Bull Earthquake Eng (2010) 8:955–972
DOI 10.1007/s10518-010-9176-8

ORIGINAL RESEARCH PAPER

Elastic period of sub-standard reinforced concrete moment resisting frame buildings

Gerardo M. Verderame · Iunio Iervolino · Gaetano Manfredi

Received: 1 May 2009 / Accepted: 21 February 2010 / Published online: 10 March 2010

Abstract The fundamental period has a primary role in seismic design and assessment as it is the main feature of the structure that allows one to determine the elastic demand and, indirectly, the required inelastic performance in static procedures. In fact, the definition of easy to manage relationships for the assessment of the elastic period has been the subject of a significant deal of both experimental and numerical/analytical studies, some of which have been acknowledged by codes and guidelines worldwide. Moreover, this kind of information is useful for territorial-scale seismic loss assessment methodologies. In the majority of cases, the assessment of the period is considered as function of the structural system classification and number of storeys or height. Reinforced concrete structures, comprising most of the building stock in Italy and in seismic prone areas in Europe and in the Mediterranean region, were built after the Second World War and are designed with obsolete seismic codes, if not for gravity loads only. Therefore, a class of buildings featuring the same height and/or number of storeys may show a significant variability of the structural system. This, along with the contribution of the stair module, may affect the elastic periods in the two main directions of a three-dimensional building. In the study presented these issues are investigated with reference to a population of existing RC structures designed acknowledging the practice at the time of supposed construction (e.g., simulated design) and with reference to the relative enforced code. The elastic period is evaluated for both main directions of the buildings of the considered sample, and regression analysis is employed to capture the dependency of the elastic dynamic properties of the structures as a function of mass and stiffness.

Keywords Elastic period · Sub-standard · Reinforced concrete · Buildings · Stairs

G. M. Verderame (✉) · I. Iervolino · G. Manfredi
Dipartimento di Ingegneria Strutturale, Università degli Studi di Napoli Federico II, Via Claudio 21, 80125 Naples, Italy
e-mail: verderam@unina.it

1 Introduction

In static procedures for seismic structural assessment, the fundamental period is one of the global characteristics to determine effects of seismic action expressed in terms of horizontal forces. In the case of dynamic procedures (e.g., nonlinear) it is necessary to select the appropriate hazard information and input ground motions, especially for first-mode dominated structures. Generally speaking, the period relates seismic demand to capacity allowing to determine the seismic performance and therefore the safety level.

Most of seismic codes worldwide propose easy-to-apply relationships to determine the elastic period as a function of height or number of storeys given the structural typology (SEAOC 1996; CEN 2005). Such relationships, especially those for reinforced concrete (RC) moment-resisting frames (MRF), have been calibrated on experimental studies, which have become a standard reference at an international level (ATC 1978; Goel and Chopra 1997). These important studies are based on seismic monitoring of buildings subjected to seismic actions, eventually repeated because more than one earthquakes hit the structure, in significantly seismic prone areas in which earthquake resistant design has been well established for a long time.

Recently, research effort was devoted to the attempt of developing similar relationships for European "typical" frames (not buildings), for the so-called effective- or yield-period, which is that of interest for determining the non-linear demand in those cases where the capacity derives from static push-over analysis (Crowley and Pinho 2004). This period, which is larger than the elastic one, has more to do with the yielding and/or cracked stiffness of the structure and is more easily evaluated via analytical/numerical procedures, although the results strictly depend on the structural modeling assumptions.

It is well known that the existing RC buildings in Italy and all of the Mediterranean region, a significant portion of the building stock, have been designed and constructed mainly after the second world war (e.g., in the 1940–1980 period), when only a fraction of the territory was considered as seismically prone. Design was mainly carried out for gravity loads only, and also when consideration of seismic action was required, it resulted in the application of period-independent horizontal forces without regard to *capacity design*, which is a fundamental concept of earthquake resistant design nowadays.

A class of gravity-load designed buildings may feature a heterogeneous structural system because the plan distribution of resisting frames may not follow the regularity principle established by modern seismic design guidelines. It is also useful to point out that it may be not appropriate, when analyzing these buildings, to refer to frames as the two main directions may have dissimilar dynamic properties reflecting the variability of the structural system. Moreover, the stair-module cannot also be neglected when investigating this issue. Similarly, the "seismic" buildings of the decades considered, although presenting a more rational structural system with respect to seismic actions, are expected to not show the stiffness and regularity features of a modern earthquake resistant building.

The study presented in this paper investigated these issues for classes of existing RC buildings and how they reflect on the elastic properties. In particular, the variability of the elastic period in the two directions is assessed analytically with respect to the variability of some parameters characterizing the structural configuration. To this aim rectangular buildings with a number of storeys between 2 and 8, which are very common in Italy, have been considered. The buildings are bare-frames for which the stair is also considered, thus the effects of other factors as infillings and finishes are not considered.

The study refers to the numerical analysis of classes of buildings (or populations) the models of which were built by simulated design. In other words, design was carried out via an automatic procedure which implements the design rules and professional practice at the time when the buildings are supposed to date (Verderame et al. 2010). In particular, this study refers to Italian design principles, which represent the European and Mediterranean practice (Bal et al. 2007; Verderame et al. 2009).

Two different populations of buildings have been considered: (1) gravity-loads only; and (2) sub-standard seismic design accounting for seismic action via statically-equivalent horizontal forces. Therefore, two different kinds of design were carried out accordingly. In the case (2) three different intensities of seismic actions were considered compatible with the classification enforced in Italy between 1935 and 1975. In fact, the first Italian seismic code (R.D.L. 640 1935) divides the National territory in two zones assigning 0.10 and 0.07 g as reference horizontal accelerations for design (*seismic coefficients* in the following). The latter value, associated to the second category, was subsequently changed into 0.05 g (R.D.L. 2105 1937). According to the code, the buildings constructed in the first category associated to the larger acceleration cannot be over 16 m in height, and 20 m in the second, except in special cases. More recent codes (Legge 1684 1962) adopted back the 0.07 g criteria for the second category, increasing the maximum admissible height to 24.5 m. During this period, the seismic actions were represented by a distribution of forces proportional to storey mass and design acceleration. Only in 1975 (D.M. 1684 1975) the linear distribution of forces was introduced.

These two populations have been split into 14 classes of buildings with a fixed number of storeys. The periods in the two main directions have been regressed versus the height, as suggested by codes and existing literature on the topic, to compare and to see how much of the variability is captured by the independent variable. Subsequently, other covariates which may explain the variability of mass and stiffness, related to the global dimensions of the building, have been included to better assess the influence on the elastic period of the design practice and structural characteristics.

In the following, after a state-of-the-art review of simplified relationships for elastic period estimation for RC buildings, the analytical procedure considered is described along with the population of buildings considered as a sample for the analyses. Finally, the results are presented and discussed with the aim of assessing the elastic period and explaining its variability with respect to existing and code approaches.

