deterministically over time, with a rate that varies randomly from event to event. Rate is modeled as a gamma-distributed random variable. The earthquake occurs once a threshold is reached. Then, the system resets itself until the next event, Figure 3.

It is possible to show that these hypotheses lead to a renewal process characterized by an inverse-gamma-distributed (IG) interarrival time (Pandey and van Noortwijk, 2004). The latter is given in Equation (6), where γ and β are the shape and scale parameters, respectively. In the equation the mean and variance of the RV are also given as a function of the parameters.

$$
\begin{cases}
f_T(t)=\dfrac{\beta^\gamma}{\Gamma(\gamma)}\cdot\left(\dfrac{1}{t}\right)^{\gamma+1}\cdot e^{-\frac{\beta}{t}} \\
E[T]=\dfrac{\beta}{\gamma-1}, & \gamma>1 \\
Var[T]=\dfrac{\beta^2}{(\gamma-1)^2\cdot(\gamma-2)}, & \gamma>2
\end{cases}
\tag{6}
$$

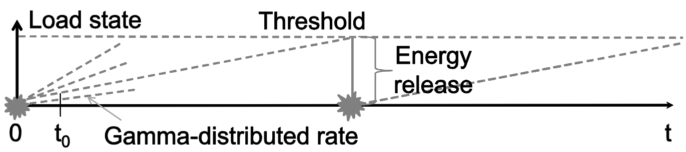

Figure 3. Representation of loading in the renewal process with Gamma-distributed load rate.

2.4 *Homogeneous Poisson process*

It is to note that the HPP model may also be seen as a renewal process with exponential interarrival time, Equation (7), with mean and standard deviation equal to μ and Poisson distribution for the increments of the associated counting process, Equation (8). The latter has independent and stationary increments that render the process *memory-less*.

$$
f_T(t)=\frac{1}{\mu}\cdot e^{-\frac{t}{\mu}}
\tag{7}
$$

$$
\begin{aligned}
&P\left[N(t)=n \mid N(t_0)=0\right]= \\
&=P\left[N(t-t_0)=n\right]=\frac{\left[(t-t_0)/\mu\right]^n}{n!}\cdot e^{-(t-t_0)/\mu}
\end{aligned}
\tag{8}
$$

3 MARKOV RENEWAL PROCESSES

The models reviewed in this section are of particular earthquake engineering interest, as they allow modeling the relationship between the time and the magnitude of the earthquake (i.e., correlation between these RVs). Two simple examples of these Markov renewal processes (MRPs) are herein considered: the *time-predictable* and the *slip-predictable* models.

3.1 *Slip-predictable model*

The slip-predictable model (SPM), similarly to those in Section 2, may represent the case in which the stress accumulates starting from some initial level for a random period of time until an earthquake occurs (Kiremidjian and Anagnos, 1984).

Interarrival times are modeled as Weibull independent and identically distributed RVs. The PDF, along with mean and variance, are given in Equation (9), where b and $1/a$ are the shape and scale parameters, respectively.

$$\begin{cases} f_T(t) = (a \cdot b) \cdot (a \cdot t)^{b-1} \cdot e^{-(a \cdot t)^b} \\ E[T] = \dfrac{1}{a} \cdot \Gamma\left(1 + b^{-1}\right) \\ Var[T] = \left(\dfrac{1}{a}\right)^2 \cdot \left[\Gamma\left(1 + 2 \cdot b^{-1}\right) - \Gamma^2\left(1 + b^{-1}\right)\right] \end{cases} \tag{9}$$

In particular, in SPM, the magnitude (M) of the next event depends on the time since the last earthquake (Figure 4) via the functional relationship, $m = g(t)$, taken deterministic herein. Hence, assuming that the next event will occur in the interval (t_0, t), the PDF of M depends on t_0 and t as in Equation (10).

