Chapter 3
Design Earthquakes and Conditional Hazard

Iunio Iervolino

Abstract   Disaggregating seismic hazard in terms of some ground motion or source parameters as magnitude and distance to the site of interest, allows to identify the earthquakes giving the largest contribution to the occurrence or exceedance of a specific value of a ground motion intensity measure (IM). If mapped, such information may be of useful to engineers in better defining the seismic threat for the structure of interest, for example in case of record selection for nonlinear seismic structural analysis. Because disaggregation results change with the spectral ordinate and return period, and more than a single event may dominate the hazard, the issues related to mapping design earthquakes (DE) are discussed in the paper with respect to Italy. Secondly, it is shown how DEs may be useful in rapidly determining conditional hazard of a secondary ground motion intensity measure with respect to a primary IM. As shown by the application for a vector comprised of peak ground acceleration as the primary IM and the Cosenza and Manfredi Index as the secondary IM, conditional hazard may be used for a more refined and consistent definition of design seismic actions on structures. The conditional hazard approach may be applied, in principle, to any other vector of IMs, providing the advantages of vector-valued hazard analysis requiring much less effort, and therefore rendering it ready for implementation in practice.

Keywords   Disaggregation • Scenario • Performance-based earthquake engineering • Record selection • REXEL
3.1 Introduction

The basis for definition of seismic actions on structures in codes is the design spectrum. Because a rational design target should be representative of the seismic hazard at the construction site, the uniform hazard spectrum (UHS) or an approximation of it, is often used as the reference for design. The UHS is built entering the elastic spectral acceleration or $Sa$ (for several structural periods, $T$) hazard curves with a specified probability of exceedance of (e.g., 10% in 50 years), and plotting the corresponding $Sa$ versus $T$.

Given the UHS, the most of codes also require to determine the relevant scenarios for the site of interest in terms of seismic sources, and not only in terms of ground motion represented by the UHS. For example, Eurocode 8 (CEN 2003), regarding input ground motion selection for structural analysis states\footnote{Other minor requirements apply; see (Iervolino et al. 2008) for details.}: in the range of periods between $0.2T_1$ and $2T_1$, where $T_1$ is the fundamental period of the structure in the direction where the accelerogram will be applied, no value of the mean 5% damping elastic spectrum, calculated from all time histories, should be less than 90% of the corresponding value of the 5% damping elastic response spectrum. Moreover, accelerograms should be adequately qualified with regard to the seismogenetic features of the sources\[...\]. Records used in the structural analysis may be: real, artificial or obtained by simulation of seismic source, propagation and site effects.

It is clear that seismic source features may be required applying code prescriptions; however, it is unlikely that the engineer is able to qualify the input ground motions with respect to the seismological features of the sources. On the other hand, the most important events may be identified via disaggregation of seismic hazard (e.g. Convertito et al. 2009). In fact, once the UHS has been defined, it is possible to identify one or more earthquakes; i.e. the values of magnitude ($M$), source to site distance ($R$) and $\epsilon$ (number of standard deviations that the ground motion parameter is away from its median value estimated by the assumed attenuation relationship) providing the largest contributions to the hazard of exceeding a specified IM threshold. These events may be referred to as the earthquakes dominating the seismic hazard in a probabilistic sense, and may be used as design earthquakes (DEs) if appropriately mapped.

In the following, disaggregation of spectral acceleration hazard for all Italian sites for structural periods equal to 0s (PGA) and 1s is presented. In a lager study four different exceedance return periods ($Tr$) were considered (2,475, 975, 475 and 50 years) corresponding to the main limit states for civil and strategic structures, however, herein only results for $Tr$ equal to 475 years will be shown; other results shall be available by Iervolino et al. (2010a).

The study shows how mapping DEs may be helpful, not only in ground motion record selection, but also in the low-effort definition of seismic actions for vectors of IM. In fact, vector-valued ground motion intensity measures have been recently...
investigated thoroughly (e.g. Baker 2007). Proposed measures are mainly function of spectral ordinates which have been shown to be useful in the assessment of structural response enhancing the confidence in estimations with respect to scalar IMs.

