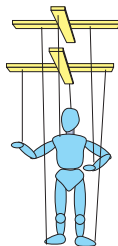# Robotic Telemanipulation:
A Brief Introduction on Modelling and Control Issues

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Telemanipulation

Summary

1. Passivity Theory

2. Modelling a teleoperation system

3. Control schemes
   - Traditional force reflection
   - Shared compliance control
   - Passive control
   - 4-Channel control

Passivity Theory

Passivity:

- represents a powerful and elegant tool for the analysis and control design for both linear and non linear dynamic systems;
- is a mathematical description of the physical concepts of power and energy;
- is very closely related to the stability theory (Lyapunov, energy functions);
- allows the design of control strategies for the interaction with arbitrary passive environments without too concern on modelling and estimation;
- human operators can easily deal with passive systems;
- transient phases are not well described;
- not easy to satisfy design constraints (performance).
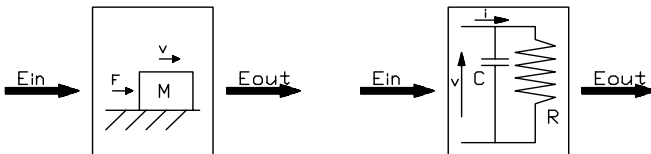
Passivity: some definitions

Let us introduce the following notations:

- $\mathcal{T}$, a subset of $\mathbb{R}_+$ (in general the time domain);
- $\mathcal{V}$, a vector space with the usual Euclidean norm $\|\cdot\|$ (usually $\mathbb{R}^n$);
- $\mathcal{F}$, the set of all functions mapping $\mathcal{T}$ in $\mathcal{V}$: $\{\mathbf{f} : \mathcal{T} \mapsto \mathcal{V}\}$, with the properties of a linear space;

and the *causal truncation* operator (defined in the case of infinite time functions)

$$P_T\mathbf{f}(t) = \left\{ \begin{array}{ll} \mathbf{f}(t) & t \leq T \\ \mathbf{0} & t > T \end{array} \right. \qquad\qquad \mathbf{f}(t) \in \mathcal{F}, \;\; t, T \in \mathcal{T}$$

Passivity: some definitions

Moreover, let us consider:

1. the normed linear subspace, $\mathcal{L}_2^n$, of the linear space $\mathcal{F}$ (the Hilbert space of the functions measurable in the Lebesgue sense):

$$\mathcal{L}_2^n(\mathcal{T}) = \left\{ \mathbf{f} : \mathcal{T} \mapsto \mathcal{V}, \left(\int_0^\infty \|\mathbf{f}\|^2 dt \right)^{1/2} < \infty \right\}$$

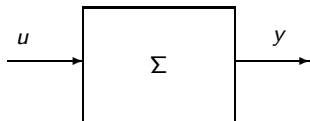2. the extended space $\mathcal{L}_{2e}^n$ associated to $\mathcal{L}_2^n$:

$$\mathcal{L}_{2e}^n(\mathcal{T}) = \{ \mathbf{f}(t) : P_\mathcal{T} \mathbf{f}(t) \in \mathcal{L}_2^n(\mathcal{T}) \}$$

$\mathcal{L}_{2e}^n(\mathcal{T})$ is the space of functions whose causal truncation belongs to $\mathcal{L}_2^n(\mathcal{T})$.

Moreover:

- $\forall \mathbf{f} \in \mathcal{L}_{2e}^n(\mathcal{T})$, the map $\mathcal{T} \mapsto \|P_\mathcal{T} \mathbf{f}\|$ is monotonically increasing,
- $\forall \mathbf{f} \in \mathcal{L}_2^n(\mathcal{T})$, the map $\|P_\mathcal{T} \mathbf{f}\| \mapsto \|\mathbf{f}\|$ as $\mathcal{T} \mapsto \infty$.

Passivity: some definitions

Consider a dynamic system described in the space state as

$$\begin{cases} \dot{\mathbf{x}}(t) &= \mathbf{f}(\mathbf{x}(t)) + \mathbf{g}(\mathbf{x}(t))\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{h}(\mathbf{x}(t)) \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathcal{L}_{2e}^n(\mathcal{T})$, $\mathbf{y} \in \mathcal{L}_{2e}^m(\mathcal{T})$, $\mathbf{u} \in \mathcal{L}_{2e}^m(\mathcal{T})$.

Assume the functions $\mathbf{f}$, $\mathbf{g}$, $\mathbf{h}$ smooth in $\mathbf{x}$, with $\mathbf{f(0)} = \mathbf{0}$, $\mathbf{h(0)} = \mathbf{0}$.

Passivity: some definitions

A function $w(t)$, called *SUPPLY RATE*, of the input $\mathbf{u}(t)$ and of the output $\mathbf{y}(t)$ is defined as

$$w(t) = w(\mathbf{u}(t), \mathbf{y}(t))$$

The function $w(t)$ is assumed to be locally integrable, i.e.

$$\int_{t_0}^{t_1} \|w(t)\| dt < \infty, \quad \forall t_0, t_1 \in \mathbb{R}_+$$

In the following, this function is assumed of the form

$$w(t) = \mathbf{y}^T(t)\mathbf{u}(t)$$

Passivity: some definitions

Definition: The system (1) is said to be *PASSIVE* if there exists a continuous function, called *storage function*, $V(\mathbf{x}) \geq 0$, $V : \mathcal{L}_{2e}^n(\mathcal{T}) \to \mathbb{R}_+$, which satisfies $V(\mathbf{0}) = 0$, such that

$$\int_{t_0}^{t} \mathbf{y}^T(\tau)\mathbf{u}(\tau)d\tau \geq V(\mathbf{x}(t)) - V(\mathbf{x}(t_0)) \qquad (2)$$

This equation is obviously equivalent to:

$$V(t) \leq V(t_0) + \int_{t_0}^{t} \mathbf{y}^T(\tau)\mathbf{u}(\tau)d\tau$$

$V(t)$ can be interpreted as the *energy* of the system: physically, a passive system cannot produce energy.