This will be more clearly addressed in the application discussed in Section 4; it is to note here, however, that the SPM implies to not assume a fixed threshold for earthquake-related energy release.

$$f_{M|N(t_0)=0 \cap N(t)=1}(m) = \\ = \begin{cases} \dfrac{f_T\left(g^{-1}(m)\right)}{\int_{t_0}^{t} f_T(z) \cdot dz} \cdot \dfrac{dg^{-1}(m)}{dm}, & m \in \left]g(t_0), g(t)\right] \\ 0, & m \notin \left]g(t_0), g(t)\right] \end{cases} \tag{10}$$

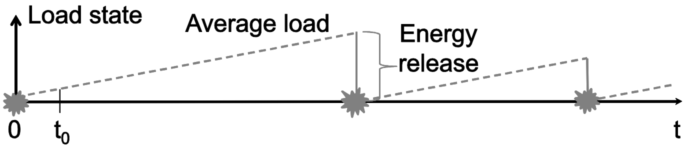

Figure 4. Loading and energy release in the SPM.

3.2 *Time-predictable model*

The time-predictable model (TPM) assumes that the time of occurrence of the next earthquake depends on the size and the time of occurrence of the last event (Anagnos and Kiremidjian, 1984). In fact, the larger the last earthquake, the longer is, on average, the time to the next event.

This hypothesis is different from the slip-predictable assumption, which implies that the size of the preceding event does not affect the occurrence time of the next earthquake.

TPM may represent the stress buildup until a threshold at which an earthquake occurs and a random part of the accumulated energy is released (Figure 5).

The magnitudes of events are assumed to be independent and identically distributed random variables. On the other hand, the interarrival times are Weibull-distributed RVs, conditional on the size of the last earthquake, Equation (11).

The PDF is the same as in Equation (9), except that its parameters depend on the magnitude, M_0,of the last event or, in other words, on time that is needed to accumulate sufficient stress to reach again the threshold.

$$f_{T|M_0}(t) = a_m \cdot b_m \cdot (a_m \cdot t)^{b_m - 1} \cdot e^{-(a_m \cdot t)^{b_m}} \tag{11}$$

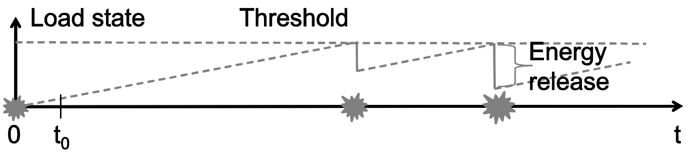

Figure 5. Loading and energy release in the TPM.

4 PROBABILISTIC SEISMIC HAZARD ANALYSIS IN THE CASE OF HYSTORY-DEPENDENT EARTHQUAKE OCCURRENCE PROCESS

Considering each of the models above, the probability that the ground motion intensity measure exceeds a certain threshold, at least once in the next $t - t_0$ years (with $t > t_0$) given t_0 years passed since the last event, indicated as $P\left[IM > im \mid N(t_0) = 0\right]$ for simplicity, can be written as in Equation (12). In the equation, the term $P\left[IM > im \mid m, r^*\right]$ represents the probability that intensity threshold is exceeded given an earthquake of magnitude m on the considered source. The latter is assumed to be separated from the site of interest by a distance equal to R; in the equation a fixed R value, r^*, is considered. This probability may be computed via ground motion prediction equations, or GMPEs.

$$
\begin{aligned}
&P\left[IM > im \mid N(t_0)=0\right] = \\
&= \sum_{n=1}^{+\infty} P\left[IM > im \mid N(t_0)=0 \cap N(t)=n\right] \times \\
&\times P\left[N(t)=n \mid N(t_0)=0\right] = \\
&\approx P\left[IM > im \mid N(t_0)=0 \cap N(t)=1\right] \times \\
&\times P\left[N(t)=1 \mid N(t_0)=0\right] = \\
&\approx P\left[IM > im \mid N(t_0)=0 \cap N(t)=1\right] \times \\
&\times P\left[N(t)\geq 1 \mid N(t_0)=0\right] = \\
&= P\left[IM > im \mid N(t_0)=0 \cap N(t)=1\right] \times \\
&\times \left\{1 - P\left[N(t)=0 \mid N(t_0)=0\right]\right\} = \\
&= \left\{1 - P\left[N(t)=0 \mid N(t_0)=0\right]\right\} \times \\
&\times \int_m P\left[IM > im \mid m, r^*\right] \cdot f_{M|N(t_0)=0\cap N(t)=1}(m) \cdot dm
\end{aligned}
\tag{12}
$$

It is to note that Equation (12) avails of some approximations allowed by results of Section 2. It was found that in most of the cases of engineering interest, the interval of concern is much smaller than the return period of the characteristic event. Therefore, the probability in question can be computed considering only one term. Furthermore, the probability of occurrence of one event is about equal to that of at least one event (see Figure 2), which is relatively easy to compute.