2 Research background

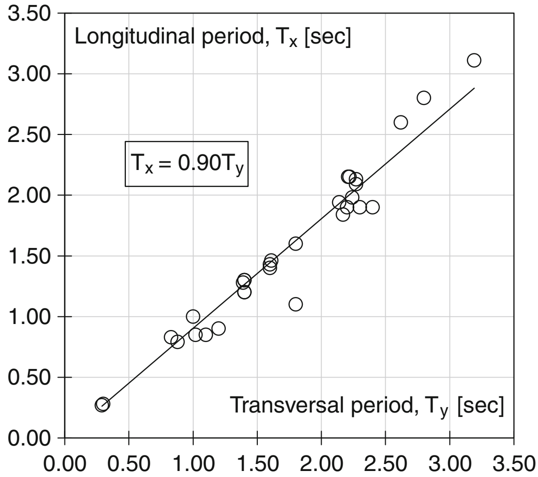
The period of vibration depends on those factors directly affecting structural mass and stiffness. Globally, proxies for the mass may be global dimensions of the building (e.g., plan area and number of floors); the stiffness may be related to structural features and height.

Most of the relationships to estimate the period are a function of global height (H) as it is a simple parameter, known before detailed design, which may explain the ratio between stiffness and mass of the building. Herein, the most common formulation proposed by research for MRF are discussed; the background on period formulas also for other structural typologies (e.g., buildings with concrete shear walls) may be found in Crowley and Pinho (2009).

The formulation of period-height relationships is typically of the type in Eq. (1) where α depends on the structural system.

$$T = \alpha H^{\beta} \tag{1}$$

Fig. 1 Correlation of elastic periods measured in the two main directions of the buildings of the study from Goel and Chopra (1997)



It appeared first in ATC3-06 (ATC 1978) with β equal to 0.75, while α was calibrated as 0.06 (if H is measured in meters or 0.025 if it is in feet), based on periods measured on some buildings during the 1971 San Fernando earthquake.

A similar relationship may be computed via the Rayleigh method (Chopra 1995) with the following seismic design assumption: (1) horizontal forces linearly distributed along the height of the building; (2) mass distribution constant along the height, (3) linear deformed shape; (4) base shear proportional to $1/T^{\gamma}$. If these conditions are met the period is expressed as:

$$T = \alpha H^{1/(2-\gamma)} \tag{2}$$

If γ equals 2/3, as established in US codes (UBC 1997):

$$T = \alpha H^{0.75} \tag{3}$$

In SEAOC-88 commentary (SEAOC 1996) α is 0.073 (if H in measured in meters or 0.030 if it is in feet). The formulation with these values of the parameters was adopted by International codes such as Eurocode 8 (CEN 2005) by rounding α to 0.075.

Alternatively, NEHRP (1994) includes a relationship as a function of the number of storeys (N), $T = 0.1N$, limited to buildings up to 12 storeys with inter-storey height not smaller than 3 m. This relationship was frequently adopted by codes before Eq. (1).

More recently, calibration of coefficients has been based on experimental data; e.g., via the monitoring of buildings during earthquakes. Goel and Chopra (1997), collected data on 37 reinforced concrete buildings, featuring seismic design and with height ranging from 10 to 100 m. For each of the buildings the periods in the two principal directions were measured. Results show that the periods in the two directions are very similar, showing an average 10% difference, see Fig. 1. This may most likely be attributed to earthquake resistant design of the buildings, which should give uniform lateral stiffness in both directions.

Note that this kind of approach renders, of course, the estimation of period depending on the history of the shaking at the site for each structure for two reasons: (1) if the shaking is strong enough to crack the structure the period measured is longer than if the structure remains uncracked; (2) if the buildings are subjected to multiple earthquakes the period

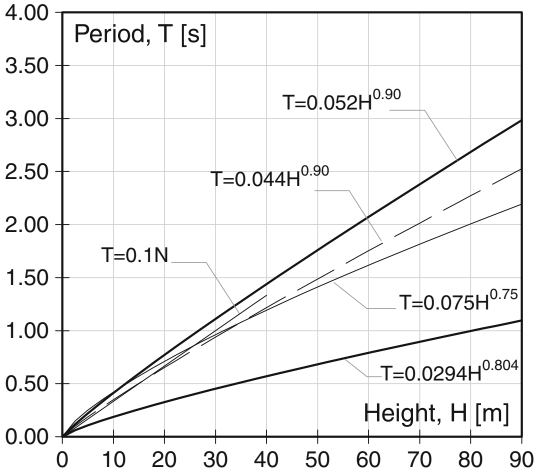


Fig. 2 Literature period–height relationships

measured after the first cracking ground motion is always related to cracked stiffness also in subsequent lower intensity shaking.

The buildings of that study were subjected to eight main Californian earthquakes from San Fernando (1971) to Northridge (1994). According to the authors, 22 buildings experienced a peak ground acceleration (PGA) lower than 0.15 g, while the others were subjected to larger acceleration at the base. As expected, the latter buildings show a larger period given height.

Comparing experimental results to those deriving from Eq. (3) an underestimation of the period is observed, especially for larger height buildings and for those which experienced a PGA larger than 0.15 g. Therefore alternative formulas were proposed resulting from a semi-empirical analysis. One of those, Eq. (4), features the best fit coefficient for Eqs. (1), (5), is that fitting data plus one standard deviation; and Eq. (6) is that obtained by conservatively fitting at minus one standard deviation and, therefore, is proposed for estimation of the period in seismic design.

$$T = 0.052H^{0.9} \tag{4}$$

$$T = 0.065H^{0.9} \tag{5}$$

$$T = 0.044H^{0.9} \tag{6}$$

A similar study was carried out by Hong and Hwang (2000) for 21 RC seismic buildings in Taiwan subjected to four events claimed to not yield the structures. The coefficients proposed in that study lead to the following expression:

$$T = 0.029H^{0.804} \tag{7}$$

Comparing semi-empirical relationships significant differences in estimations are found. In Fig. 2 such comparison is given, and it may be noted how the estimation according to Eq. (4) leads to a 130% average overestimation with respect to Eq. (7). Such a difference may be related to the different design criteria and construction practice in the two countries. Closer agreement is found between code-suggested relationships (i.e., Eqs. (3), (6) and that as a function of the number of storeys) for $H \leq 40$ m, which is the range of interest for the building stock in southern Europe (e.g., Italy).

From the comparison it may be argued that the calibration of Eq. (1), via numerical or experimental approaches, is affected by the assumptions on the dynamic response and on characteristics of seismic design. It is, therefore, necessary to investigate whether period-height relationships for sub-standard seismic design or gravity loads design buildings, could lead to different estimation with respect to that of codes and/or existing literature. These may be common conditions for existing buildings and, therefore, it is the focus of the following analyses.