In a pair of IMs, it is often the case one is considered of primary importance with respect to the other. For example, it is generally believed that integral ground motion IMs, associated with duration, are less important with respect to the peak parameters of the record; nevertheless, there may be cases in which the cumulative damage potential of the earthquake is also of concern. For these IMs, it seems appropriate to develop conditional hazard maps; i.e., maps of percentiles of a secondary IM given the occurrence or exceedance of a primary parameter for which a design hazard map is often already available. In the presented study, this concept is illustrated and conditional hazard is developed for a parameter which may account for the cumulative damage potential of ground motion, the so-called Cosenza and Manfredi index (ID), given peak ground acceleration (PGA). The study shows how easily it is possible to obtain analytical distributions of ID conditional on PGA and on the corresponding DEs. Mapping of conditional hazard seems useful to complement design hazard maps for acceleration.

3.2 Italian Design Earthquakes

Given the characterization of seismic sources and once an IM is chosen, probabilistic seismic hazard analysis (PSHA) allows to identify, for each considered site, the probability of exceedance of different IM values in a time interval of interest. For its integral nature, PSHA, combines the contribution to the hazard from all considered sources. The event most important may be identified via disaggregation, and may be referred to as those providing the largest contributions to the hazard in terms of exceeding a specified IM threshold. Analytically, result of disaggregation is the joint probability density function (PDF) of \( \{M, R, e\} \) conditional to the exceedance of the IM\( _0 \) threshold, Eq. 3.1.

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\[
 f(m, r, e | IM > IM_0) = \frac{\sum_{i=1}^{n} v_i \cdot f(IM > IM_0 | m, r, e) \cdot f(m, r, e)}{\lambda_{IM_0}}
\]  

(3.1)

In Eq. 3.1 I is an indicator function that equals 1 if IM is larger than IM\( _0 \) for the specific values of \( r, m, \) and \( e; n \) is the number of seismic sources relevant for the hazard at the site, \( v_i \) is the earthquake occurrence probability for the fault \( i; f(m, r, e) \) is the joint PDF of \( \{M, R, e\}; \) and \( \lambda_{IM_0} \) is the hazard for IM\( _0 \). From the equation it is possible to observe that disaggregation depends on IM\( _0 \) (i.e., the hazard level being disaggregated, or the return period of the IM) and on the definition of the IM itself. If the IM of interest is Sa(T), then disaggregation, and therefore DEs, also depend on T. In fact, UHS for different return periods is
characterized by different DEs, and, within a given UHS, short and long period ranges may display different M, R and $\varepsilon$ from disaggregation (e.g. Reiter 1990).

In Italy, recent seismic code (CS.LL.PP 2008) introduced a new seismic classification of national territory, which was discretized by a grid of more than $10^4$ nodes where the seismic hazard has been computed in terms of acceleration spectral ordinates from 0s to 2s, along with PGA disaggregation (see http://esse1.mi.ingv.it/). In this way, the code provides design spectra very close to the UHSs and software tools have been developed to automatically select sets of ground motion records compatible to them; i.e., REXEL (Iervolino et al. 2010b). However, PGA hazard disaggregation may be insufficient to define DEs for structures in the moderate period range. To overcome this gap, herein hazard is developed for PGA (considered as a benchmark) and Sa for T equal to 1s, and the disaggregation is computed for the whole country referring to the 475 years hazard.

Hazard values were computed for 30 values of the IMs equally distributed between 0.001 g and 1.5 g. All the analyses have been performed by a FORTRAN program specifically developed and also used in Convertito et al. (2009). The modeling of seismogenetic zones is that of Meletti et al. (2008), also adopted by the Istituto Nazionale di Geofisica e Vulcanologia or INGV (Fig. 3.1). Because of seismogenetic zones modeling, the hazard software assumes an uniform distribution of possible epicenters. Seismicity parameters of each zone are those used by Barani et al. (2009) (Table 3.1). The considered grid for Italy is the same of that from INGV and used in the Italian seismic code. All the analyses refer to rock site conditions according to Ambraseys et al. (1996), which is the ground motion prediction equation (GMPE) considered, magnitude is that of surface waves.