5 ILLUSTRATIVE APPLICATION

Hazard, in terms of peak ground acceleration (PGA), was computed according to the all reviewed models, considering the Paganica fault (central Italy) as a case-study (Figure 6). Hazard, here, is conditional on the time elapsed since the last event and its magnitude. Indeed, this kind of comparison is expected to highlight main differences among the reviewed models.

Models are calibrated so that they can be considered homogeneous only in terms of return period of an event of about M 6.3. Hence, more than on specific values of hazard, attention will be put on their trends.

In the case of BPT-, ERP-, and IG-RP, parameters of the interarrival time distributions were calibrated so that Tr is equal to 750 yr and the coefficient of variation is 0.43, which, according to Pace et al. (2006), characterize M 6.3 events on the Paganica fault.

For the HPP the magnitude distribution was taken as a truncated exponential defined in the $[5.8, 6.8]$ interval as in Equation (13), while the rate of occurrence of HPP-described earthquakes was assumed to be 1/750 event/yr, Figure 7.

$$
f_M(m) = \frac{2.49 \cdot e^{-2.49\cdot(m-5.8)}}{1-e^{-2.49\cdot(6.8-5.8)}} \tag{13}
$$

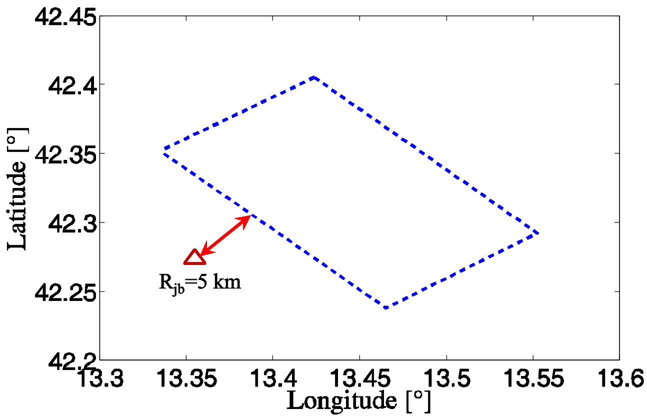

Figure 6. Source-site scheme.

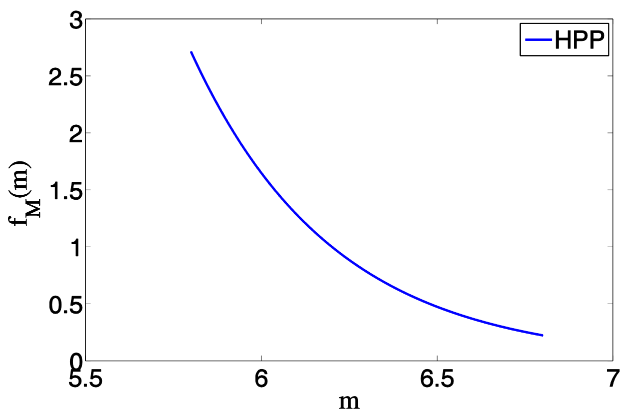

Figure 7. Exponential magnitude distribution for the HPP process on the fault.

For TPM, it was assumed that the last earthquake was a $M_0 = 6.3$ event, while M of the next characteristic event was considered to follow a truncated Gaussian distribution in the interval $[5.8, 6.8]$, that is, the mean value is set equal to 6.3, while a standard deviation equal to 0.1667 is adopted.

Finally, for SPM, all magnitudes were considered to be related to time of occurrence via Equation (14). A plot of the relationship is given in Figure 8.

$$
t = 0.039 \cdot 10^{0.68\cdot m} \tag{14}
$$

The interarrival time distribution was calibrated in such a way that the mean of the interarrival time is equal to 750 yr and CoV is still 0.43. This leads to the same parameters of the TPM interarrival time PDF; however, it is to underline that the return period of a M 6.3 event does not result in exactly 750 yr for this SPM, yet it is close to it. Indeed, even if 750 yr is the expected time to the next event, which can virtually be of any magnitude, such an event, by virtue of the time-magnitude relationship adopted, will be larger than M 5.8 with 0.91 probability.