3 Sub-standard RC buildings

Existing RC buildings do not reflect regularity of strength and stiffness which a building conceived with capacity-design principles shows. In fact, most of them may be considered sub-standard because they are designed for gravity loads only in areas subsequently considered seismically hazardous, or because they are designed with inadequate seismic provisions. Herein, samples from both categories are considered, and it is necessary to underline that there may be significant structural differences between the two. Design for gravity loads does not require plan regularity of the structural system and therefore these buildings show non-uniform distribution of the resisting substructures, which may lead to different elastic responses in the two main directions. In other words, if the frames are considered as the substructures resisting horizontal forces it has to be recalled that, in a rectangular building, columns may still be the nodes of a regular grid (although some architectural requirements may let this distribution depart from regularity), but the location of beams is driven by the principal direction of the slab on which vertical loads act (typical Italian slabs are constituted by unidirectional RC joint-slabs). Therefore, the number and orientation of frames is determined by elements carrying gravity loads. For example, in Fig. 3 sketches of two case studies are given (Cosenza et al. 2002); they clearly show how Italian existing buildings may have a non-uniform distribution of resisting systems.

Building designed also accounting for horizontal forces show a more regular distribution of frames because the action was assumed to be equal (and period-independent) in the two directions. This leads to a first three-dimensional conception of the structure with frames specifically devoted to resist seismic actions.

Within this framework, load design was carried out with approximate methods. In the gravity-load design, simplified schemes instead of plane frames were used. In fact, columns were proportioned based on the intensity of the axial load only, while the beam design reflects the continuous multiple-supports scheme. On the other hand, seismic load effects are evaluated via frame modeling, although the distribution of seismic actions to elements is still approximated by being based on the floor mass distribution or on the columns' inertia, the latter known as a shear-type model.

Seismic actions during the considered time span were determined imposing horizontal forces depending on fixed seismic coefficients: 0.10, 0.07, and 0.05 g depending on the assumed seismic potential of the site; in other words, period-independent seismic forces.

Design criteria did not consider limit-states expressed in terms of maximum inter-storey drift ratios as modern codes do, therefore, the storey-stiffness indirectly depends on the lateral load resistance which should be provided as a consequence of the horizontal forces. Therefore the following structural features are expected for this kind of buildings:

- existing buildings, independently of gravity loads or seismic design, feature a lower lateral stiffness with respect to modern code-designed structures;
- among existing buildings, those provided with seismic design have a larger global stiffness with respect to those designed for gravity-loads only, as horizontal forces should imply larger dimensions of elements constituting the seismic resisting systems (i.e., frames);

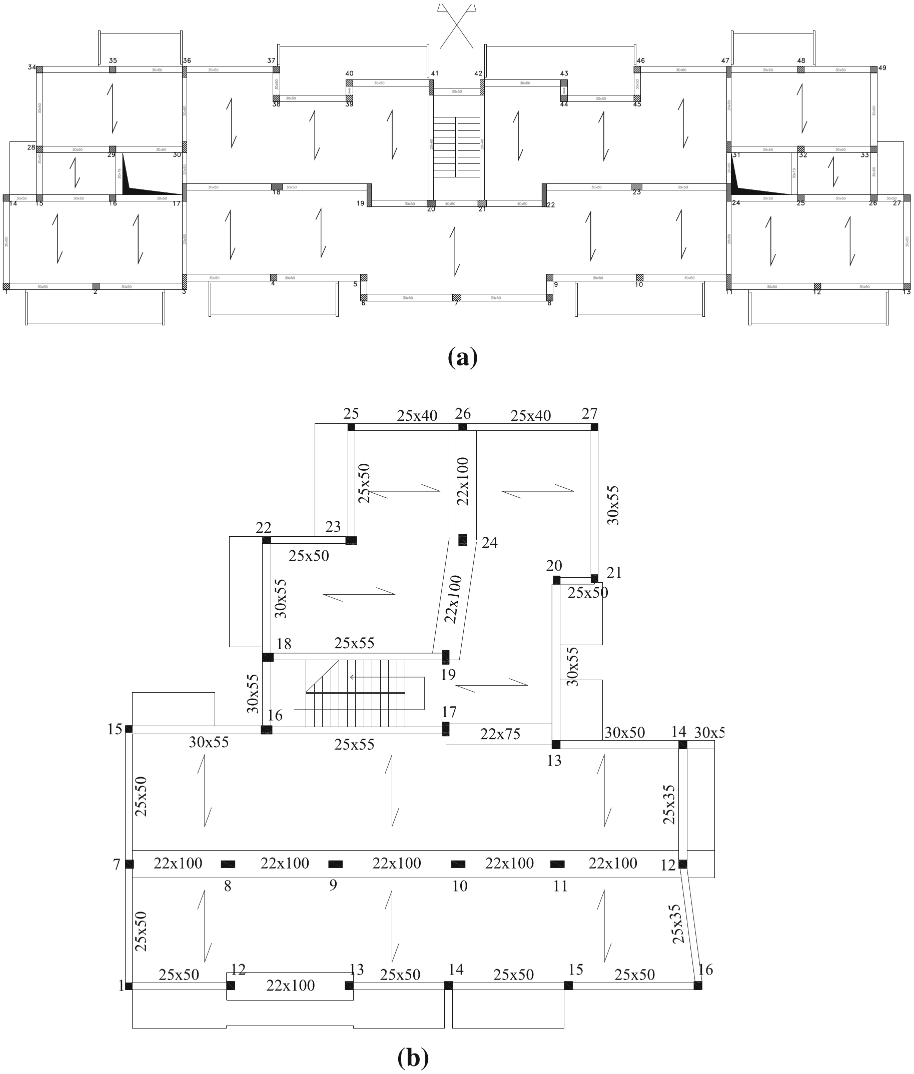


Fig. 3 Examples of gravity loads designed buildings: **a** rectangular-shaped (Cosenza et al. 2002), **b** T-shaped

- the eventual variability of structural system in gravity loads design may lead to significant differences in the fundamental period in the two principal directions of these kinds of buildings.

These issues are investigated in the following referring to a widely diffused typology of existing buildings in Italy, built approximately between 1950 and 1975.

4 Methodology

The investigation of the elastic dynamic features of the buildings described above as a function of various types of mechanical parameters was based on the simulated design of a population of buildings, which is made of the following phases:

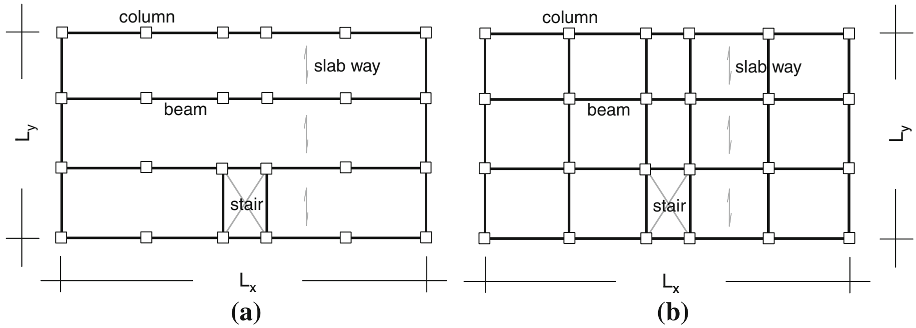


Fig. 4 Structural configurations of the considered building types: gravity-loads designed (**a**) and seismic design (**b**)

(i) *building definition*—depends only on the 3D dimension of the building;
(ii) *identification of possible structural system for the building*—at this stage the structural parameters, number of frames, bay lengths, and column orientation are defined;
(iii) *simulated design of the structural systems*—in terms of cross sections' dimensions and reinforcements, both longitudinal and transversal, of the elements;

In the case of gravity load design, the number of frames is determined by the distribution of vertical forces only, while in the case of seismic design additional frames derive from the lateral forces. Design of the elements is carried out referring to the recommendations of the codes enforced at the time in terms of reinforcement rations, material design strengths and so on, see Verderame et al. (2010) for details.

