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The effect of spatial dependence on hazard validation

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SUMMARY

In countries where best-practice probabilistic hazard studies and seismic monitoring networks are available, there is increasing interest in direct validation of hazard maps. It usually means trying to quantitatively understand whether probabilities estimated via hazard analysis are consistent with observed frequencies of exceedance of ground motion intensity thresholds. Because the exceedance events of interest are typically rare with respect to the time span covered by data from seismic networks, a common approach underlying these studies is to pool observations from different sites. The main reason for this is to collect a sample large enough to convincingly perform a statistical analysis. However, this requires accounting for the dependence among the stochastic processes counting exceedances of ground motion intensity measures thresholds at different sites. Neglecting this dependence may lead to potentially fallacious conclusions about inadequateness of probabilistic seismic hazard. This study addresses this issue revisiting a hazard validation exercise for Italy, showing that accounting for this kind of spatial dependence can change the results of formal testing.

Key words: Probabilistic forecasting; Probability distributions; Statistical methods; Seismic attenuation.

1 INTRODUCTION

Due to their underlying predictive meaning, probabilistic seismic hazard analysis or PSHA (e.g. Cornell 1968; Reiter 1990) studies are often questioned. Italy is not an exception in this sense, and there is currently an ongoing debate about consistency and adequacy of the national hazard map, which serves as a basis for the definition of seismic actions for structural design (Stucchi *et al.* 2011).

Several studies have tried to quantitatively confirm or disprove probabilistic seismic hazard estimates via observed ground motions over the years. The most sound research is based on the theory of hypothesis testing. These studies recognize that validating hazard at a single site requires a large number of earthquake observations, which are seldom available due to the very long time (on average) required to collect them; see, for example, Iervolino (2013). Therefore, they tend to pool seismic records at different sites, in the same time span, to create a sample of sufficiently large size to make the formal comparison with PSHA results.

Hazard maps are usually a collection of location-specific values of ground motion intensity measures (IMs), for example peak ground accelerations or PGAs, corresponding to a given rate of exceedance from site-specific hazard analysis (i.e. from marginal IM probabilistic distributions). As a consequence, their statistical validation should also be site-specific, collecting multiple earthquake recordings at each location in question. However, the rarity of strong ground motions records pushes toward pooling data from multiple sites in the region covered by the map. In this context, validation is only possible if it is considered that, even if the effects of different earthquakes at each given site (i.e. IM) are assumed stochastically independent, the effects of each given earthquake at different sites cannot, in general, be considered independent (Iervolino & Giorgio 2015; Giorgio & Iervolino 2016). As a consequence, validation via data recorded at different sites cannot be performed using the same statistical tools which could be used for exceedances observed at a single site. Neglecting the effect of stochastic dependence of ground motion IMs may led to fallacious conclusions, for example labelling seismic hazard estimates from PSHA as erroneous (often claimed not conservative), while they are not. This short study highlights the practical impact of this issue. In particular, the hazard validation study by Albarello & D'Amico (2008) is revised to account for the effect of the above-mentioned dependence. To this aim, the remainder of the paper is organized such that a brief review of spatial dependence of IMs is given first, highlighting that the processes counting in time exceedances of IM thresholds can rarely be considered independent among sites. Then, the considered hazard validation study and related data are described and discussed. Subsequently, the study by Albarello & D'Amico (2008) is revised, under the same hypotheses and considering the same models, except accounting for the effect of dependence existing among IMs. Finally, some remarks are given.

2 CAUSES AND EFFECTS OF SPATIAL DEPENDENCE IN MULTISITE HAZARD

In its standard form, PSHA consists of the estimate of the rate (e.g. annual) of exceedance of a given value of an IM threshold, say im^*, at a site of interest (e.g. the location where a building under design is to be constructed). The computation of this rate, which can be indicated as $\lambda_{IM>im^*}$, is often carried out considering the rate of earthquakes above a magnitude of interest occurring on the source, ν; then, the conditional probability of IM exceedance given event magnitude, M, and location, $\{X, Y\}$, as well as other parameters (to follow); and finally by averaging the product of these two terms over all possible events via the joint distribution, $f_{M,X,Y}(m,x,y)$, of M and $\{X, Y\}$:

$$\lambda_{IM>im^*} = \nu \iiint_{m,x,y} P\left(IM > im^* | m,x,y\right) \times f_{M,X,Y}(m,x,y)\; dm\, dx\, dy. \tag{1}$$