Passivity: some definitions

Definition: The system (1) is said to be *STRICTLY PASSIVE* if there exists a continuous (storage) function, $V(\mathbf{x}) \geq 0$, $V : \mathcal{L}_{2e}^n(\mathcal{T}) \to \mathbb{R}_+$, which satisfies $V(\mathbf{0}) = 0$, and a positive definite function, called *dissipation rate*, $\phi(\mathbf{x}(t))$, $\phi > 0$, $\phi : \mathcal{L}_{2e}^n(\mathcal{T}) \to \mathbb{R}_+$ such that

$$\int_{t_0}^{t} \mathbf{y}^T(\tau)\mathbf{u}(\tau)d\tau \geq V(\mathbf{x}(t)) - V(\mathbf{x}(t_0)) + \int_{t_0}^{t} \phi(\mathbf{x}(\tau))d\tau \qquad (3)$$

Or equivalently:

$$V(t) \leq V(t_0) + \int_{t_0}^{t} \mathbf{y}^T(\tau)\mathbf{u}(\tau)d\tau - \int_{t_0}^{t} \phi(\mathbf{x}(\tau))d\tau$$

Passivity: some definitions

The definition of passivity given in eq. (2) is often reported in the literature in the differential form as

$$\dot{V}(\mathbf{x}(t)) \leq \mathbf{y}^T(t)\mathbf{u}(t)$$

or, by explicitly introducing the dissipation rate function ϕ, as

$$\dot{V}(\mathbf{x}(t)) = \mathbf{y}^T(t)\mathbf{u}(t) - \phi(\mathbf{x}(t)) \tag{4}$$

which reflects the concept of the conservation of energy of a physical system.

Passivity: some definitions

As previously mentioned, passivity and Lyapunov stability are closely related concepts. In fact, the following result holds.

Lemma Let suppose the system (1) be (strictly) passive. If the storage function $V(\mathbf{x})$ is positive definite, radially unbounded, and decrescent, then, for $\mathbf{u} \equiv \mathbf{0}$, the equilibrium $\mathbf{x} = \mathbf{0}$ of (1) is globally uniformly (asymptotically) stable.
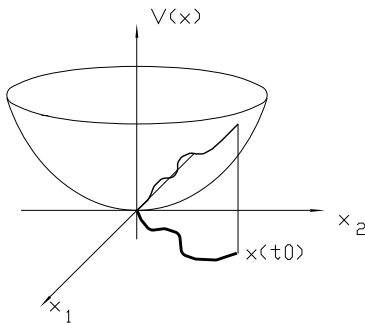
Passivity: some useful properties

1. The passivity formalism may be easily applied to the analysis of stability properties of composed systems.
   For example:
   - a combination of passive subsystems is passive,
   - if at least one subsystem is dissipative, then the overall combination is asymptotically stable.

2. If a system with input $\mathbf{u}(t)$ and output $\mathbf{y}(t)$ is passive, then a linear transformation given by a mapping $\mathbf{Au} \to \mathbf{A}^{-T}\mathbf{y}$ yields a passive system.

3. More in general, the passivity of a system is not changed by a transformation expressed by an orthogonal matrix $\mathbf{A}$ (i.e. a matrix such that $\mathbf{AA}^T = \mathbf{I}$).

Passivity concepts have been widely used in robotics:

- to study the stability properties of robots interacting with unknown environments,
- for the robustness analysis of force control schemes,
- for the development of *haptic devices*,
- for the design of *telemanipulation control systems*.

Robotic Telemanipulation:
A Brief Introduction on Modelling and Control Issues
Modelling Aspects

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Modelling a teleoperation system

Network analogy: bilateral teleoperation systems can be viewed as a cascade interconnection of two-port (master, communication channel and slave) and one-port (operator and environment) blocks.
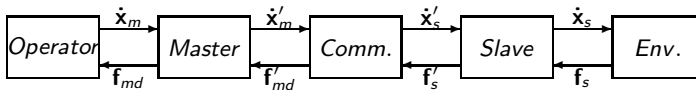
By means of the mechanical/electrical analogy and of network theory, the teleoperation system is described as interconnection of one and two-port electrical elements.
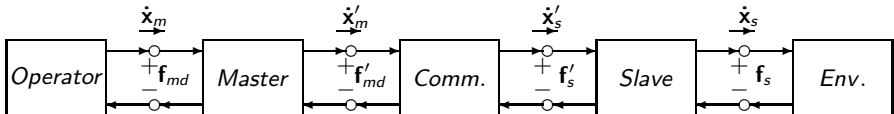
Modelling a teleoperation system

Linear Time-Invariant (LTI) continous systems can be described by the relationships between the effort and flow variables:

	effort variable	**flow variable**
mechanical system	force/torque applied to the system	linear/angular velocity of the system
electrical system	voltage across the terminals	current through the network

The analogy is based on similarities between the following variables of mechanical and electrical systems:

Electrical		$\Longleftrightarrow$	**Mechanical**	
Voltage	$V(t)$		Force	$f(t)$
Current	$I(t)$		Velocity	$\dot{x}(t)$
Resistance	R		Viscous friction	b
Inductance	L		Inertia	M
Capacitance	$1/K$		Stiffness	$f(t) = 1/K \int v(t)dt$
One-port impedance	Z		Series/parallel of previous elements	$f(s) = Z(s)\, v(s)$

Description of a teleoperation

Each two-port element and the overall teleoperation system:
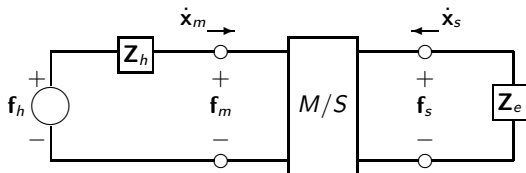
can be described in terms of:

$$
\begin{aligned}
(\mathbf{f}_m, \mathbf{f}_s, t) &= \mathbf{Z}(\dot{\mathbf{x}}_m, -\dot{\mathbf{x}}_s, t) &\quad &\textit{impedance operator} \\
(\dot{\mathbf{x}}_m, -\dot{\mathbf{x}}_s, t) &= \mathbf{Y}(\mathbf{f}_m, \mathbf{f}_s, t) &\quad &\textit{admittance operator} \\
(\mathbf{f}_m, -\dot{\mathbf{x}}_s, t) &= \mathbf{H}(\mathbf{f}_s, \dot{\mathbf{x}}_m, t) &\quad &\textit{hybrid operator} \\
(\mathbf{f}_m, \dot{\mathbf{x}}_m, t) &= \mathbf{C}(\mathbf{f}_s, -\dot{\mathbf{x}}_s, t) &\quad &\textit{chain operator} \\
(\mathbf{f} - b\,\dot{\mathbf{x}}, t) &= \mathbf{S}(\mathbf{f} + b\,\dot{\mathbf{x}}, t) &\quad &\textit{scattering operator}
\end{aligned}
$$

For LTI systems the operators $\mathbf{Z}, \mathbf{Y}, \mathbf{H}, \mathbf{C}, \mathbf{S}$ are transfer matrices.

Description of a teleoperation system

The most convenient description can be chosen considering:

- distinction between independent and dependent variables,
- generality of the description,
- *stability analysis*,
- *performance evaluation criteria* (hybrid matrix).