It is to note that, steps (ii) and (iii) lead to multiple structures being associated to a single building as several structural systems and design alternatives correspond to the same global dimensions.

4.1 Considered buildings' population

The considered buildings refer to a rectangular plan shape and a moderate number of storeys. In this type of buildings, typically, there are one or two units for each floor and one a stairway which is assumed to be centered with respect to the longitudinal direction of the building (the longer one). In generating the population of analyzed buildings the considered variable parameters are its longitudinal length (L_x), transversal length (L_y) and the global height (H) excluding the foundations. Interstorey height is constant and equal to 3.0 m.

The structural configuration adopted in the simulated design refers to structures which have to carry gravity loads only, subsequently adjusted (by adding frames) to also withstand horizontal actions. Frames, to support the slabs, are all to be erected in the same direction, which is typically the transversal one (the shorter) of the building, this leads to the definition of this resisting system as *parallel plane frames*. In the transversal direction, then, only two frames at the two ends of the building and the stair module exist. In Fig. 4a this scheme is illustrated. On the other hand, the structural configuration in the case of seismic design corresponds with an integration of the former with multiple frames in the same direction, one per bay (Fig. 4b).

For this kind of building the bay lengths have been assumed to be comprised in the 3.0–5.0 m range; this leads to a variability of the structural configuration within the population.

Gravity loads design was carried out according to the design values of dead and live loads of the codes in the time-span considered. As discussed, the design of elements refers to simple sub-structuring schemes in which the design of columns is driven by axial load and design of beams refers to a multiple supports model (or continuous beam).

Seismic design was carried out by static linear analysis, the most common tool at the time (and still widely used today by practitioners). Three seismic coefficients are considered for seismic design equal to 0.10 g, 0.07 g e 0.05 g according to the evolution of seismic classification of the territory in Italy up to 1975. The seismic forces distribution considered is that proportional to storey masses (R.D.L. 640 1935; R.D.L. 2105 1937; Legge 1684 1962). The static location of horizontal forces refers to the flexible-slab assumption; in the transversal direction the contribution of the stair is not considered.

The design of cross-sections is carried out in the *allowable stresses* framework. The design values were $\sigma_c = 5.0$ and $\sigma_c = 7.5$ MPa for axial load and axial load combined with bending, respectively (R.D.L. 2229 1939). The cylindrical compressive mean strength of concrete f_c is constant among the structures and equal to 15 MPa.

All cross sections in the beams have the same 30 cm basis and a height in the (30cm÷50cm) range. The columns may have $(30 \times 30)\text{cm}^2$ to $(45 \times 75)\text{cm}^2$ sections; this latter value associated to the first floor columns of the higher buildings.

The ranges of variability of building dimensions considered is that as follows:

- longitudinal length: $L_x = [15.0, 20.0, 25.0, 30.0]$ m;
- transversal length: $L_y = [8.0, 10.0, 12.0]$ m;
- building height (H) is comprised between $(6.0 \div 24.0)$ m corresponding to 2–8 storeys.

All possible combinations of these values lead to 84 buildings for each of the two possible design categories (*seismic* and *gravity loads*); while the structural configuration variability leads to 175 structural systems, two per building on average.

The allowable stress design leads to proportions of the elements which are the smallest admissible to comply with the actions induced by gravity and seismic loads. Therefore, the lateral stiffness in the two directions is generally to be considered a lower bound.

Considering also the four design options (one gravity loads and three seismic design levels) a population of 700 structures was analyzed. In particular, two linear analyses have been carried out for each structure to investigate the elastic period in the two principal directions of the buildings to which the structure considered corresponds.

5 Implications of structural features on elastic properties

The elastic characteristic of the different directions of each considered structures are evaluated via the classical eigenvalues analysis, i.e., Eq. (8):

$$[\mathrm{K}] - \omega^2\,[\mathrm{M}])\,\{\varphi\} = \{0\} \tag{8}$$

where [K] is the stiffness matrix of the MDOF structural system, [M] is the seismic storey masses matrix, $\{\phi\}$ is the displacement vector of the vibration mode, and ω is the associated circular frequency. [K] is determined starting from the cross section stiffness ($\mathrm{E_c I}$), which has been evaluated referring to the inertia (I) of the uncracked section and the elastic modulus of concrete, $\mathrm{E_c}$, defined as in Eq. (9), from Eurocode2 (CEN 2004), in which f_c is the cylindrical compressive strength of concrete expressed in MPa .

$$\mathrm{E_c} = 22000\,(f_c/10)^{0.3}\ (\mathrm{MPa}) \tag{9}$$

The storey masses in the diagonal [M] matrix have been evaluated according to Eurocode 8 based on the analysis of dead and live loads for the structure for which the elastic periods are evaluated.

From Eq. 8 the ω_i, $\{\phi\}_i$, and m_i^*, associated to the i-th mode defined as the *effective mass* and evaluated as:

$$m_i^* = \sum m_k \underset{k,i}{\phi} \tag{10}$$

where m_k is the seismic mass of the k-th storey, $\phi_{k,i}$ is the displacement of the k-th floor in the i-th mode.

The periods considered are those corresponding only to the fundamental modes, primarily translational, associated to the principal directions of the structure. Therefore, for each of the two directions the fundamental $\{\phi\}$, ω, and m^* parameters are determined and the corresponding elastic period, T_{el}, is defined as:

$$T_{\mathrm{el}} = \frac{2\pi}{\omega} \tag{11}$$

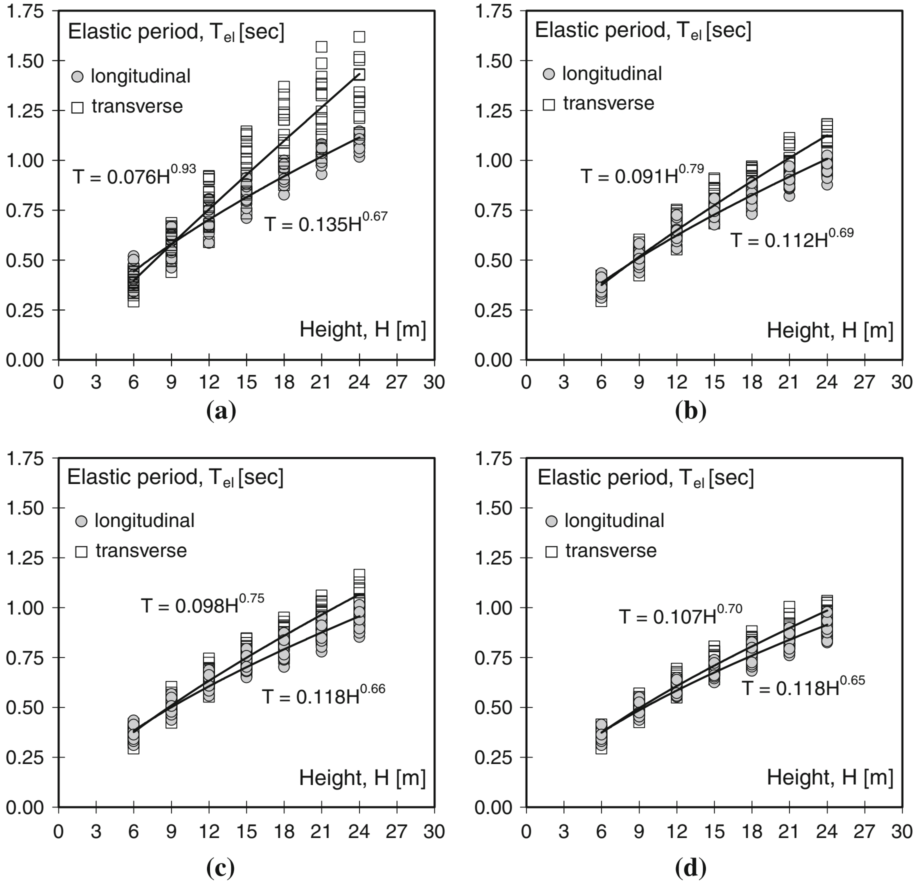


Fig. 5 Period-height relationships for the analyzed populations: **a** gravity-loads design, **b** seismic design (0.05 g), **c** seismic design (0.07 g) and **d** seismic design (0.10 g)

Figure 5 shows the trends of the periods as a function of the height for the four categories of buildings investigated. As expected the design options affect the elastic periods. In fact, those corresponding to gravity loads design are generally smaller than those of structures proportioned to resist also to horizontal force, among which to the higher reference acceleration corresponds to the lower natural periods. Moreover, the longitudinal direction is generally stiffer than the transverse, although the transverse to longitudinal period ratio is larger in the case of design for gravity loads with respect to seismic.

5.1 Gravity loads designed buildings

In these structures, the period in the short direction is more variable with respect to the longitudinal one at a given height. This depends on the particular structural configuration, Fig. 4a, which significantly affects, the ratio between *effective mass* m^* and *lateral stiffness*, K_{el}, of the building. In fact, in Fig. 6, the trends of these quantities are given as a function of the height for the two directions.

The effective mass relates almost linearly with the height, and it is variable in both directions because of the ranges of global dimensions considered, L_x and L_y, which lead to a wide range of plan areas for the populations analyzed, as can be observed in Fig. 7a. In general, the buildings feature effective masses similar in the two directions, therefore the differences in elastic periods depend on the different lateral stiffness.

As observed in Fig. 6, the long direction has lateral stiffness larger than that in the short direction; this discrepancy increases with height and is up to 50% for buildings with more than 3 storeys. Vice-versa, the longitudinal direction has a larger variability of stiffness with respect to the transversal one. This has to be attributed to the different lateral resisting system of the two directions (Fig. 4a).

The stair sub-structure significantly affects the stiffness in the short direction. Despite the fact that this direction has only the two frames on the perimeter, this is magnified in buildings with three storey or less (H equal or less than 9 m), for which the transverse stiffness ins larger than the longitudinal. The stair module effect, rapidly decreases with height.

The variability of the area covered by the buildings is differently reflected in the elastic properties of the two directions (Fig. 7b). In the longitudinal direction the increase of L_x implies an increase in the number of bays, while an increase in L_y leads to an increase in the number of longitudinal frames. Therefore, the longitudinal direction has an increasing stiffness with area. Vice versa, the resisting system in the transversal direction is only marginally affected by an increase in the area. In fact, it does not imply modification in the stair module and the slight trend observed is due to the variation of number of bays and of the proportions of the elements of the two frames on the perimeter.

It may be concluded that for gravity loads buildings, the stair module plays a determinant role in the determination of the transversal period. The lower stiffness in the short direction with respect to the longer and its comparatively small variability given height clarify the trend of the period reported in Fig. 5a.

5.2 Seismic buildings

Seismic buildings have a resisting system also in the transversal direction and are also characterized by a more uniform distribution of structural sub-systems, e.g., Fig. 4b. This is reflected in different trends of stiffness with respect to gravity loads designed buildings.

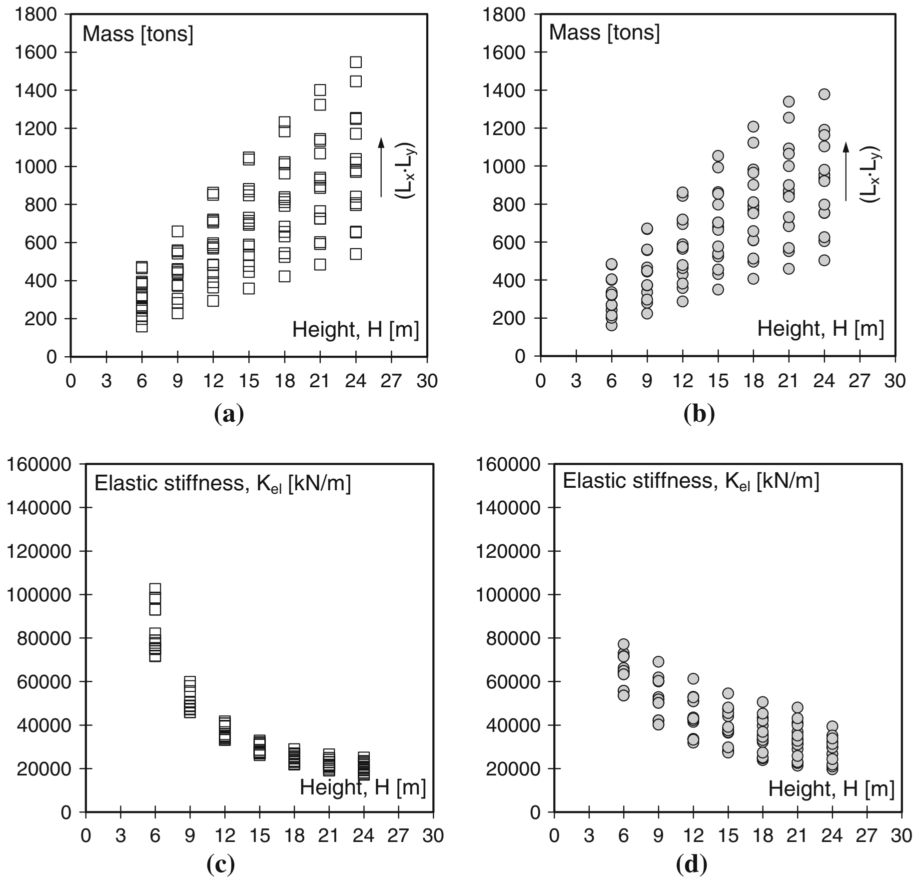


Fig. 6 Gravity-loads design: **a** transverse elastic stiffness versus height; **b** longitudinal elastic stiffness versus height; **c** transverse effective mass versus height; **d** longitudinal effective mass versus height