This formulation is convenient, because the $P(IM > im^*|m, x, y)$ term is obtained from ground motion prediction equations or GMPEs (usually a function of source-to-site distance, $R(X, Y)$), while ν and $f_{M,X,Y}(m, x, y)$ are obtained based on seismicity – historical or instrumental – and geological information about the source.[1]

It is possible to show that, under the hypotheses of classical PSHA, the process describing the occurrence of events determining exceedance of im^* at the site of interest follows a homogeneous Poisson process (HPP). This means that the stochastic process counting the number of exceedances of im^* at the site is completely defined by the rate in eq. (1) and that the probability that the number of exceedances in the $(t, t + \Delta t)$ time interval at the site, $N_{IM>im^*}(t, t + \Delta t)$, results exactly equal to k can be computed as

$$P\left[N_{IM>im^*}(t, t+\Delta t) = k\right] = \frac{\left(\lambda_{IM>im^*}\,\Delta t\right)^k}{k!}\, e^{-\lambda_{IM>im^*}\,\Delta t}. \tag{2}$$

If the analysis as per eq. (1) is repeated for all IM-values in a range of interest, a curve for $\lambda_{IM>im^*}$, as a function of im^*, is obtained. The graph of this function is termed hazard curve, and for each IM-value it provides the rate of the specific HPP regulating the occurrence of its exceedances at the site of interest.

When multiple sites are concerned, that is, in the case of multi-site hazard, one may want to calculate the rate of exceedance of a vector of IM values. For example, if two sites $\{j, h\}$ are considered, let the objective of multi-site hazard be to compute the annual rate of earthquakes causing exceedance jointly at the two sites, $\lambda_{IM_j>im_j^* \cap IM_h>im_h^*}$; it could be carried out by eq. (3). In the equation, the probability $P(IM_j > im_j^* \cap IM_h > im_h^*|m, x, y)$ shows that, to compute $\lambda_{IM_j>im_j^* \cap IM_h>im_h^*}$, it is required to account for the possible simultaneous exceedances in a specific earthquake of given magnitude and location:

$$\lambda_{IM_j>im_j^* \cap IM_h>im_h^*} = \nu \iiint_{m,x,y} P\left(IM_j > im_j^* \cap IM_h > im_h^* | m,x,y\right) \times f_{M,X,Y}(m,x,y)\; dm\, dx\, dy. \tag{3}$$

The nature and form of stochastic dependence existing among the processes counting in time exceedances of ground motion thresholds at multiple sites is related to the probabilistic characterization of the effects of a common earthquake at the different sites. To provide insights into this correlation it is worthwhile to recall that common GMPEs, providing the $P(IM > im^*|m, x, y)$ term of eq. (1), model the logs of IM, at the generic site j due to earthquake i, $\log IM_{j,i}$, as:

$$\log IM_{j,i} = E\left(\log IM \,|m_i, r_{j,i}, \boldsymbol{\theta}\right) + \eta_i + \varepsilon_{j,i}, \tag{4}$$

where $E(\log IM|m_i, r_{j,i}, \boldsymbol{\theta})$ is the mean of $\log IM_{j,i}$ conditional on parameters such as magnitude, source-to-site distance, and others ($\boldsymbol{\theta}$). It is computed considering the IMs in multiple events featuring the same magnitude, recorded at the same distance from the source, in the same $\boldsymbol{\theta}$ conditions. η_i denotes the inter-event residual, which is a constant term for all sites in the ith earthquake; and $\varepsilon_{j,i}$ is the intra-event residual of the log of IM at site j in earthquake i. The inter-event residual is used to account for the fact that the mean of logs of IMs, recorded in the same event, at different sites featuring the same distance from the source, even for the same magnitude, varies from event-to-event because of factors unexplained by the GMPE model. The intra-event residual accounts for the fact that in a single earthquake: (a) observed IMs are different among sites at the same distance from the source; (b) and that, for sites that are close, recorded IMs tend to be similar. Interevent residuals are usually assumed to be stochastically independent from intra-event residuals, and both are usually assumed to be normally distributed at each site, with zero mean and with variance σ^2_{inter} and σ^2_{intra}, respectively. Then, for site-specific hazard, IM is modelled as a lognormal random variable where the log has variance $\sigma_T^2 = \sigma^2_{\text{inter}} + \sigma^2_{\text{intra}}$.