### 3.2.1 Disaggregation Results

The hazard results, computed in terms of PGA and Sa at T = 1s, are in fair agreement with those of INGV, and they are considered as the basis for disaggregation analyses presented in this section. The joint PDFs of M, R and $\varepsilon$ given the exceedance of IM$_0$ corresponding to the four return periods considered, were computed, for each site of the grid, via simulation and using bins of M, R and $\varepsilon$ equal to 0.05, 1.0 and 0.5, respectively. Minimum and maximum values used for $\varepsilon$ are $-3$ and $+3$. Subsequently the first two modes of the joint PDF from disaggregation were extracted. The first mode is identified as the M, R and $\varepsilon$ vector giving the largest contribution to the hazard, while the second mode corresponds to second higher relative largest contribution, identified if the differences between first and second mode are 5.0 km and 0.25 in terms of R and M.

In Figs. 3.2 and 3.3 modes of disaggregation distributions are shown. In the map referring to the second mode, white zones indicate that the hazard contribution of second mode is zero or negligible (i.e., providing a contribution to hazard lower than $10^{-4}$). Analyses show that almost all sites are characterized by at least two DEs. This means that, from a design point of view, for each site it may be useful to
know not only the first mode, but also the second one, in definition of seismic action on structures. Moreover it is possible to identify some general trends: (i) the first mode corresponds to an earthquake caused by the closer source (or the source the site is enclosed into) and with low-to-moderate magnitude, (ii) the second mode accounts for the influence of the more distant zones usually with larger magnitude, and (iii) moving from PGA to Sa, the number of sites with two DEs increases and the contribution of the second mode also increases (see following section). As consequence of (ii) and (iii), it can be inferred that the influence of more distant zones is higher for Sa(1s) than for PGA.

For a few sites, the particular combination of geometrical condition and seismic parameters of each source can determine an inversion of disaggregation results, and in such sites the sources influencing the first mode can be more distant than that related to the second mode. Other exceptions are represented by sites with a single mode; i.e., one DE. These sites are enclosed, or are close to, zones with high seismicity with respect to the surrounding zones which give negligible hazard contribution.
### Table 3.1  Seismic sources’ parameters

<table>
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<tr>
<th>Zone</th>
<th>$M_{\text{min}}$</th>
<th>$M_{\text{max}}$</th>
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<tr>
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<td>6.1</td>
<td>0.043</td>
<td>0.778</td>
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<tr>
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<td>0.609</td>
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<tr>
<td>936</td>
<td>3.7</td>
<td>5.2</td>
<td>0.448</td>
<td>1.219</td>
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</table>

For each zone it is provided: minimum ($M_{\text{min}}$) and maximum magnitude ($M_{\text{max}}$); annual rate of earthquake occurrence above $M_{\text{min}}$ ($v$); and negative slope of Gutenberg-Richter relationship ($b$)

### 3.2.2  An Example of Multimodal Disaggregation

In Convertito et al. (2009) and in Barani et al. (2009) some interesting examples of disaggregation results for individual sites have already been presented: most of those cases are characterized by two different modal values with comparable contributions. In the work presented in this paper the site of Lecce (southern Italy)
Fig. 3.2 First (left) and second (right) DEs for PGA and $\text{Tr} = 475$ years
Fig. 3.3 First (left) and second (right) DEs for Sa(1s) and Tr = 475 years
is considered (latitude 40.338° N, longitude 18.147° E) and disaggregation results are shown for PGA and Sa(1 s) and for Tr = 475 years. In particular the joint PDF obtained from Eq. 3.1 is represented in Fig. 3.4 showing the marginal PDFs of R and M and of R and e.

The considered site is not enclosed in any seismic source and its hazard is affected by sources 931 and 926 with a minimum distance lower than 100 km, and by sources 925, 927, 929 and 930 with a minimum distance between 100 and 200 km. For the combination of the characteristics of all the sources around the site, the PGA and Sa(1s) hazards for 475 years both correspond to 0.053 g.

Disaggregation of PGA hazard shows that the site is characterized by two different modal values: the first one due to R and M equal to about 80 km and 6.8, and the second one due to R equal to 180 km and M equal to 7.3. Hazard contribution of the second mode is much lower.

The same modes are computed for Sa(1s) but, as expected, the increment of spectral period determines increment of hazard contribution of more distant sources and, as consequence, the second mode becomes comparable with the first one.