Impedance matrix
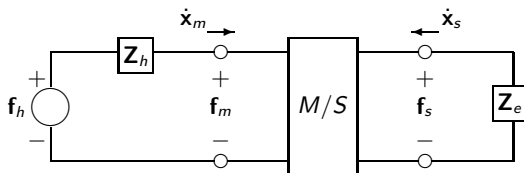
Considering a LTI two-port element, it is possible to introduce the *impedance matrix* $\mathbf{Z}(s)$ which relates effort ($\mathbf{f}$) and flow ($\dot{\mathbf{x}}$) variables as:

$$\mathbf{f}(s) = \mathbf{Z}(s)\,\dot{\mathbf{x}}(s)$$

Considering the master/slave system as a connection of consecutive two-port elements, the impedance matrix of the overall system is given by:

$$\mathbf{f}(s) = \left[\begin{array}{c} \mathbf{f}_m(s) \\ \mathbf{f}_s(s) \end{array} \right] = \left[\begin{array}{cc} \mathbf{Z}_{mm}(s) & \mathbf{Z}_{ms}(s) \\ \mathbf{Z}_{sm}(s) & \mathbf{Z}_{ss}(s) \end{array} \right] \left[\begin{array}{c} \dot{\mathbf{x}}_m(s) \\ \dot{\mathbf{x}}_s(s) \end{array} \right] = \mathbf{Z}(s)\dot{\mathbf{x}}(s)$$

Impedance matrix

$$\mathbf{Z}(s) = \left[\begin{array}{cc} \mathbf{Z}_{mm}(s) & \mathbf{Z}_{ms}(s) \\ \mathbf{Z}_{sm}(s) & \mathbf{Z}_{ss}(s) \end{array} \right]$$

With respect to the block structure of the impedance matrix $\mathbf{Z}(s)$, two particular cases are of interest:

- Bilateral Teleoperation: the impedance matrix (the teleoperator) is defined <u>bilateral</u> when both the off-diagonal blocks of $\mathbf{Z}(s)$, $\mathbf{Z}_{ms}$ and $\mathbf{Z}_{sm}$, are not null:
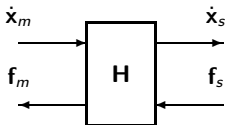
$$\mathbf{Z}_{ms} \neq \mathbf{0} \qquad \mathbf{Z}_{sm} \neq \mathbf{0}$$

- Reciprocal Teleoperation: the impedance matrix (the teleoperator) is defined <u>reciprocal</u> when the off-diagonal blocks of $\mathbf{Z}(s)$, $\mathbf{Z}_{ms}$ and $\mathbf{Z}_{sm}$, are equal:

$$\mathbf{Z}_{ms} = \mathbf{Z}_{sm}$$

Hybrid matrix

A more general description for teleoperation systems is given by the *hybrid matrix*, defined according to the following sign convention:

$$\left[\begin{array}{c} \mathbf{f}_m(s) \\ -\dot{\mathbf{x}}_s(s) \end{array} \right] = \left[\begin{array}{cc} \mathbf{h}_{11}(s) & \mathbf{h}_{12}(s) \\ \mathbf{h}_{21}(s) & \mathbf{h}_{22}(s) \end{array} \right] \left[\begin{array}{c} \dot{\mathbf{x}}_m(s) \\ \mathbf{f}_s(s) \end{array} \right] = \mathbf{H}(s) \left[\begin{array}{c} \dot{\mathbf{x}}_m(s) \\ \mathbf{f}_s(s) \end{array} \right]$$

where:

$$\mathbf{h}_{11} = \left. \left[\frac{\partial \mathbf{f}_m}{\partial \dot{\mathbf{x}}_m} \right] \right|_{\mathbf{f}_s=0} \qquad\qquad \mathbf{h}_{12} = \left. \left[\frac{\partial \mathbf{f}_m}{\partial \mathbf{f}_s} \right] \right|_{\dot{\mathbf{x}}_m=0}$$

$$\mathbf{h}_{21} = \left. \left[-\frac{\partial \dot{\mathbf{x}}_s}{\partial \dot{\mathbf{x}}_m} \right] \right|_{\mathbf{f}_s=0} \qquad\qquad \mathbf{h}_{22} = \left. \left[-\frac{\partial \dot{\mathbf{x}}_s}{\partial \mathbf{f}_s} \right] \right|_{\dot{\mathbf{x}}_m=0}$$

Hybrid matrix

Physical meaning of the hybrid matrix elements:

$$\left[\begin{array}{c} \mathbf{f}_m \\ -\dot{\mathbf{x}}_s \end{array} \right] = \left[\begin{array}{cc} \mathbf{Z}_{in} & \text{force ratio} \\ \text{velocity ratio} & \mathbf{Z}_{out}^{-1} \end{array} \right] \left[\begin{array}{c} \dot{\mathbf{x}}_m \\ \mathbf{f}_s \end{array} \right]$$

$\mathbf{Z}_{in}$ and $\mathbf{Z}_{out}$: input and output teleoperator impedances.

**IDEAL HYBRID MATRIX**. In case of ideal telepresence, forces and velocities of master and slave are equal, and therefore:

$$\left\{ \begin{array}{ccc} \mathbf{f}_s & = & \mathbf{f}_m \\ \dot{\mathbf{x}}_s & = & \dot{\mathbf{x}}_m \end{array} \right. \qquad \Longrightarrow \qquad \mathbf{H}_{ideal}(s) = \left[\begin{array}{cc} \mathbf{0}_n & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0}_n \end{array} \right]$$

Scattering representation - Wave variables

Let consider the total power flow in a two-port element as composed of two terms, the *input power P_{in}* and *output power P_{out}*:

$$
\begin{aligned}
P &= P_{in} - P_{out} = \mathbf{f}^T \dot{\mathbf{x}} \\
&= [\mathbf{f}_m^T, \mathbf{f}_s^T] \begin{bmatrix} \dot{\mathbf{x}}_m \\ -\dot{\mathbf{x}}_s \end{bmatrix} \\
&= \frac{1}{2}\left(\mathbf{u}^T\mathbf{u} - \mathbf{v}^T\mathbf{v}\right)
\end{aligned}
$$
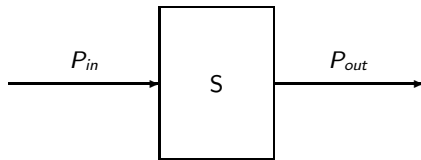
where:

$$\mathbf{u} = [\mathbf{u}_m^T \ \mathbf{u}_s^T]^T \qquad\qquad \mathbf{v} = [\mathbf{v}_m^T \ \mathbf{v}_s^T]^T$$

$$\mathbf{u}_m = \frac{1}{\sqrt{2b}}(\mathbf{f}_m + b\dot{\mathbf{x}}_m) \qquad \mathbf{u}_s = \frac{1}{\sqrt{2b}}(\mathbf{f}_s - b\dot{\mathbf{x}}_s)$$

$$\mathbf{v}_m = \frac{1}{\sqrt{2b}}(\mathbf{f}_m - b\dot{\mathbf{x}}_m) \qquad \mathbf{v}_s = \frac{1}{\sqrt{2b}}(\mathbf{f}_s + b\dot{\mathbf{x}}_s)$$
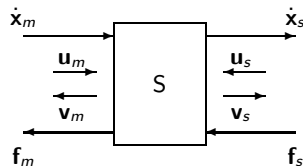
NOTE: a scaling factor b is assumed between forces and velocities