It is generally assumed that the logs IMs at multiple sites (say $j = 1, 2, \ldots, s$) form a Gaussian random field (GRF), given M and $\{X, Y\}$; for example, Park *et al.* (2007); Malhotra (2008). This means that the logs of IMs have a multivariate normal distribution where the components of the mean vector are given by the $E(\log IM|m_i, r_{j,i}, \boldsymbol{\theta})$ terms (one for each j) and the covariance matrix, Σ , is

$$\Sigma = \sigma^2_{\text{inter}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} + \sigma^2_{\text{intra}} \begin{bmatrix} 1 & \rho_{1,2} & \cdots & \rho_{1,s} \\ \rho_{2,1} & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{s,1} & \rho_{s,2} & \cdots & 1 \end{bmatrix}. \tag{5}$$

In eq. (5), $\rho_{j,h}$, is the correlation coefficient between intra-event residuals at two generic sites $\{j, h\}$ among those considered. Assigning the mean vector and the covariance matrix completely defines the GRF.

With regard to correlation of IMs, it should be noted that

(i) correlation/dependence among IMs is generated by the fact that all components of the mean vector share the same event features such as magnitude and location;

(ii) correlation/dependence among IMs is also generated by the fact that the first term at the right hand side of eq. (5) produces inter-event residuals that are perfectly-correlated at all sites in one event (given earthquake i the interevent term is the same for each j);

(iii) the last source of correlation/dependence among IMs in a single earthquake is the second term at the right hand side of eq. (5); the latter matrix produces non-perfectly correlated intra-event residuals.

In other words, the primary source of dependence among IMs, generated at different sites by the same event, stems from the fact that ground motions at all sites share the same rupture's features, including the interevent residual. Another source of stochastic

[1] In the case of multiple sources, the integral of eq. (1) is repeated for each source and the resulting rates added.

dependence is that represented by intra-event residuals; usually, correlation among these residuals is assumed to reduce as the separation distance between sites increases (e.g. Esposito & Iervolino 2011).

Because of the kind of dependence of these IMs, it is possible to show that if the earthquakes can cause exceedances at the two sites j and h (i.e. exceedances are not mutually exclusive in each event), then the counting processes $N_{IM_j>im_j^*}(t)$ and $N_{IM_h>im_h^*}(t)$, see eq. (2), are stochastically dependent. Indeed, it was demonstrated in Giorgio & Iervolino (2016) that the processes $N_{IM_j>im_j^*}(t)$ and $N_{IM_h>im_h^*}(t)$ are stochastically independent if and only if the probability of exceedance at multiple sites in one earthquake is zero: $P(IM_j > im_j^* \cap IM_h > im_h^* | m, x, y) = 0$; whereas they are positively correlated otherwise. The only way not to observe multiple exceedances in any specific earthquake is for the inter-site distance to be large enough. This condition is hardly satisfied in the case of monitoring via dense seismic networks; therefore, stochastic spatial dependence should always be considered in hazard validation, as proven in the following.

3 A HAZARD VALIDATION STUDY FOR ITALY

In this section, the validation study developed in Albarello & D'Amico (2008) is revised based on the arguments discussed so far. The aim of the considered study was to validate the Italian hazard map reporting the PGA with 10 per cent in 30 yr exceedance probability on A-type local site conditions (according to the Eurocode 8 classification; CEN 2003). This IM-value will be indicated as PGA(10/30) hereafter. The cited authors gathered data from sixty-eight seismic stations operating during a 30 yr period across the country. For these stations, it was observed that the PGA(10/30) values from the official hazard study for Italy (Stucchi *et al.* 2011) were collectively exceeded thirteen times. In Fig. 1, the year of earthquake occurrence and/or of PGA(10/30) exceedance, is given.