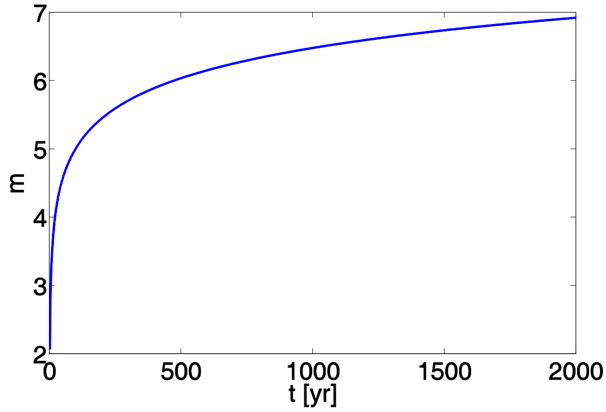

Figure 8. Time-magnitude relationship assumed.

In Table 1, the resulting parameters are given for all the models; note that in the Erlang case the mean value ant the CoV are slightly different because of the integer shape parameter. Figure 9 shows the PDFs (that in this section are all indicated as $f_T(t)$, also in the case of TPM) computed via these values.

Table 1. Parameters of time to next event PDFs.

Model	Distribution Parameters		Tr [yr]	CoV
BPT	μ = 750	α = 0.43	750	0.43
Erlang	k = 5	λ = 0.0072	693	0.45
IG	γ = 7.3	β = 4725	750	0.43
SPM	a = 0.00118	b = 2.5	752	0.43
TPM	$a_{6.3}$ = 0.00118	$b_{6.3}$ = 2.5	752	0.43

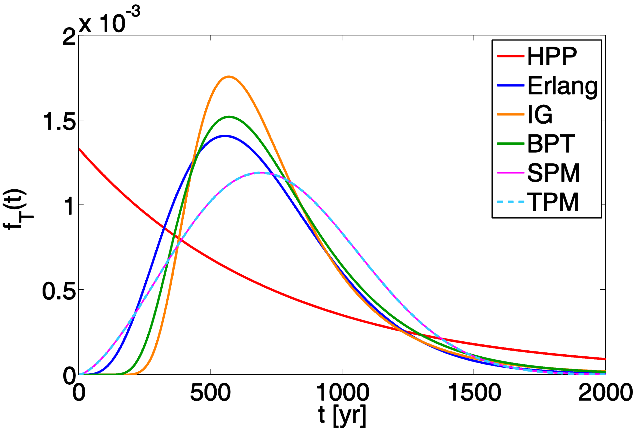

Figure 9. PDFs of interarrival time according to the considered processes.

5.1 *Results and discussion*

Figure 10 shows the probability of at least one event in a 50 yr time interval, calculated adopting $f_T(t)$ defined in Table 1.

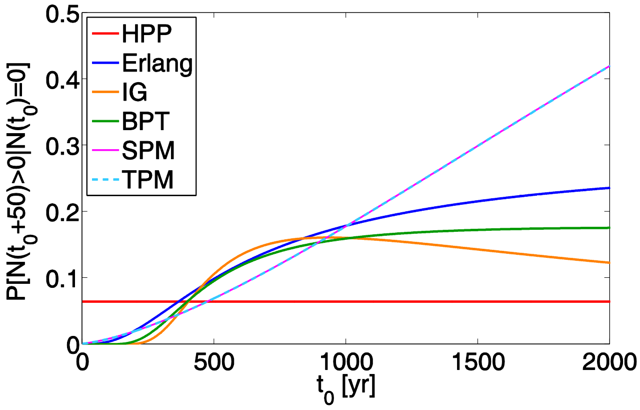

Figure 10. Probability of at least one event in 50 years as a function of the time since the last earthquake.

Trend observed in figure strictly depends on the shape of the hazard-rate function, Equation (15), which gives the instantaneous probability of an event occurrence given that no event had occurred until *t*.

$$h(t) = \frac{f_T(t)}{1 - F_T(t)} \tag{15}$$

It is noteworthy that, for some processes, after a certain time spent since the last earthquake, probability computed in figure tends to decrease. This depends on the fact that the hazard-rate function, associated to some of the considered $f_T(t)$, has a non-monotonic trend. In fact, as shown in Figure 11, BPT and IG models may have a non-monotonic hazard-rate functions that increase after the last earthquake, then decrease eventually (Matthews et al., 2002; Glen, 2011). Erlang RP with shape parameter $k > 1$ has a (bounded) increasing hazard-rate. Finally, SPM and TPM, with shape parameter of the Weibull distribution $b > 1$, feature a diverging hazard-rate (Matthews et al., 2002). HPP has a constant hazard rate which is 1/750.