Scattering representation - Wave variables

With

$$\mathbf{u} \rightarrow \text{input wave} \qquad \mathbf{v} \rightarrow \text{output wave}$$

- Wave variables can be applied both to linear and non-linear systems.
- From physical intuition, in stable system the "amplitude" of $\mathbf{v}$ is less than the "amplitute" of $\mathbf{u}$:

$$\frac{1}{2} \int \mathbf{v}^T \mathbf{v} dt \leq \frac{1}{2} \int \mathbf{u}^T \mathbf{u} dt$$

  i.e. the "gain" of the system is less than one.
- In the definition of wave variables, time-delay is not considered: time-delay does not affect the "amplitude" of the wave.
- Different I/O properties may be achieved introducing passive elements.

Scattering matrix

The Scattering operator (or matrix) relates the input/output *wave variables*, $\mathbf{u}$ and $\mathbf{v}$, at each port of the teleoperator instead of the power variables, $\dot{\mathbf{x}}$ and $\mathbf{f}$.

Definition. Given an *n*-port system, the *scattering matrix* (or *scattering operator*) is defined as the operator which relates input and output wave variables as:

$$\mathbf{v}(t) = \mathbf{S}(t)\mathbf{u}(t) \qquad \Longleftrightarrow \qquad \mathbf{f}(t) - b\dot{\mathbf{x}}(t) = \mathbf{S}(t)\left[\mathbf{f}(t) + b\dot{\mathbf{x}}(t)\right]$$

for LTI system:

$$\mathbf{v}(s) = \mathbf{S}(s)\mathbf{u}(s) \qquad \Longleftrightarrow \qquad \mathbf{f}(s) - b\dot{\mathbf{x}}(s) = \mathbf{S}(s)\left[\mathbf{f}(s) + b\dot{\mathbf{x}}(s)\right]$$

Scattering and hybrid representation are related by the equation:

$$\mathbf{S}(s) = \left[\begin{array}{cc} \mathbf{I}_n & \mathbf{0}_n \\ \mathbf{0}_n & -\mathbf{I}_n \end{array}\right]\left[\mathbf{H}(s) - \mathbf{I}_{2n}\right]\left[\mathbf{H}(s) + \mathbf{I}_{2n}\right]^{-1}$$

Scattering vs Passivity

Theorem. An LTI n-port element with scattering matrix $\mathbf{S}(s)$ is passive if and only if

$$\|\mathbf{S}(s)\| \leq 1$$

Corollary. An LTI n-port element described by the scattering matrix $\mathbf{S}(s)$ is passive if and only if

$$\|\mathbf{S}(s)\| = \sup_{\omega} \lambda_{max}^{1/2} \{\mathbf{S}^*(j\omega)\mathbf{S}(j\omega)\} \leq 1$$

where $\lambda_{max}\{\mathbf{S}(j\omega)\}$ is the maximum eigenvalue and $\mathbf{S}^*(j\omega)$ the transpose conjugate of $\mathbf{S}(j\omega)$.

Scattering vs Passivity

Considerations.

- Connection of passive *n*-port preserves passivity.

- In traditional force reflection teleoperation systems time-delay instabilities are originated by the non-passive features of the communication line (local controllers assuming to stabilize the respective subsystems).

- The use of a particular communication channel based on the analogous of a lossless transmission line results in a passive communication channel (Passivity Based Teleoperation).

Passivity Theory
Modelling a teleoperation system
Control schemes

Traditional force reflection
Shared compliance control
Passive control
4-Channel control

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Some Control Schemes

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Some control schemes

Some general considerations:

1. In telemanipulation, a relevant control problem is given by the time-delay introduced by the communication channel (more than robots control or slave/environment interaction features);

2. Presence of time-delay has to be considered for the stability problems;

3. Force feedback to the operator and local compliance control at the slave site have to be designed for avoiding excessive contact forces;

4. Telemanipulation systems where only position information are transmitted between master and slave result in very stiff devices with poor performances;

5. In reliable teleoperation a coordination signal (force reflection) is required;

6. Direct reflection of the force signal can result in unstable systems.

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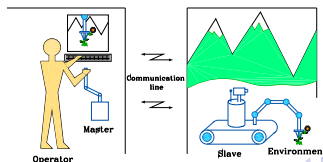
Some control schemes

Some well known bilateral teleoperation systems:

1. "Traditional" force relection teleoperation (TFR);
2. "Shared Compliance Control" teleoperation (SCC);
3. Passivity-Based teleoperation;
4. Predictive control.

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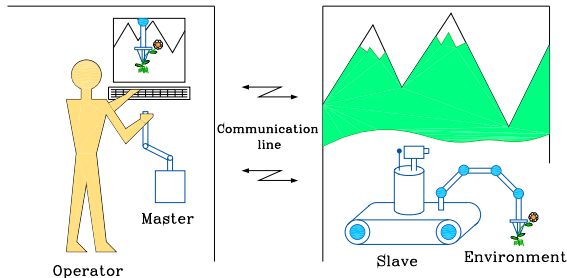
Some control schemes

These control schemes are investigated considering:

1. **Presence of time-delays.** The control schemes will be described and their features in the presence of time-delays discussed.

2. **Scattering theory.** The scattering analysis will be adopted in order to investigate the passivity properties of the control methodologies (hybrid and scattering matrices).

3. **Communication line.** The control schemes are mainly based on different methodologies for the computation of the coordination signal, without dealing with the local controllers.
   The analysis considers the communication line properties as a key factor for a suitable definition of the coordination in presence of time-delays.

4. **Limitations.** In the following analysis, the human operator and the environment model are not considered.

5. **Experimental activity.** A simple 1 dof teleoperation device (two one-dof "robots" position and force sensorized) is used for implementing the different control methodologies.