These results, as noted already in Convertito et al. (2009), point out that even if hazard contribution of the second mode is comparatively low (like in the case of

![Disaggregation results for Lecce for PGA (left) and Sa(1s) (right) and Tr = 475 years](image-url)
a characterization of DEs should prudently account for it. In fact, when looking at spectral ordinates close to the fundamental period of the most common structures, the second mode may become significant. This has engineering consequences because, for example, although given a response spectrum the displacement structural response may not be very sensitive to magnitude and distance (Iervolino and Cornell 2005), ground motions characterized by different magnitudes and source-to-site distances can display different seismic demand, for example, in terms of cyclic structural response (e.g. Iervolino et al. 2006).

### 3.3 Conditional Hazard

Acceleration-based IMs (e.g., spectral ordinates) have been shown to be important and useful in the assessment of structural response of buildings. However, there are cases in which it is desirable to account for additional IMs while defining seismic actions. For example, although it is generally believed that, under some hypotheses, integral ground motion parameters associated to duration are less important for structural demand assessment with respect to peak quantities of ground motion, there are cases in which the cumulative damage potential of the earthquake is also of concern.

An easy yet hazard-consistent way of including secondary IMs in record selection is represented by the conditional hazard maps; i.e., maps of secondary ground motion intensity measures conditional, in a probabilistic sense, to the design hazard for the primary parameter. To illustrate the conditional hazard concept, in Iervolino et al. (2010c) the joint distribution of PGA and a parameter which may account for the cumulative damage potential of ground motion, was investigated. The chosen energy related measure is the so-called *Cosenza and Manfredi index*, the ratio of the integral of the acceleration squared to the PGA and peak ground velocity (PGV). $I_D$ has proven to be a good proxy for cyclic structural response (Manfredi 2001). It is defined in Eq. 3.2 where $a(t)$ is the acceleration time-history, $t_E$ is the total duration of the seismic event. Therefore, the numerator of $I_D$ is proportional to the *Arias Intensity* and it will be referred to as $I_A$.

The best candidates to be ground motion intensity measures are those for which hazard analysis is easy to compute, which requires a ground motion prediction equation (GMPE) to be available. Therefore, a GMPE was developed for $I_D$. The dataset used consists of 190 horizontal components from 95 recordings of Italian earthquakes used by Sabetta and Pugliese (1987). For the purposes of the present study the records were obtained by the *European Strong-motion Database* (ESD) (Ambraseys et al. 2000, Ambraseys et al. 2004). The dataset in terms of magnitude, distance, and site conditions is given in Fig. 3.5 (left).

\[
I_D = \frac{\int_0^{t_E} a^2(t)dt}{\text{PGA} \cdot \text{PGV}} = \frac{I_A}{\text{PGA} \cdot \text{PGV}} \quad (3.2)
\]
The empirical predictive equations for the logs of the IMs (the generic one is indicated as $Y$) appearing in the definition of $I_D$ were fitted by regression using the same functional form of Sabetta and Pugliese (1996), Eq. 3.3, as a function of moment magnitude, epicentral distance (in km), and recording site geology. In this form, $h$ is a fictitious depth, the dummy variables $S_1$ and $S_2$ refer to the site classification and take the value of 1 for shallow and deep alluvium sites, respectively, and zero otherwise. The residual, $\varepsilon_{\log_{10} Y}$, is a random variable which in ordinary least squares regressions, is implicitly assumed to be Gaussian with zero mean and a standard deviation $\sigma_{\log_{10} Y}$.

$$\log_{10} Y = a + b \, M + c \, \log_{10} (R^2 + h^2)^{1/2} + d \, S_1 + e \, S_2 + \varepsilon_{\log_{10} Y} \quad (3.3)$$

The estimates for the coefficients for PGA, PGV and $I_A$, obtained using the ordinary least-squares regression, are given in Table 3.2. In the same table also the estimated standard deviations of the respective residuals, are also given. $h$ values were not estimated and assumed to be coincident to those provided by Sabetta and Pugliese (1996).