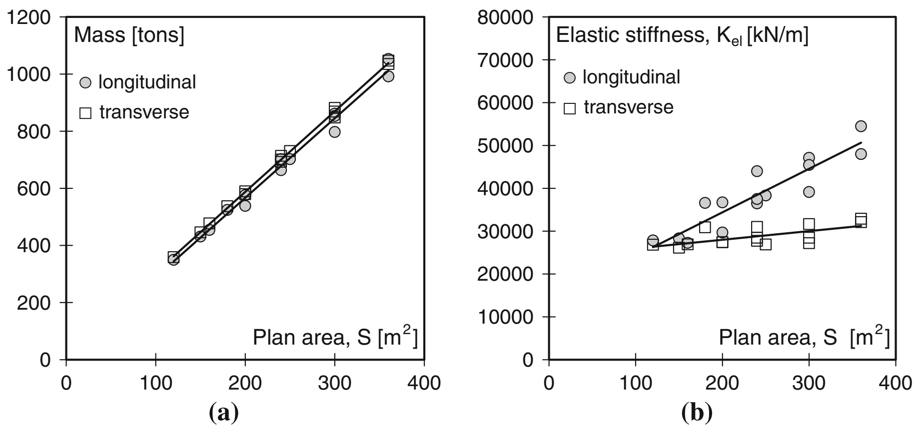


Fig. 7 Effective mass (**a**) and elastic stiffness (**b**) versus area for a 15.0 m buildings designed for gravity loads only

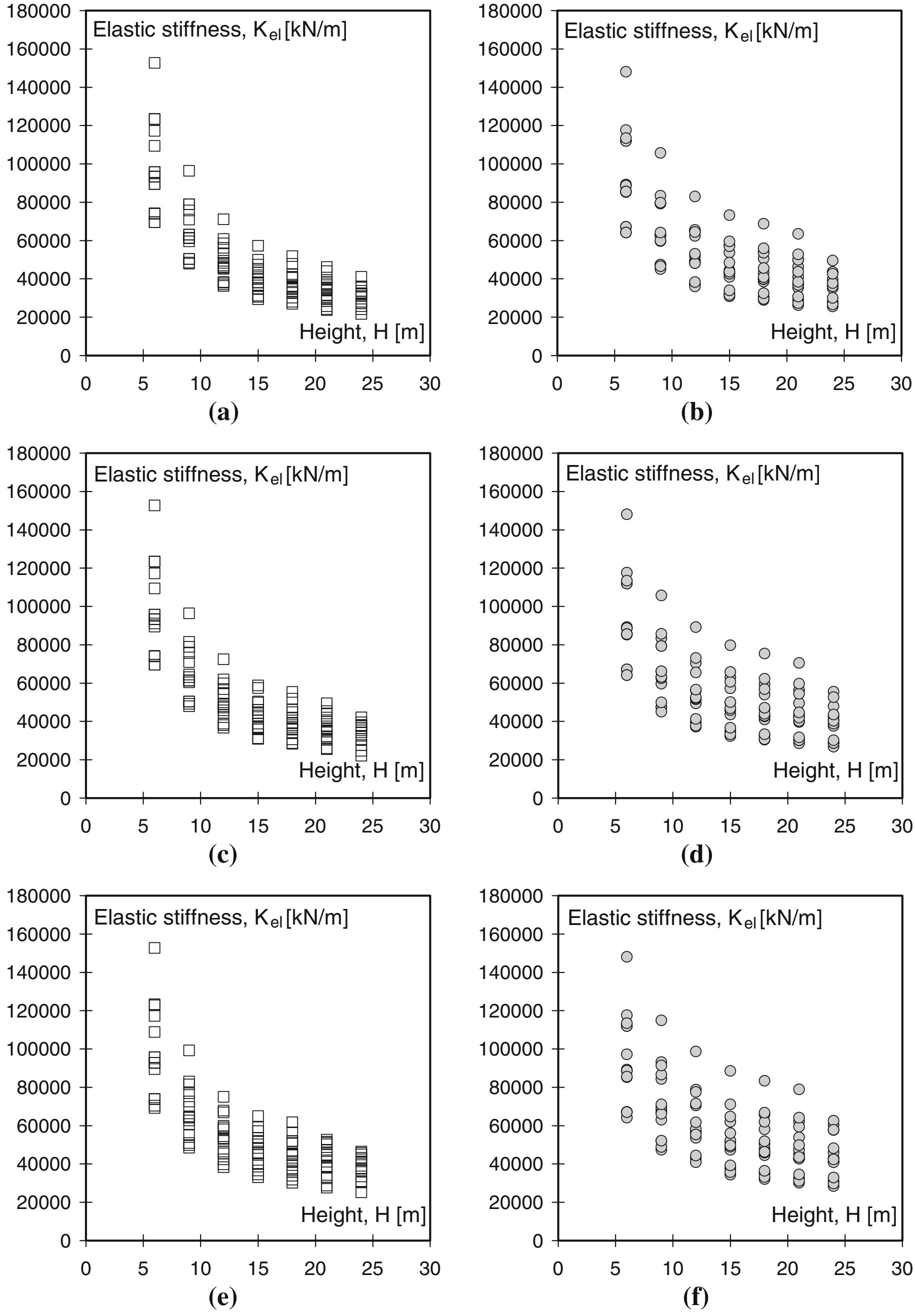


Fig. 8 Elastic stiffness versus height in transverse (*left*) and longitudinal (*right*) directions for seismic buildings with seismic coefficients of 0.05 g (**a**, **b**), 0.07 g (**c**, **d**), 0.10 g (**e**, **f**)

In Fig. 8 the stiffness as a function of height is given for the two directions of the three seismic categories considered. The stiffness in the two directions is similar. In fact, the stiffness in the longitudinal direction is never larger than 20% more with respect to the transversal direction. In particular, two storey buildings have a stiffness that is constant, on average, with respect to the design acceleration. The moderate height leads to actions that can be taken by the minimum proportions of structural elements. Therefore, the structures result generally similar independently of the design acceleration. Moreover, for this kind of buildings the stiffness reduces with global height in both directions and variability is also comparable. The more uniform distribution of resisting systems with respect to the gravity loads designed is also reflected in a different trend of stiffness as a function of plan area which now affects also the transversal direction. In Fig. 9a and b K_{el} is given for the two directions of buildings with H equal to 15.0 m and designed for 0.05 and 0.10 g, respectively.