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Traditional force reflection
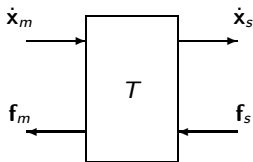
Consider the master/slave systems connected by the simple communication "law":

$$\begin{cases} \mathbf{f}_{md}(t) & = & \mathbf{f}_s(t - T) \\ \dot{\mathbf{x}}_{sd}(t) & = & \dot{\mathbf{x}}_m(t - T) \end{cases}$$

where T is the time-delay due to the communication network.

Considerations:

- even in presence of small time-delay (limited bandwidth) instabilities appear;
- the insertion of a force reflection gain $G_{fr} < 1$, i.e.

$$\mathbf{f}_{md}(t) = G_{fr}\ \mathbf{f}_s(t - T) \qquad\qquad G_{fr} < 1$$

  reduces the performances without producing a valuable improvement of the stability properties;
- the communication channel does not result passive, and this originates instability;
- the non-passive communication channel introduces power contributions in the overall system.
  These contributions have to be compensated by introducing attenuation in the local controllers.

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Modelling the TFR

Given:

1. the master dynamics and communication variables (local master controller):

$$\begin{cases} \mathbf{M}_m\ddot{\mathbf{x}}_m(t) &=& -\mathbf{f}_{md}(t) - \mathbf{B}_m\dot{\mathbf{x}}_m(t) - \mathbf{K}_h\mathbf{x}_m(t) \\ \mathbf{f}_{md}(t) &=& \mathbf{f}_s(t-T) \end{cases}$$

2. the slave dynamics, communication variables and slave local controller:

$$\begin{cases} \mathbf{M}_s\ddot{\mathbf{x}}_s(t) &=& \mathbf{f}_s(t) - \mathbf{B}_s\dot{\mathbf{x}}_s(t) \\ \dot{\mathbf{x}}_{sd}(t) &=& \dot{\mathbf{x}}_m(t-T) \\ \mathbf{f}_s(t) &=& \mathbf{K}_p[\mathbf{x}_{sd}(t) - \mathbf{x}_s(t)] \end{cases}$$

where:

- $\mathbf{M}_i$, $\mathbf{B}_i$ master/slave masses and damping factors,
- $\mathbf{K}_h$ human operator model (stiffness),
- $\mathbf{K}_p$ slave position controller.

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Modelling the TFR

From

$$\left[\begin{array}{c} \mathbf{f}_m \\ -\dot{\mathbf{x}}_s \end{array} \right] = \mathbf{H} \left[\begin{array}{c} \dot{\mathbf{x}}_m \\ \mathbf{f}_s \end{array} \right]$$

the following hybrid matrix is obtained:

$$\mathbf{H}(s) = \left[\begin{array}{cc} 0 & e^{-sT} \\ -e^{-sT} & 0 \end{array} \right]$$

By using the hybrid/scattering relationship, the scattering matrix $\mathbf{S}(s)$, computed for $s = j\omega$, is

$$\mathbf{S}(j\omega) = \left[\begin{array}{cc} -j\tan(\omega T) & \sec(\omega T) \\ \sec(\omega T) & j\tan(\omega T) \end{array} \right]$$

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Modelling the TFR

The norm of the scattering matrix for TFR is:

$$\|\mathbf{S}(j\omega)\| = \sup_{\omega}\{ \ |\tan(\omega T)| \ + \ |\sec(\omega T)| \ \}$$

The norm of the scattering matrix results infinite and the passivity conditions are not verified even for very low time-delays T.

In practical applications, stability can be achieved only by inserting attenuation at the local controllers in order to compensate the power components introduced by the communication channel.

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TFR: scattering analysis

The maximum singular value of the scattering matrix, $\sigma_{max}\{\mathbf{S}(j\omega)\}$, of TFR teleoperators is reported in the figure:

- as a function of ωT,
- for different values of the force reflection gain G_{fr}.
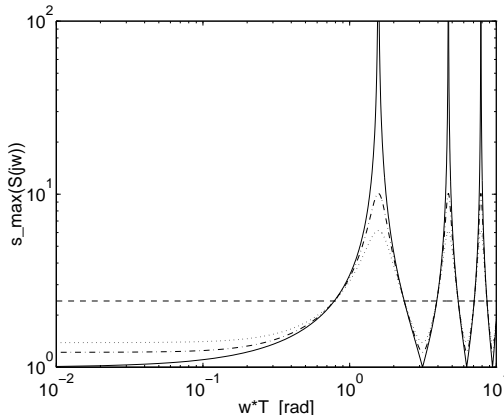
- $G_{fr} = 0.0$ (dashed),
- $G_{fr} = 0.5$ (dotted),
- $G_{fr} = 1.0$ (solid),
- $G_{fr} = 1.5$ (dashdot).

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TFR: scattering analysis
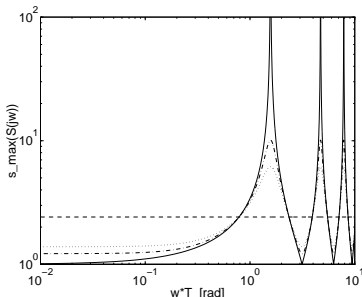
- $\|\mathbf{S}(j\omega)\| \geq 1$, $\forall\ G_{fr} \geq 0.0$, i.e. the system is not passive $\forall G_{fr}$.
- The norm of the scattering matrix is unbounded for $G_{fr} = 1.0$, bounded for $G_{fr} \neq 1.0$.
- The non-passivity features of the TFR do not change by reducing the force reflection gain.

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TFR Verification

- Repetitive operator actions (force impulses) on the master,
- without interaction at the slave site,
- time-delays programmed to:

$T = 0.01\ s$

(a) operator/environment forces
(b) master/slave positions

(c) master/slave forces
(d) master/slave torques

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TFR Verification

- Repetitive operator actions (force impulses) on the master,
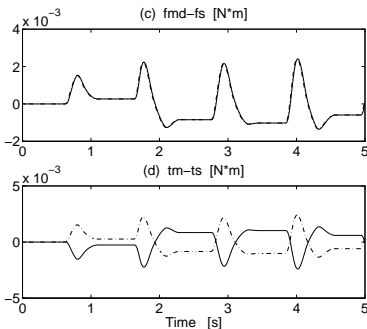- without interaction at the slave site,
- time-delays programmed to:

$T = 0.1\ s$

(a) operator/environment forces
(b) master/slave positions

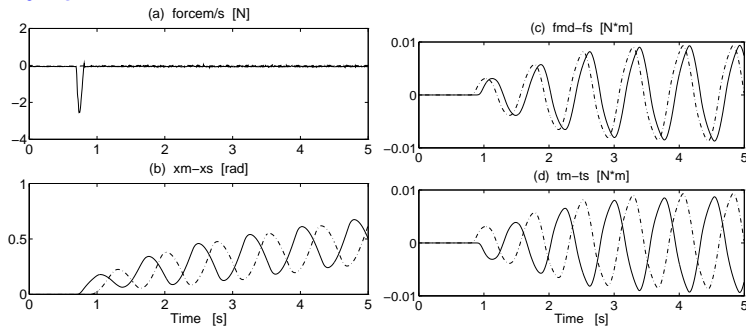(c) master/slave forces
(d) master/slave torques

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Shared Compliance Control (SCC)

Shared Compliance Control deals with:

- interaction of the slave with the environment;
- the time-delay problems.