In order to obtain an attenuation relation for the logs of $I_D$ as a function of $M$, $R$ and local site conditions it is possible to derive its coefficients as linear combinations of those for $\log_{10} \text{PGA}$, $\log_{10} \text{PGV}$ and $\log_{10} I_A$ leading to the expression of Eq. 3.4, in which subscripts 1, 2 and 3 for $c$ coefficient and $h$ refer to PGA, PGV and $I_A$, respectively.
\[ \log_{10} I_D = a + bM + \log_{10} \left( \frac{(R^2 + h_1^2)^{c_1}(R^2 + h_2^2)^{c_2}}{(R^2 + h_3^2)^{c_3}} \right)^{\frac{1}{2}} + dS_1 + eS_2 + \varepsilon_{\log_{10}I_D} \]  

(3.4)

The coefficients of Eq. 3.4 are listed in Table 3.3. A plot of ID versus epicentral distance is given in Fig. 3.5 (right) where the typical increasing trend with distance of duration-related measures is shown.

Because, if the vector comprised of logs of PGA and ID can be considered jointly normally distributed, all the possible marginal and conditional distributions obtained from the joint distribution are still Gaussian, The skewness and kurtosis’ tests of Mardia (1985) were used to test multivariate normality of the vector made of \( \varepsilon_{\log_{10}PGA} \) and \( \varepsilon_{\log_{10}ID} \). With a given significance level of 0.05, the multivariate skewness and the multivariate kurtosis resulted non-significant.

The residuals of the prediction relationships for the logs of PGA and ID have been also tested for correlation in order to compute \( f(\log_{10}I_D|\log_{10}PGA) \), that is, the conditional PDF of the logs of ID given the logs of PGA. The estimated correlation coefficient (\( r \)) between \( \varepsilon_{\log_{10}PGA} \) and \( \varepsilon_{\log_{10}ID} \) (equal to \(-0.25\)) has been tested for statistical significance using a Student-T statistic Mood et al. (1974) and assuming as the null hypothesis \( r = 0 \) (\( r \) is the “true” correlation coefficient), which was rejected at 0.05 significance level.

Because of bivariate normality, the conditional PDF for one of the variables given a known value of the other, is normally distributed. The conditional mean, \( \mu_{\log_{10}I_D|\log_{10}PGA,M,R} \), and standard deviation of \( \log_{10} I_D \), \( \sigma_{\log_{10}I_D|\log_{10}PGA} \), given that \( \log_{10} PGA = z \), are given in Eq. 3.5 where \( \mu_{\log_{10}I_D|M,R} \) and \( \sigma_{\log_{10}I_D} \) are the mean and the standard deviation from the ID GMPE.; \( \mu_{\log_{10}PGA|M,R} \) and \( \sigma_{\log_{10}PGA} \) are the mean and the standard deviation from the PGA GMPE.

Because the joint distribution of ID and PGA depends on the ID attenuation and from the PGA attenuation, therefore also on magnitude and distance, to obtain the conditional distribution of the logs of ID conditional on PGA only, the marginalization in Eq. 3.6 is required.

\[
\begin{align*}
\mu_{\log_{10}I_D|\log_{10}PGA,M,R} &= \mu_{\log_{10}I_D|M,R} + \rho \, \frac{\mu_{\log_{10}PGA|M,R} - z}{\sigma_{\log_{10}PGA}} \\
\sigma_{\log_{10}I_D|\log_{10}PGA} &= \sigma_{\log_{10}I_D} \sqrt{1 - \rho^2} 
\end{align*}
\]

(3.5)

\[
f(\log_{10}I_D|\log_{10}PGA) = \int \int f(\log_{10}I_D|\log_{10}PGA,M,R)f(M,R|\log_{10}PGA)dm \, dr
\]

(3.6)
It is easy to recognize that the \( f(M, R | \log_{10}\text{PGA}) \) term in Eq. 3.6 is the PDF of \( M \) and \( R \) given the occurrence of \( \log_{10}\text{PGA} \); i.e., the result of disaggregation of seismic hazard. As an approximation of the integral, the modal values \( M^* \) and \( R^* \) (i.e., the first DE) may be plugged in Eq. 3.5; i.e., Eq. 3.7.

\[
\mu_{\log_{10}\text{ID} | \log_{10}\text{PGA}} \approx \mu_{\log_{10}\text{ID} | M^*, R^*} + \rho \sigma_{\log_{10}\text{ID}} \frac{z - \mu_{\log_{10}\text{PGA} | M^*, R^*}}{\sigma_{\log_{10}\text{PGA}}} \tag{3.7}
\]

### 3.4 Illustrative Application

An example of the possible use of conditional hazard is given in Fig. 3.6 which shows the maps of seismic hazard in terms of \( \text{ID} \) conditional to the PGA with a 10\% exceedance probability in 50 years in Italy. In particular, Fig. 3.6a,b are the 50th and 90th percentiles of the conditional \( \text{ID} \) PDF, respectively.