To compute seismic hazard expressed in terms of probability of exceedance of an IM-value in 50 yr, the approximation in Equation (12), whose suitability was shown for the Erlang renewal model, was assumed for all the other processes because of the similarity of the PDFs of the time to the next event (Figure 9).

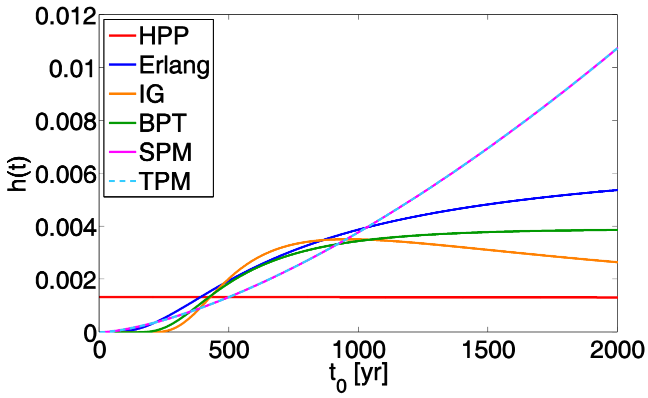

Figure 11. Hazard rate function for the different models.

To evaluate the $P\left[IM > im \mid m, r^{*}\right]$ term, the Sabetta and Pugliese (1996) GMPE was considered. The site was set at fixed R_{jb} distance (Joyner and Boore, 1981) equal to 5 Km (Figure 6).

In Figure 12 the probability that the PGA exceeds a certain threshold is plotted versus the time passed since the last event. The IM-threshold was assumed, as an example, equal to 0.447g. It is the median PGA given M 6.3 and r^* = 5 km for shallow alluvium site according to the considered GMPE.

All history-dependent models, especially RPs, provide similar results for a time spent since the last earthquake of about one half of the return period of the event. Conversely, probabilities start to be increasingly different as t_0 gets significantly large. This may render critical the selection of which one of the models to choose for a specific fault when the last known event is not recent.

The non-monotonic hazard-rate function of some of them also shows up in the results given in Figure 12, which indicates that the probability of exceedance of IM may decrease after a certain time since the last event, a behavior that may not be easy to justify.

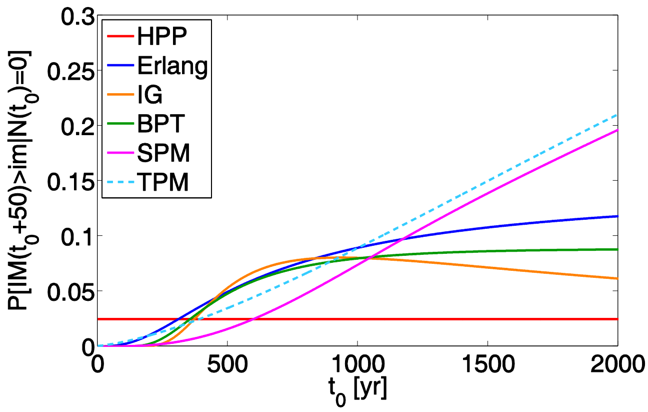

Figure 12. Hazard for PGA = 0.447 g.

6 CONCLUSIONS

The memory-less homogeneous Poisson process, where interarrival times are independent and identically distributed exponential random variables, is often used in hazard assessment for engineering seismic risk analysis. However, when a single fault is of concern and/or the time scale is different from that of the long term, history-dependent processes may be considered. In this paper, models for mainshock occurrence on an individual source, were reviewed with working examples. The models considered refer to the renewal, and Markov renewal point processes.

The Paganica fault (in central Italy) was considered to compute both the probability of occurrence of one event in the time interval of interest, as well as the seismic hazard, expressed in terms of (conditional) probability of exceedance of an intensity value in a given time-frame.