To validate the hazard map, Albarello & D'Amico (2008) adopt two approaches, both based on the hypothesis that the number of exceedances observed at different sites in 30 yr, given that these are *sufficiently far away from each other,* can be considered stochastically independent. In particular, in the first approach (i.e. the one considered in this paper), termed the *counting approach*, a Bernoulli random variable (RV) is associated to the exceedance event of PGA(10/30) at each site. Such a RV assumes value zero if exceedance is not observed, and one if $\mathrm{PGA}_j(10/30)$ is exceeded *at least once* in 30 yr at site $j = \{1, 2, \ldots, 68\}$. This RV is characterized by 0.1 probability of observing the exceedance, which is a direct consequence of the definition of PGA(10/30). Indeed, the Bernoulli RVs associated to different sites are equally distributed with parameter $p = 0.1$ (obviously, each site has a different $\mathrm{PGA}_j(10/30)$, $j = \{1, 2, \ldots, 68\}$, corresponding to the same exceedance probability). Then, under the independence hypothesis, the probability that at exactly k of the sixty-eight considered stations is observed at least one exceedance in 30 yr can be computed via the binomial probability mass function in eq. (6), where the number of trials, s, is 68 and $p = 0.1$:

$$P\,[k \text{ exceedances of PGA(10/30) across 68 sites in 30 yr}] = \binom{68}{k}\, 0.1^k\, 0.9^{68-k}. \quad (6)$$

Consequently, Albarello & D'Amico (2008) computed the mean and the variance of the number of sites in which at least one exceedance is observed as $s \cdot p = 68 \cdot 0.1 = 6.8$ and $s \cdot p \cdot (1-p) = 68 \cdot 0.1 \cdot (1-0.1) = 6.12$, respectively. Finally, they performed a formal statistical test to check the (null) hypothesis that the exceedance probability at the generic site is 0.1, as suggested by the Italian hazard map, against the (alternative) hypothesis that this probability differs from 0.1. Indeed, considering that from available data it results that exceedance has been observed in thirteen of the

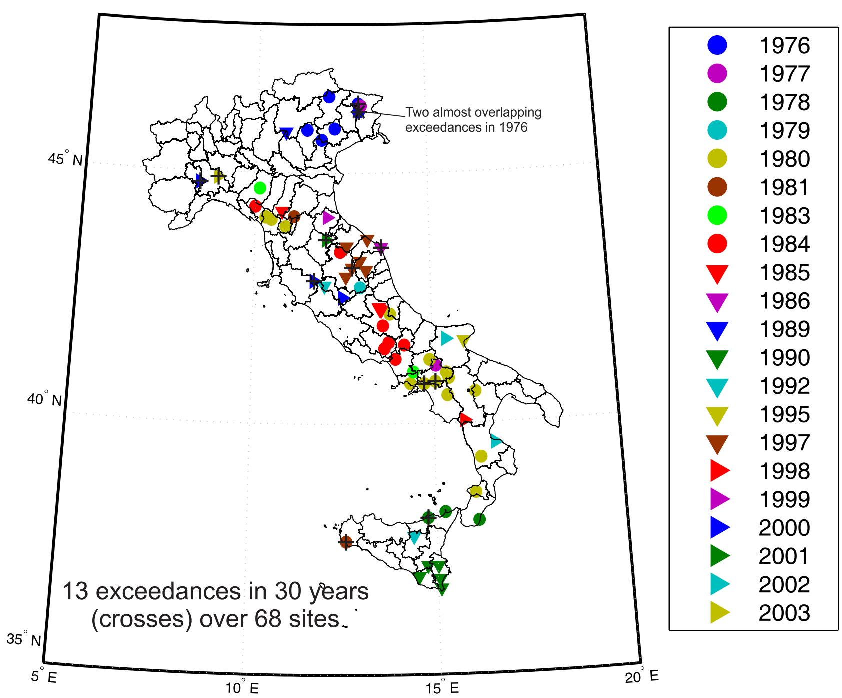

Figure 1. Seismic stations continuously operating for 30 yr considered by Albarello & D'Amico (2008), with earthquake occurrence years and observed exceedances of PGA(10/30) (crosses).

sixty-eight sites in 30 yr, and recalling that for the central limit theorem (i.e. via a Gaussian approximation; e.g. Mood *et al.* 1974) it can be assumed that

$$P\left[|\text{number of exceedances} - 6.8| > 1.96\sqrt{6.12}\right] \cong 0.05 \qquad (7)$$

they infer that, being $13 - 6.8 = 6.2 > 1.96\sqrt{6.12} = 4.85$, the observed number of exceedances gives evidence that the null hypothesis has to be rejected at the 0.05 significance level. Finally, although recognizing the possible biasing effect of the independence assumption, they concluded that the values from the hazard map *apparently tend to underestimate the actual hazard*. It is noted that, based on the approach adopted by Albarello & D'Amico (2008), the test rejects the null hypothesis in the case twelve or more exceedances are observed. In fact, the upper limit of the acceptance region of the considered two-tailed hypothesis test at 0.05 significance level is: $6.8 + 1.96\sqrt{6.12} = 11.65$ (in this approach, the p-values for twelve and thirteen exceedances are 0.035 and 0.012, respectively).