Seismic designed buildings are generally stiffer that those designed for gravity loads only. In Fig. 10 the ratio of the average seismic to gravity loads stiffness is given for the two directions. This ratio increases in the transversal direction while it is almost constant in the

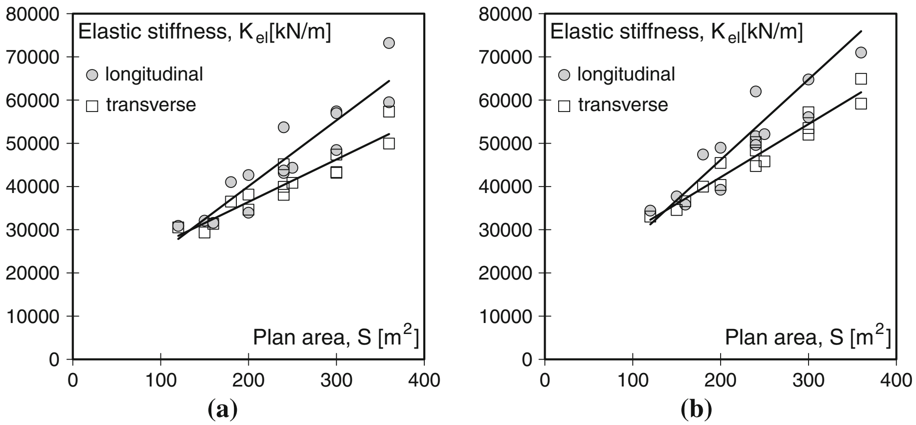


Fig. 9 Seismic buildings: elastic stiffness versus area for a 15 m building designed with acceleration equal to 0.05 g (**a**) and 0.10 g (**b**)

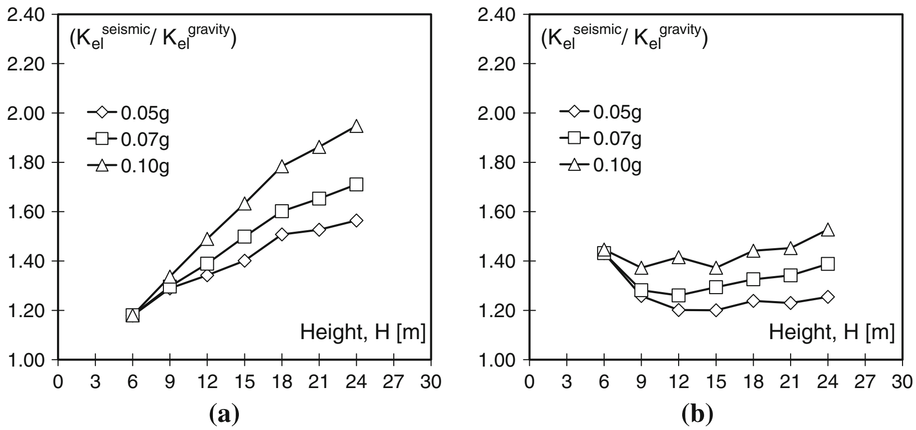


Fig. 10 Average seismic to gravity elastic stiffness ratio in **a** transverse and **b** longitudinal direction

longitudinal. This is mainly because the different structural systems affects more the former direction rather than the latter. The plane frames added in the seismic design contribute to the stiffness in an increasing manner with respect to the height of the building.

In the longitudinal direction, an increase in the design accelerations also implies a increment in the stiffness leading, on average, to a ratio with respect to the gravity loads design case of 1.25, 1.35, and 1.45 for 0.05, 0.07, and 0.10 g. In the transverse direction, 20% is the minimum stiffness increment, a number found for the two storey buildings, while 95% is scored by eight storey buildings, corresponding to the maximum global height considered in the study, and designed for 0.10 g. These results are reflected in the trends of Fig. 5b–d.

6 Period predictors from regression analysis

Although it is not an experimental sample, simple regression analysis on the analyzed populations allows one to see how results in terms of fundamental period display with respect to height. For comparative purposes, the same power-law formulation of Eq. (1) was assumed and the coefficients were estimated via ordinary least square regression.

For the case of gravity loads design the relationships for transverse and longitudinal directions are, respectively:

$$T_{\mathrm{el}} = 0.076H^{0.93} \qquad T_{\mathrm{el}} = 0.135H^{0.67} \tag{12}$$

Moreover, for the transverse direction, the same relationship was retrieved also not considering the contribution of stair sub-structure, Eq. (13).

$$T_{\mathrm{el}} = 0.105H^{0.94} \tag{13}$$

Analogous relationships were determined for the three populations of seismic buildings: *seismic coefficient* 0.05 g

$$T_{\mathrm{el}} = 0.091H^{0.79} \qquad T_{\mathrm{el}} = 0.112H^{0.69} \tag{14}$$

seismic coefficient 0.07 g

$$T_{\mathrm{el}} = 0.098H^{0.75} \qquad T_{\mathrm{el}} = 0.118H^{0.66} \tag{15}$$

seismic coefficient 0.10 g

$$T_{\mathrm{el}} = 0.107H^{0.70} \qquad T_{\mathrm{el}} = 0.118H^{0.65} \tag{16}$$

Figure 11 serves to compare the relationships found for the two considered directions. The trends generally reflect what observed for stiffness. The largest periods is observed for gravity-loads designed buildings while it decreases if the design acceleration is increased. In the longitudinal direction the period of seismic buildings is lower by 10–20% with respect to gravity loads. In the transversal direction, the reduction in period or seismic buildings may be as large as 45% for the eight storey buildings.

Comparing the period-height relationship including and not-including the stair module for gravity loads design buildings, the contribution of the sub-structure may be appreciated as the stair leads to a reduction of the latter as large as 40%.

Finally, only as a reference, the Eurocode 8 (CEN 2005) period-height curve is also given in the figure. The Eurocode 8 period is systematically lower than that found because it is a lower bound already (Goel and Chopra 1997) and also because it is expected to refer to buildings featuring a different design philosophy.

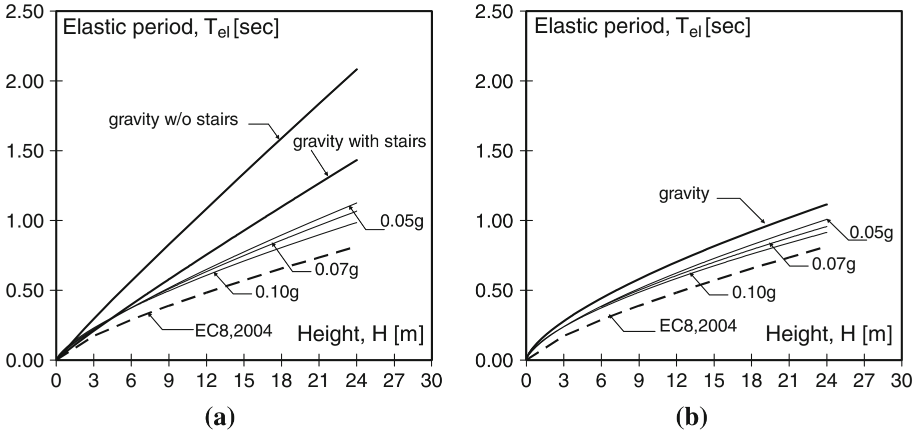


Fig. 11 Comparison of period-height relationships for the buildings analyzed in the transverse (**a**) and longitudinal (**b**) directions