Two basic features:

- the particular coordination signal;
- sharing the teleoperated capability with some degree of slave autonomy.

Coordination signal: based on the *Position-Error Based Force Reflection*

$$\mathbf{f}_{md}(t) = G_{fr} \left[\mathbf{x}_m(t) - \mathbf{x}_s(t - T) \right]$$

It is proportional to the error of the actual master posture and the delayed slave one, through the force reflection gain G_{fr}.

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Shared Compliance Control (SCC)

$$\mathbf{f}_{md}(t) = G_{fr} \left[\mathbf{x}_m(t) - \mathbf{x}_s(t - T) \right]$$

The force reflection can be originated by:

1. interaction at the slave site,
2. time-delays.

Note that the coordination strategy introduces a *compliance* between the robot positions.

Shared control: an autonomous compliance controller is realized at the slave site.
Shared compliance is a key-factor for overcoming instabilities due to time-delays.

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Shared Compliance Control (SCC)

Overall control scheme:
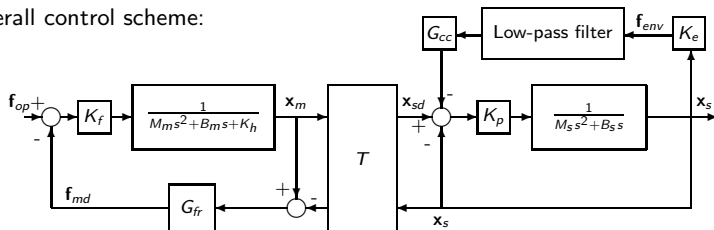
master dynamics	$1/(M_m s^2 + B_m s + K_h)$
slave dynamics	$1/(M_s s^2 + B_s s)$
force reflection gain	G_{fr}
shared compliance controller	G_{cc}
environment model	K_e

- Force measurements are used at the slave site (compliance controller).
- Position errors for deriving the coordination signal.

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SCC: Stability Analysis

The time-domain description of the network is:

$$\left\{ \begin{array}{rcl} \mathbf{f}_{md}(t) &=& G_{fr} \left[\mathbf{x}_m(t) - \mathbf{x}_s(t-T)\right] \\ \mathbf{x}_{sd}(t) &=& \mathbf{x}_m(t-T) \end{array} \right.$$

In order to apply the network theory and compute the hybrid matrix, we consider the following scheme:
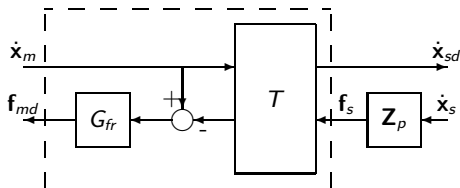
The impedance $\mathbf{Z}_p$ is introduced in order to represent the slave variables as power factors $\dot{\mathbf{x}}_{sd}$ and $\mathbf{f}_s$, thus allowing the hybrid representation.

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SCC: Stability Analysis

If an unitary value for $\mathbf{Z}_p$ is considered, the system is described by:

$$\mathbf{H}(s) = \left[\begin{array}{cc} \frac{G_{fr}}{s} & -G_{fr}e^{-sT} \\ -e^{-sT} & 0 \end{array} \right]$$

and the scattering matrix is:

$$\mathbf{S}(j\omega) = \left[\begin{array}{cc} \dfrac{j\omega - G_{fr} + G_{fr}e^{-2j\omega T}j\omega}{-j\omega - G_{fr} + G_{fr}e^{-2j\omega T}j\omega} & \dfrac{-2G_{fr}e^{-j\omega T}j\omega}{-j\omega - G_{fr} + G_{fr}e^{-2j\omega T}j\omega} \\[4mm] \dfrac{-2e^{-j\omega T}j\omega}{-j\omega - G_{fr} + G_{fr}e^{-2j\omega T}j\omega} & -\dfrac{j\omega + G_{fr} + G_{fr}e^{-2j\omega T}j\omega}{-j\omega - G_{fr} + G_{fr}e^{-2j\omega T}j\omega} \end{array} \right]$$

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Shared compliance control
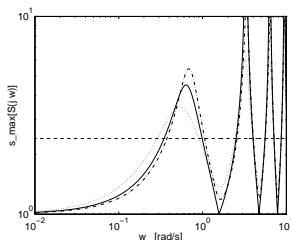Passive control
4-Channel control

SCC: Stability Analysis

The maximum singular value, $\sigma_{max} \{\mathbf{S}(j\omega)\}$, of the scattering matrix of SCC is reported in the figure:

- for different G_{fr} values,
- for $T = 1$ s.
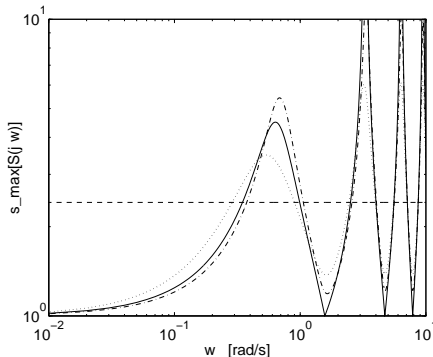
- $G_{fr} = 0.0$ (dashed),
- $G_{fr} = 0.5$ (dashdot),
- $G_{fr} = 1.0$ (solid),
- $G_{fr} = 1.5$ (dotted).

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SCC: Stability Analysis

- SCC is not passive for any value of G_{fr} and for any T;
- Even if at low frequency $\sigma_{max}\{\mathbf{S}(j\omega)\} \simeq 1.0$, local controllers have to be considered for attenuation of energy contributions of the communication network.

In any case, stability may be achieved depending on:

- the time-delay;
- G_{fr};
- proper local controllers.

It results that higher (TFR) values of G_{fr} can be imposed for a given T.

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SCC - Experimental Verification

- Repetitive operator actions (force impulses) on the master,
- force reflection gain approximately equal to TFR.