3.1 Reviewing exceedance data

It may be worthwhile for the following discussion to look more carefully at the data considered in Albarello & D'Amico (2008). In particular, the observed exceedances are reported in Table 1. The first two columns are the original data, while the event information is retrieved by the *station search* option of the Italian accelerometric archive or ITACA (Luzi *et al*. 2008) online database (http://itaca.mi.ingv.it/; last accessed December 2016). The last column is the PGA(10/30) from the national hazard map; that is, from the study described in Stucchi *et al.* (2011). The latter does not directly provide PGA(10/30) (see http://zonesismiche.mi.ingv.it/; last accessed December 2016). The closest available PGAs are PGA(6/30) and PGA(14/30). A log-linear interpolation between these allows the obtaining of the PGA(10/30) values for the sites.

At this point it is necessary to discuss a few issues. The seismic source model discussed in Stucchi *et al*. (2011) computes exceedance rates of PGA due to magnitude larger than 4.15, except one seismic zone where the minimum magnitude is 3.55 (the Mount Etna zone, in Sicily). Therefore, exceedances considered for validation should be those caused by earthquakes of magnitude larger than 4.15. The exceedance at the RASRL station (Sirolo, Ancona, central Italy) is an M3.7 event, which occurred far from the Etna's zone. Moreover, the maximum recorded PGA at RASRL from ITACA is slightly lower than that available to Albarello & D'Amico (2008), and it is below the PGA(10/30) of the site from the hazard map. In fact, IM values are sensitive to waveform processing, and such a variability, for the same record among different data sources, is not exceptional, and should be accounted for as an additional source of uncertainty. In any case, these two issues put in question whether the datum from RASRL should be considered an exceedance or not in the considered validation study.

It should also be noted that records collected in early history of seismic monitoring (around the seventies in Italy) are from analog instruments, and cannot be considered homogeneous with respect to more recent measurements, typically from digital instruments. Moreover, the given soil conditions in the ITACA database for the considered sites are, in some case, far from A-type soil. This may be relevant as, in the case of the tested hazard model that relies on GMPEs with linear soil terms, the PGA(10/30) values computed for soil conditions different than A-type would be necessarily larger. Finally, data used for validation are part of those used to calibrate the models employed in PSHA being validated (e.g. earthquake rates and GMPEs), which constitutes a further issue. Nevertheless, the impact on validation results may be considered negligible when data are a relatively small fraction of the bulk at the basis of the tested hazard map. In any case, these issues are not addressed further herein, where the focus is on the effect of spatial dependence.

4 ACCOUNTING FOR DEPENDENCE OF IMs IN HAZARD VALIDATION

In this section, the study of Albarello & D'Amico (2008) is revised to account for the effect of dependence of IMs at multiple sites caused by a common seismic event (see Section 2). The final target is to build the distribution of the number of sites, among the sixty-eight considered, experiencing at least one exceedance of $\text{PGA}_j(10/30)$, $j = \{1, 2, \ldots, 68\}$ in 30 yr. It is easy to recognize that such a distribution (hereinafter referred to as the *exact* distribution) is the sum of the same Bernoulli random variables Albarello & D'Amico (2008) considered to obtain the binomial RV of eq. (6) (i.e. the RVs that are equal to zero if $\text{PGA}_j(10/30)$ is not exceeded *at least once* in 30 yr at site $j = \{1, 2, \ldots, 68\}$, and are equal to one otherwise). The two distributions, that of Section 3 and the one discussed in this section, only differ because the latter accounts for the fact that the considered Bernoulli random variables are stochastically dependent, whereas the former neglects that.

Table 1. Exceedance data in the study Albarello & D'Amico (2008) and PGA(10/30) from the hazard map.