Because Figs. 7 and 9, show that the effective mass and translational stiffness is also correlated with the plan extension of the building, it is expected that this variable has some prediction power with respect to the period. Therefore, an expression which includes also the plan area is considered Eq. (17):

$$T = \alpha H^{\beta} S^{\gamma} \tag{17}$$

where S is the product of the two principal plan dimensions of the building L_x and L_y. For the case of gravity loads design, least squares regression leads to the relationships for the transversal and longitudinal directions, respectively, given in Eq. (18):

$$T_{\mathrm{el}} = 0.009 H^{0.93} S^{0.39} \qquad T_{\mathrm{el}} = 0.044 H^{0.67} S^{0.21} \tag{18}$$

the same kind of analysis for the seismic directions provides:

seismic coefficient 0.05 g

$$T_{\mathrm{el}} = 0.029 H^{0.79} S^{0.21} \qquad T_{\mathrm{el}} = 0.059 H^{0.69} S^{0.14} \tag{19}$$

seismic coefficient 0.07 g

$$T_{\mathrm{el}} = 0.033 H^{0.75} S^{0.20} \qquad T_{\mathrm{el}} = 0.062 H^{0.66} S^{0.12} \tag{20}$$

seismic coefficient 0.10 g

$$T_{\mathrm{el}} = 0.039 H^{0.70} S^{0.19} \qquad T_{\mathrm{el}} = 0.068 H^{0.65} S^{0.10} \tag{21}$$

How much S contributes to explain the period is assessed simply analyzing the standard error for the regressions' residuals, σ_T:

$$\sigma_T = \sqrt{\left[\sum_{i=1}^{n} \left(\log \overline{T} - \log T_i\right)^2\right] \Big/ (n-1)} \tag{22}$$

In Eq. (22) $\overline{T}$ is the period from the regression model for the building having the computed value T_i and n is the size of the sample. In Table 1 the values of σ_T for all cases analyzed are reported, for the two conditions with and without the plan dimension factor, S. The results

Table 1 Standard error for the regressions' residuals

Design type	Direction	Standard error, σ_T	
		$T = \alpha H^{\beta}$	$T = \alpha H^{\beta} S^{\gamma}$
Gravity	Transverse	0.131	0.051
	Longitudinal	0.078	0.045
seismic 0.05g	Transverse	0.086	0.055
	Longitudinal	0.072	0.058
Seismic 0.07g	Transverse	0.082	0.051
	Longitudinal	0.066	0.055
Seismic 0.10g	Transverse	0.073	0.044
	Longitudinal	0.059	0.050

lead one to conclude that S adds information on the period for gravity loads design only as for this building typology, adding S leads to a 60% reduction in the standard error with respect to the formulation which only accounts for the height.

7 Conclusions

The elastic period has a primary role in the seismic design and assessment of buildings. Many codes propose simplified equations, retrieved on semi-empirical basis, expressing the fundamental period as a function of height, which represents the relationship between mass and stiffness of the structure. Nevertheless, the majority of these relationships are based on data of buildings reflecting seismic design criteria very different from those of the European existing structures.

In the study presented herein, two populations of typical Euro-Mediterranean reinforced concrete buildings have been investigated: the first one being designed for gravity loads only, the second one designed with obsolete seismic design criteria. The structures considered are bare-frames and assumed to be restrained at the base.

Modal analyses allowed to assess the influence of design criteria, structural system and global dimensions (area and height) on the elastic stiffness, the effective mass, and the elastic period for both principal axes of the buildings to be assessed. The results are different for the two classes:

- Gravity load designed buildings feature periods in the two directions which have an increasing difference with height that can be as large as 50%. The period shows large variability in the short direction due to the variability of plan area.
- Seismic designed buildings show a lower period in the longitudinal direction with respect to the corresponding gravity load buildings; this reduction, obviously, increases with design acceleration and is up to 20%. In the short direction the reduction of fundamental period is more significant (50%) because it is due not only to different design criteria, but also (mainly) to the different structural system.

Finally, based on the results of the analyses a power-law regression was carried out as a function of height. In the comparison with Eurocode 8 formulas existing buildings show systematically larger periods for those herein analyzed. In particular, gravity loads designed buildings, featuring a 3D structural system, seem to require a twofold definition of period referring to the two directions. Therefore, height alone seems inadequate to explain period variability and the results of this study suggest that. Also a global parameter (e.g., plan area) should be added in simplified relationships for rapid period evaluation.

Acknowledgments This work was supported by the Italian *Dipartimento della Protezione Civile* as part of the 2005–2008 ReLUIS project.

References

Applied Technological Council (1978) Tentative provisions for the development of seismic regulation for buildings. Rep. No, ATC3-06, Applied Technological Council, Palo Alto

Bal IE, Crowley H, Pinho R, Gulay FG (2007) Structural characteristics Of Turkish RC building stock in northern Marmara region for loss assessment applications. IUSS Press, Pavia

CEN (2004) Eurocode 2: design of concrete structures—part 1–1: general rules and rules for buildings. European Prestandard ENV 1992-1-1. Comite European de Normalisation, Brussels

CEN (2005) Eurocode 8: design provisions for earthquake of structures—part 1–4: strengthening and repair of buildings. European Prestandard ENV 1998-1-4. Comite European de Normalisation, Brussels

Chopra AK (1995) Dynamics of structures: theory and applications to earthquake engineering. Pretince-Hall, Upper Saddle River

Cosenza E, Manfredi G, Verderame GM (2002) Seismic assessment of gravity loads designed r.c. frames: critical issues in structural modeling. J Earthq Eng 6(1):101–122

Crowley H, Pinho R (2004) Period-height relationship for existing European reinforced concrete buildings. J Earthq Eng 8(1):93–119

Crowley H, Pinho R (2009) Revisiting Eurocode 8 formulae for periods of vibration and their employment in linear seismic analysis. Earthq Eng Struct Dyn. doi:10.1002/eqe.949 (in press)

D.M. n 1684 (1975) Approvazione delle norme tecniche per le costruzioni in zone sismiche (in italian)

Goel RK, Chopra AK (1997) Period formulas for moment-resisting frame buildings. Struct Eng Div ASCE 123:1454–1461

Hong L, Hwang W (2000) Empirical formula for fundamental vibration periods of reinforced concrete buildings in Taiwan. Earthq Eng Struct Dyn 29:327–333

Legge n 1684 (1962) Provvedimenti per l'edilizia, con particolari prescrizioni per le zone sismiche (in Italian)

NEHRP (1994) Recommended provisions for the development of seismic regulations for new buildings. Building Seismic Safety Council, Washington, DC

R.D.L. n 2105 (1937) Norme tecniche di edilizia con speciali prescrizioni per le località colpite dai terremoti (in Italian)

R.D.L. n 640 (1935) Norme tecniche di edilizia con speciali prescrizioni per le località colpite dai terremoti (in Italian)

R.D.L. n 2229 (1939) Norme per la esecuzione delle opere in conglomerate cementizio semplice od armato (in Italian)

SEAOC (1996) Recommended lateral force requirements and commentary. Seismological Engineers Association of California, San Francisco

UBC-Uniform Building Code (1997) International conference of building officials. Whittier, CA

Verderame GM, Polese M, Mariniello C, Manfredi G (2010) A simulated design procedure for the assessment of seismic capacity of existing RC buildings. Adv Eng Softw 41(2):323–335

Verderame GM, Polese M, Cosenza E (2009) Vulnerability of existing RC buildings under gravity loads: A simplified approach for non sway structures. Eng Struct 31(9):2141–2151