The conditional \( \text{ID} \) maps were obtained using the distribution of parameters in Eq. 3.5 in which the \( z \) (log of PGA) values are those computed in the hazard analysis described in Sect. 2, while the values of magnitude and distance (\( M^* \) and \( R^* \)) to plug in the \( \mu_{\log_{10}\text{PGA} | M, R} \) and \( \mu_{\log_{10}\text{ID} | M, R} \) terms of Eq. 3.7 are those shown in Fig. 3.2a, c; i.e., the design earthquakes\(^2\) corresponding to the PGA on which the \( \text{ID} \) distribution is conditional.

![Fig. 3.6 Maps of ID in terms of 50th (a) and 90th (b) percentiles conditional to PGA with a 475 years return period and using first DEs of Fig. 3.2](image)

\(^2\)For the purposes of the illustrative application it is assumed that moment magnitude can be approximated by surface-wave magnitude. Moreover, Eq. 3.5 requires disaggregation for the occurrence of PGA, while herein DEs from disaggregation of exceedance hazard are considered.
3.5 Conclusions

Disaggregation can be considered as a useful tool to address the definition of design scenarios to be used in engineering practice. In the work presented in this paper design earthquakes from disaggregation of all Italian sites for structural periods equal to 0s and 1s was presented referring to hazard with a 475 years exceedance return period. First and second modal values are used here as synthetic identifiers of DEs.

Results show that, usually, the modal value with the largest contribution to hazard corresponds to a moderate-magnitude earthquake caused by the closer source, while the influence of the more distant zones is accounted for by the second mode. Moreover, because in most cases Italian sites are located inside seismogenetic zones assumed for hazard analysis, first mode of disaggregation is characterized by a source-to-site distance lower than 10 km. For Sa at $T = 1s$, the contribution of more distant sources is higher than in the PGA case. Finally it is to conclude that only a few sites are characterized by a single DE and this is particularly evident from disaggregation of Sa(1s) hazard, which is more representative than PGA for ordinary structures.

An immediately intelligible use of design earthquakes is ground motion record selection for nonlinear dynamic analysis of structures. However, there are other possible uses, one of which occurs when more than one ground motion parameter has to be taken into account in seismic structural assessment. For example, although it is generally believed that integral ground-motion parameters are secondary for structural demand assessment with respect to peak quantities of ground motion, sometimes the cumulative damage potential of the earthquake is also of concern. For these cases it could be useful to have a distribution of secondary intensity measures conditional on the primary parameter used to define the seismic action on structures (e.g., accelerations). Under some hypotheses, this can be carried out in close-form and was called conditional hazard.

This approach has the advantages of vector-valued seismic hazard analysis without the computational effort required by PSHA for vectors of IMs. To explore such a concept, in this paper the distribution of a parameter which may account for the cumulative damage potential of ground-motion, conditional to peak ground acceleration (PGA), was investigated. The chosen secondary measure is the so called Cosenza and Manfredi index ($I_D$). A ground-motion prediction relationship has been retrieved for the log of $I_D$ on the basis of an empirical dataset of Italian records already used for well known prediction equations proposed in the past by other researchers. Subsequently, the residuals of prediction relationships have been tested for correlation and for joint normality. The study allowed to obtain analytical distributions of $I_D$ conditional on PGA and the corresponding design earthquakes. Results of the study have been used to compute, as an illustrative example, the distributions of $I_D$ conditional on PGA with a return period of 475 years for each node of a regular grid having about 2 km spacing and covering Italy.
The presented conditional hazard maps provide information on the values of ID which, for example, should be taken into account along with the hazard in terms of PGA at the site. In fact, conditional hazard can complement the hazard curves or maps produced for the primary IM.

Conditional hazard may be extended, in principle, to any vector of IMs.

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References

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