Station ID	Max recorded PGA (g)	Max recorded PGA (g) (ITACA)	Event ID (ITACA)	ML (M_w) (ITACA)	Soil (ITACA)	PGA(10/30) (g) (A-type soil)
RATLM1	0.342	0.346	IT-1976-0002	6.4	B	0.193
SRC0	0.249	0.250	IT-1976-0030	6.0	B[a]	0.202
FRC	0.352	0.349	IT-1976-0030	6.0	B	0.202
RANAS	0.149	0.148	IT-1978-0004	5.5 (6.0)	C	0.145
RAMRT	0.140	0.141	IT-1980-0012	6.5 (6.9)	B	0.115
RABGI	0.189	0.187	IT-1980-0012	6.5(6.9)	B	0.153
RAMZR	0.193	0.193	IT-1981-0006	(4.9)	B	0.051
RASRL	0.143	0.137	IT-1986-0001	3.7	C	0.140
RANCR	0.500	0.502	IT-1997-0006	5.8 (6.0)	E	0.193
RAPNC	0.160	0.150	IT-2000-0001	3.0 (4.5)	B[a]	0.117
RANZZ	0.131	0.132	IT-2000-0008	4.3 (4.8)	C	0.041
RAPVS	0.196	0.186	IT-2001-0041	4.4 (4.7)	B	0.179
RATRT	0.087	0.086	IT-2003-0023	4.7 (4.8)	E	0.067

[a]Local site condition inferred from large-scale geology rather than measured average shear wave velocity in the upper 30 m of soil.

To compute the sought distribution, it is required to simulate random fields of IMs at the sixty-eight sites. To this aim, the model at the basis of the hazard map of Stucchi *et al.* (2011) is considered; it is made of thirty-six areal seismic source zones. As it regards the seismic features of each zone (e.g. earthquake rates and magnitude distributions), it should be recalled that the work of Stucchi *et al.* (2011) considers a logic tree made of several branches. Herein, the branch named 921 is considered, because it is the one producing results claimed to be the closest to those provided by the full logic tree results. In this branch the GMPE is that of Ambraseys *et al.* (1996), which is also used herein. Note that this GMPE has only one residual term, not distinguishing between inter- and intra-event. It is treated herein as an intra-event residual with $\sigma_{\text{intra}} = 0.25$. Moreover, no spatial correlation is introduced among the residuals (e.g. Esposito & Iervolino 2011). This means that only source (i) of spatial correlation from Section 2 is explicitly considered. It can be shown that accounting for the other two sources of correlation can only amplify the dependence between the considered Bernoulli RVs (Giorgio & Iervolino 2016).

Given the described hazard model, which is consistent with the work of Albarello & D'Amico (2008), the seismic history spanning 30 yr at the sixty-eight sites was simulated one-hundred-thousand times via the following steps, using the software of Iervolino *et al.* (2016).

(i) A realization of the number of earthquakes, say n, occurring in the country in 30 yr was obtained by sampling from a Poisson distribution with rate $30 \cdot \nu_{\text{tot}}$, where ν_{tot} is the sum of the annual rates of occurrence of earthquakes of the thirty-six zones: $\nu_{\text{tot}} = \sum_{z=1}^{36} \nu_z$.

(ii) For each earthquake $i = \{1, 2, \ldots, n\}$ the seismic source zone where it occurs is sampled according to the probability that when a seismic event occurs it is from zone z: $P[\text{Zone} = z|\text{earthquake } i \text{ occurs}] = \nu_z/\nu_{\text{tot}}$.

(iii) Once the zone generating event i is established, earthquake magnitude and location are simulated. The M-value is sampled from the magnitude distribution for the source in question, while $\{x, y\}$ are simulated assuming that these are uniformly distributed over the source zone.

(iv) The magnitude and location of the ith earthquake are used, in turn, to compute the mean of the logs of PGA on rock at each of the sixty-eight sites. Moreover, sixty-eight values of the residual of the GMPE are sampled independently from a Gaussian distribution with zero mean and variance σ^2_{intra}. The mean plus the residual of the GMPE is used to compute, in the simulated event, the PGA at each site (zero intensity is assigned to sites more distant than 200 km from the earthquake location, because of the applicability limits of the considered GMPE). These IMs allow the counting of how many exceedances are observed at the sites in the ith earthquake of the simulated thirty years.

(v) Repeating steps (i–iv) for the n earthquakes allow the counting, in the thirty years under simulation, of how many different sites, among the sixty-eight considered, experienced at least one exceedance of their PGA(10/30), say they are k.

(vi) Repeating steps (i–v) one-hundred-thousand times allows the collection of the k-values to build the exact distribution of the number of sites experiencing at least one exceedance of PGA(10/30) in thirty years. This distribution, clearly, is the probability mass function (PMF) of a discrete random variable taking values between zero and sixty-eight and it is shown in Fig. 2.