$T = 0.1\ s$

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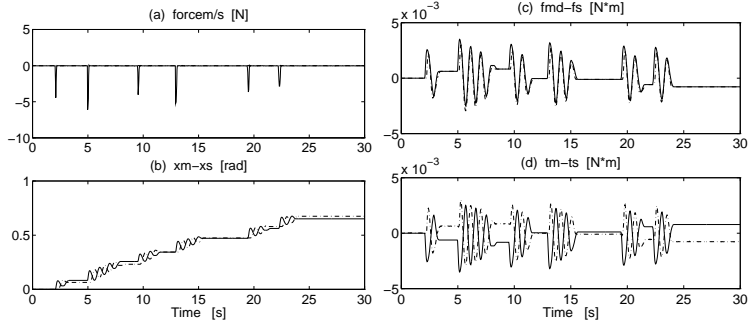
SCC - Experimental Verification

- Repetitive operator actions (force impulses) on the master,
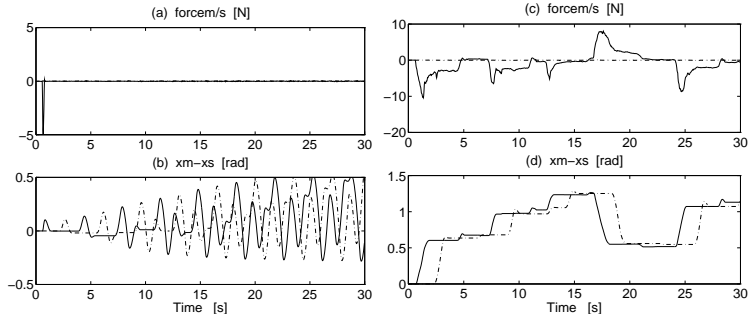- force reflection gain approximately equal to TFR.

$T = 1.0\ s$

- For a given time-delay, the force reflection gain G_{fr} can be larger than in the TFR scheme.
- For larger time-delays the force reflection gain should be accordingly reduced to guarantee stability.

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SCC and phase-lag controller

In SCC teleoperation for a given time-delay T a proper limitation of the force reflection gain G_{fr} should be introduced.

For large time-delys the maximum G_{fr} assuring stability is noticeably reduced.

In order to increase the maximum G_{fr} value it is possible to introduce the following control law in the feedback flow of the teleoperator, in place of the G_{fr} gain:

$$D(s) = G_{fr} \frac{1 + s/z}{1 + s/p}$$

The zero (z) and the pole (p) of the phase-lag network design can be based on frequency techniques (Michailov hodographs, Domain subdivision).

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SCC and phase-lag controller
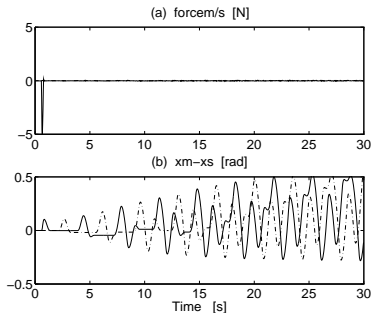
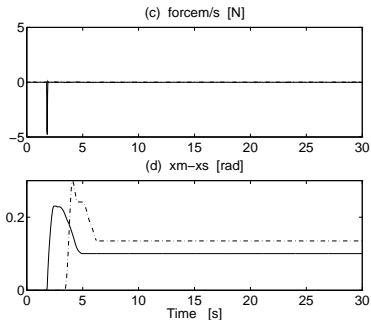Experimental comparison between the original SCC scheme and the phase-lag scheme.

- Approximately equal steady-state force reflection,
- similar initial operator force action on master, (in both cases time-delay $T = 1.0\ s$).

SCC scheme

SCC scheme and phase-lag network

(a) operator/environment forces
(b) master/slave positions

(c) operator/environment forces
(d) master/slave positions

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Robotic Telemanipulation:
A Brief Introduction on Modelling and Control Issues
Passive Control

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Passivity based teleoperation

Instabilities in time-delay teleoperation is produced by the non-passive features of the communication network (i.e. traditional force reflection).

Goal of passivity-based teleoperation is to obtain passivity for the communication network

$$\implies \quad stability \quad \forall \quad time\text{-}delay \quad T$$

The communication channel is designed on the basis of the lossless transmission line and the electro-mechanical analogy.

Two control schemes have been introduced a few years ago (Anderson & Spong, Niemeyer & Slotine) corresponding to different levels of complexity and phenomena taken into account:

1. the *"lossless transmission line"*,

2. the *"passivity-based with impedance adaptation"*.

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Passivity based teleoperation - Lossless TL

The lossless transmission line is described by the two-port:

$$\begin{cases} \mathbf{f}_m(s) &= \tanh(sT)\,\dot{\mathbf{x}}_m(s) + \operatorname{sech}(sT)\,\mathbf{f}_s(s) \\ -\dot{\mathbf{x}}_s(s) &= -\operatorname{sech}(sT)\,\dot{\mathbf{x}}_m(s) + \tanh(sT)\,\mathbf{f}_s(s) \end{cases}$$

resulting in the hybrid matrix:

$$\mathbf{H}(s) = \left[\begin{array}{cc} \tanh(sT) & \operatorname{sech}(sT) \\ -\operatorname{sech}(sT) & \tanh(sT) \end{array} \right]$$

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Passivity based teleoperation - Lossless TL

The scattering operator $\mathbf{S}(j\omega)$ is:
$$\mathbf{S}(j\omega) = \left[\begin{array}{cc} 0 & e^{-j\omega T} \\ e^{-j\omega T} & 0 \end{array} \right]$$

from which the following alternative representation of the lossless transmission:

$$\left[\begin{array}{c} \mathbf{f}_{md}(t) - \dot{\mathbf{x}}_m(t) \\ \mathbf{f}_s(t) + \dot{\mathbf{x}}_{sd}(t) \end{array} \right] = \left[\begin{array}{c} \mathbf{f}_s(t-T) - \dot{\mathbf{x}}_{sd}(t-T) \\ \mathbf{f}_{md}(t-T) + \dot{\mathbf{x}}_m(t-T) \end{array} \right]$$

- In real applications a scaling (impedance) should be introduced in the previous equations between velocities and forces.
- Introduction of scaling factors between velocities and forces should be carefully considered in order to maintain the passivity of the network.
- Introduction of scaling without altering the passivity properties is obtained by means of two transformers (passive two-port elements) at both the master and the slave site.
- The two transformers should introduce respectively the scaling factors:

master transformer ratio	B
slave transformer ratio	$1/B$

- B is the *Characteristic Impedance* of the communication network.
- A proper choice of B is essential in order to exploit the performances of the device.