The magnitude is considered to be that of characteristic events, that is when the considered source generates almost fixed-magnitude earthquakes. To homogenize the models, these were calibrated to have mean and variance of time to next event distributions as similar as possible.

Considering the time intervals of common engineering interest, it was assumed that the probability of more than one event is negligible (showed for the Erlang renewal process), simplifying hazard calculations.

It was also observed that because of the hazard-rate function, some processes show a decreasing probability of occurrence after a certain time has passed since the last event. This appears not to be the result of explicit representation of actual earthquake physics, while rather a collateral effect of the mathematics of the assumed models.

Engineering hazard analysis show that history-dependent models have a similar trend, especially renewal processes, until a time of about a half of the mean return period of the event, and that the results from all models tend to relatively diverge as the elapsed time since the last event increases.

This means that the longer is the time spent since the last known earthquake on the source, the more critical is the selection of the process which is considered to be appropriate to represent earthquake occurrence.

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REFERENCES

Anagnos, T. and Kiremidjian, A.S. 1984. Stochastic time-predictable model for earthquake occurrences. *Bulletin of the Seismological Society of America* 74(6): 2593-2611.

Anagnos, T. and Kiremidjian, A.S. 1988. A review of earthquake occurrence models for seismic hazard analysis. *Probabilistic Engineering Mechanics* 3(1): 3-11.

Cornell, C.A. and Winterstein, S.R. 1988. Temporal and magnitude dependence in earthquake recurrence models. *Bulletin of the Seismological Society of America* 78(4): 1522-1537.

Glen, A.G. 2011. On the Inverse Gamma as a Survival Distribution. *Journal of Quality technology* 43(2): 158-166.

Joyner, W.B. and Boore, D.M. 1981. Peak horizontal acceleration and velocity from strong motion records including records from the 1979 Imperial Valley, California, earthquake. *Bulletin of the Seismological Society of America* 71:2011-38.

Jordan, T., Chen, Y., Gasparini, P., Madariaga, R., Main, I., Marzocchi, W., Papadopoulos, G., Sobolev, G., Yamaoka, K. and & Zschau, J. 2011. *Operational earthquake forecasting. State of Knowledge and Guidelines for Utilization.* Annals of Geophysics 54(4): 319-384.

Kiremidjian, A.S. and Anagnos, T. 1984. Stochastic slip-predictable model for earthquake occurrences. *Bulletin of the Seismological Society of America* 74(2): 739-755.

Matthews, V.M., Ellsworth, L.W. and Reasenberg, A.P. 2002. A Brownian model for recurrent earthquakes. *Bulletin of the Seismological Society of America* 92(6): 2233-2250.

McGuire, R.K. (2004). Seismic Hazard and Risk Analysis, *Earthquake Engineering Research Institute*, MNO-10, Oakland, California, 178 pp.

Pace, B., Peruzza, L., Lavecchia and G., Boncio P. 2006. Layered seismogenetic source model and probabilistic seismic-hazard analyses in central Italy. *Bulletin of the Seismological Society of America* 96(1): 107-132.

Pandey, M.D. and van Noortwijk, J.M. 2004. Gamma process model for time-dependent structural reliability analysis. In E. Watanabe, D. M. Frangopol, and T. Utsonomiya, editors. Bridge Maintenance, Safety, Management and Cost, Proceedings of the Second International Conference on Bridge Maintenance, Safety and Management (IABMAS), Kyoto, Japan, Taylor & Francis Group, London.

Reid, H.F. 1910. *The Mechanics of the Earthquake, The California Earthquake of April 18, 1906*, Report of the State Investigation Commission, Vol.2, Carnegie Institution of Washington, Washington, D.C.

Sabetta, F. and Pugliese, A. 1996. Estimation of response spectra and simulation of nonstationarity earthquake ground motion, *Bulletin of the Seismological Society of America* 86: 337–352.

Takahashi, Y., Der Kiureghian, A. and Ang, H-S A. 2004. Life-cycle cost analysis based on a renewal model of earthquake occurrences. *Earthquake Engineering Structural Dynamics* 33: 859-880.

Yeo, G.L. and Cornell, C.A. 2005. *Stochastic Characterization and Decision Bases under Time-Dependent Aftershock Risk Performance-Based Earthquake Engineering*, PEER Report 2005/13. Pacific Earthquake Engineering Research Center, Berkeley, CA.