For comparison, the figure also reports the binomial distribution of eq. (6), which Albarello & D'Amico (2008) obtained neglecting dependence existing among the above-mentioned site-specific Bernoulli random variables. The two distributions have the same mean, 6.8, as the average total number of exceedances in thirty years is not affected by the presence of dependence. However, the binomial distribution has variance equal to 6.12, while the exact distribution has a larger variance, equal to 8.43 (such variance would further increase considering a GMPE modelling inter-event residuals and the spatial correlation of intra-event residuals; see Giorgio & Iervolino 2016).

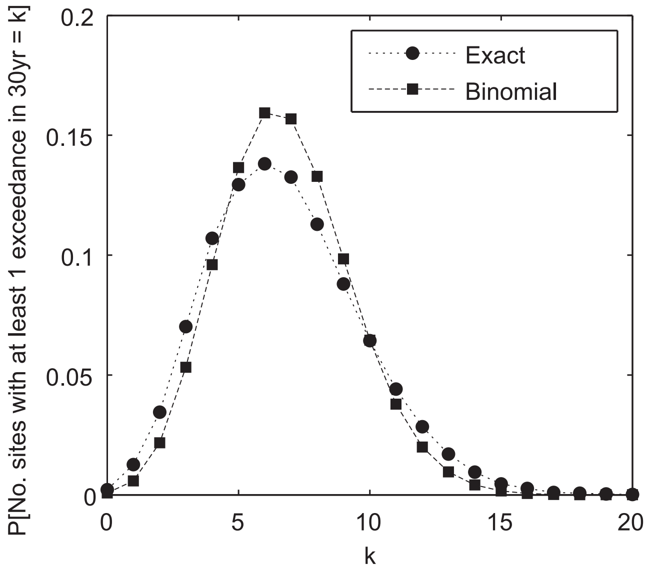

Figure 2. Probability distribution of the number of sites with at least one exceedance of PGA(10/30) in 30 yr on A-type soil.

At this point the same formal hypothesis test discussed in Section 3 can be carried out using the exact PMF of Fig. 2. To do so it is necessary to build the acceptance region $[k_1, k_2]$ for the two-tailed test at the significance level 0.05 via the approximated relationships in eq. (8).

$$\begin{cases} k_1 = 6.8 - 1.96\sqrt{8.43} = 1.11 \\ k_2 = 6.8 + 1.96\sqrt{8.43} = 12.49 \end{cases} \tag{8}$$

Because $k_2 = 12.49$, it means that also in this exact case thirteen observed exceedances lead to the conclusion that the null hypothesis, that PGA(10/30) for the sites have been appropriately evaluated via PSHA, has to be rejected at the level of significance 0.05.

The test results show that sites, on average, are distant enough to almost satisfy the independence assumption discussed above. Nevertheless, the effect of the spatial dependence is not negligible as the variance increase of the exact case leads twelve exceedances within thirty years to fall in the acceptance region, while this number would be out of the acceptance region in the case exceedances are modelled as in Albarello & D'Amico (2008); that is, using eq. (6). Eq. (8) refers to a Gaussian approximation as in the revised study (the p-values for twelve and thirteen exceedances, computed with the same approach of Albarello & D'Amico 2008, are 0.073 and 0.033, respectively). Nevertheless, the same test results are obtained by building the exact acceptance region from the PMF of Fig. 2. At this point it has to be recalled that, as discussed in Section 3.1, at least one exceedance remains doubtful. If it would be not considered, then exceedances would become twelve, a number that leads not to reject the hypothesis that observed exceedances are in agreement with the hazard map in the exact case only. Indeed, twelve is still in the rejection region according to the binomial distribution, giving evidence that spatial dependence may have a large impact on hazard validation and it should be always accounted for, even if the stations seems far enough of each other. (This discussion does not consider the actual soil conditions of the stations from Table 1, which would further lead rise the PGA(10/30) values for the site and in turn reduce the number of observed exceedances.)

5 CONCLUSIONS

This short paper discussed the critical role of spatial dependence among strong motion IMs in attempts to validate probabilistic seismic hazard studies via observed ground motions. Indeed, such a stochastic dependence implies, in general, dependence of stochastic processes counting in time the exceedances of IM thresholds at multiple sites. The only possibility for the number of exceedances that occur in a common time span at different sites to be independent is that each earthquake can cause exceedance at one site at the most, which is a condition rarely occurring in the case of modern (i.e. relatively dense) seismic monitoring networks. As a consequence, the test statistic to validate hazard cannot rely on models not able to account for this form of dependence.

To quantitatively evaluate the effect of spatial dependence, a previous hazard validation was revised. It considers sixty-eight sites in Italy and the corresponding PGAs with marginal exceedance probability equal to 10 per cent in thirty years from the national hazard map. It was found that, although the considered sites were somewhat sparse, and the effect of dependence could be considered negligible at a first glance, it may be relevant. This is evident in the case of the revisited study, where one of the counted exceedances is questionable. Excluding it, the small difference brought in by spatial dependence, would lead to revert the decision whether the observed exceedances are in agreement with the tested hazard map.

Two side conclusions also emerged from the study: (i) neglecting the effect of spatial dependence among strong motion IMs is expected to be more detrimental for recent hazard validation studies, because they will likely involve data from dense seismic networks; (ii) it is of paramount importance to carefully evaluate data, to make sure they come from shaking fully consistent with what is contemplated by the tested hazard map, because even small changes in the number of observed exceedances can be relevant for the validation results.

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REFERENCES

Albarello, D. & D'Amico, V., 2008. Testing probabilistic seismic hazard estimates by comparison with observations: an example in Italy, *Geophys. J. Int.,* **175,** 1088–1094.

Ambraseys, N.N., Simpson, K.A. & Bommer, J.J., 1996. Prediction of horizontal response spectra in Europe, *Earthq. Eng. Struct. Dyn.,* **25,** 371–400.

CEN, European Committee for Standardisation TC250/SC8, 2003. *Eurocode 8: Design Provisions for Earthquake Resistance of Structures,* Part 1.1: General rules, seismic actions and rules for buildings, PrEN1998-1.

Cornell, C.A., 1968. Engineering seismic risk analysis, *Bull. seism. Soc. Am.,* **58,** 1583–1606.

Esposito, S. & Iervolino, I., 2011. PGA and PGV spatial correlation models based on European multi-event datasets, *Bull. seism. Soc. Am.,* **101,** 2532–2541.

Giorgio, M. & Iervolino, I., 2016. On multi-site probabilistic seismic hazard analysis, *Bull. seism. Soc. Am.,* **106,** 1223–1234.

Iervolino, I., 2013. Probabilities and fallacies: why hazard maps cannot be validated by individual earthquakes, *Earthq. Spectra,* **29,** 1125–1136.

Iervolino, I. & Giorgio, M., 2015. The effect of stochastic dependence on hazard validation studies, in *Proc. CSNI Workshop on Testing PSHA Results and Benefit of Bayesian Techniques for Seismic Hazard Assessment,* EUCENTRE Foundation, Pavia, Italy.

Iervolino, I., Chioccarelli, E. & Cito, P., 2016. REASSESS V1.0: a computationally-efficient software for probabilistic seismic hazard analysis, in *Proc. VII European Congress on Computational Methods in Applied Sciences and Engineering,* ECCOMAS, Crete Island, Greece.

Luzi, L., Hailemikael, S., Bindi, D., Pacor, F., Mele, F. & Sabetta, F., 2008. ITACA (ITalian ACcelerometric Archive): a web portal for the dissemination of Italian strong-motion data, *Seismol. Res. Lett.,* **79,** 716–722.

Malhotra, P.K., 2008. Seismic design loads from site-specific and aggregate hazard analyses, *Bull. seism. Soc. Am.,* **98,** 1849–1862.

Mood, A.M., Graybill, F.A. & Boes, D.C., 1974. *Introduction to the Theory of Statistics,* McGraw-Hill, 480 pp.

Park, J., Bazzurro, P. & Baker, J.W., 2007. Modeling spatial correlation of ground motion intensity measures for regional seismic hazard and portfolio loss estimation, in *Proc. 10th International Conference on Application of Statistic and Probability in Civil Engineering (ICASP10)*, Tokyo, Japan, 8 pp.

Reiter, R., 1990. *Earthquake Hazard Analysis: Issues and Insights,* Columbia Univ. Press, 254 pp.

Stucchi, M., Meletti, C., Montaldo, V., Crowley, H., Calvi, G.M. & Boschi, E., 2011. Seismic hazard assessment (2003–2009) for the Italian building code, *Bull. seism. Soc. Am.,* **101,** 1885–1911.