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Passivity based teleoperation - Lossless TL

The resulting network is:

$$\begin{cases} \mathbf{f}_{md}(t) &= \mathbf{f}_s(t-T) + B[\dot{\mathbf{x}}_m(t) - \dot{\mathbf{x}}_{sd}(t-T)] \\ \dot{\mathbf{x}}_{sd}(t) &= \dot{\mathbf{x}}_m(t-T) + \frac{1}{B}[\mathbf{f}_{md}(t-T) - \mathbf{f}_s(t)] \end{cases}$$

The introduction of the characteristic impedance of the communication network results in the re-definition of the wave variables:
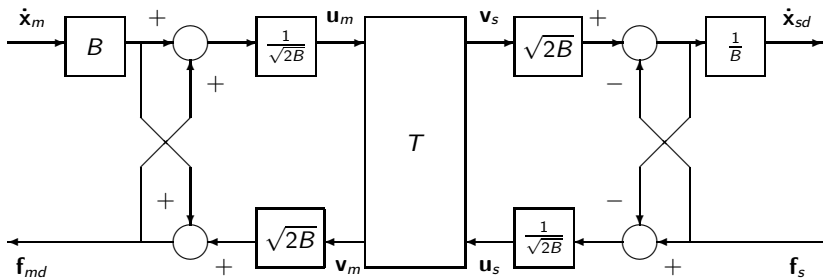
$$\mathbf{u}_m = \frac{1}{\sqrt{2B}}(\mathbf{f}_m + B\dot{\mathbf{x}}_m) \qquad\qquad \mathbf{u}_s = \frac{1}{\sqrt{2B}}(\mathbf{f}_s - B\dot{\mathbf{x}}_s)$$

$$\mathbf{v}_m = \frac{1}{\sqrt{2B}}(\mathbf{f}_m - B\dot{\mathbf{x}}_m) \qquad\qquad \mathbf{v}_s = \frac{1}{\sqrt{2B}}(\mathbf{f}_s + B\dot{\mathbf{x}}_s)$$

By considering the wave variables instead of the power ones, the following (alternative) description of the communication network is obtained:

$$\begin{cases} \mathbf{f}_{md}(t) &= B\,\dot{\mathbf{x}}_m(t) + \sqrt{2B}\,\mathbf{v}_m(t) \\ \dot{\mathbf{x}}_{sd}(t) &= -\frac{1}{B}\,[\mathbf{f}_s(t) - \sqrt{2B}\,\mathbf{v}_s(t)] \end{cases}$$

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Passivity based teleoperation - Lossless TL

The following transmission line is obtained:

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Impedance adaptation

Impedance mismatches are present at the extremities of the communication line, originating *Power Reflections* at both sites of the teleoperation system.

The non-strict passivity (note that $\|\mathbf{S}(j\omega)\| = 1.0$) of the lossless transmission line does not introduce dissipation for the possible power reflections, destabilizing the device.

Impedance adaptation at the terminations of the line is possible by means of two admittance/impedance elements whose values are tuned with the characteristic impedance of the line B:

master termination	admittance	$1/B$
slave termination	impedance	B

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Impedance adaptation

The time descriptions of the adaptation elements are:

$$
\begin{cases}
\dot{\mathbf{x}}'_m(t) &= \dot{\mathbf{x}}_m(t) - \dfrac{1}{B}\mathbf{f}_{md}(t) \\
\mathbf{f}'_s(t) &= \mathbf{f}_s(t) + B\dot{\mathbf{x}}_{sd}(t)
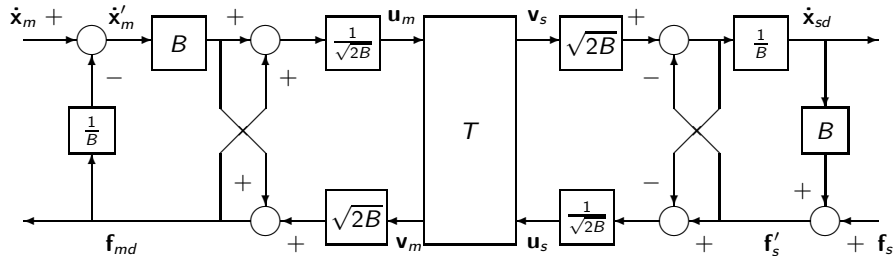\end{cases}
$$

$\dot{\mathbf{x}}'_m$ and $\mathbf{f}'_s(t)$ being the new input variables of the communication line.

The insertion of these elements results in the modification of the stability features of the network as well as of its description:

$$
\begin{cases}
\mathbf{f}_{md}(s) &= \dfrac{B}{2}\dot{\mathbf{x}}_m(s) + \dfrac{1}{2}e^{-sT}\mathbf{f}_s(s) \\
\dot{\mathbf{x}}_{sd}(s) &= \dfrac{1}{2}e^{-sT}\dot{\mathbf{x}}_m(s) - \dfrac{1}{2B}\mathbf{f}_s(s)
\end{cases}
$$

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Impedance adaptation

A scheme representing the two adaptation elements at the terminations of the communication line is:

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Traditional force reflection
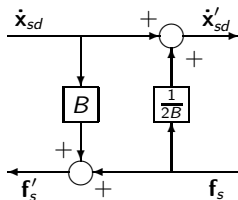Shared compliance control
Passive control
4-Channel control

Impedance adaptation

Considering the previous power variable description of the network

$$\begin{cases} \mathbf{f}_{md}(s) &= \frac{B}{2}\dot{\mathbf{x}}_m(s) + \frac{1}{2}e^{-sT}\mathbf{f}_s(s) \\ \dot{\mathbf{x}}_{sd}(s) &= \frac{1}{2}e^{-sT}\dot{\mathbf{x}}_m(s) - \frac{1}{2B}\mathbf{f}_s(s) \end{cases}$$

the following considerations can be drawn:

- termination elements introduce not unitary scaling factors in the system equation (effect not present in the non-adapted network),
- power modification in the network results in the presence of a position drift between master and slave variables when the slave is interacting with the environment, i.e. when $\mathbf{f}_s \neq 0$ (or transient phases),
- a scheme for the compensation of this effect can be obtained by adding the following further element at the slave site.


$$\dot{x}'_{sd}(s) = \frac{1}{2}e^{-sT}\dot{x}_m(s)$$

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Scattering Analysis

Maximum singular value $\sigma\{\mathbf{S}(j\omega)\}$ of the scattering matrix of passivity based teleoperation scheme is given:

- as a function of ωT,
- for different values of the force reflection gain G_{fr}.
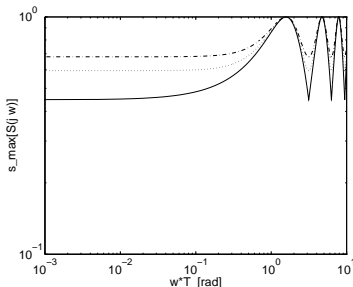

- $G_{fr} = 0.5$ (dashdot),
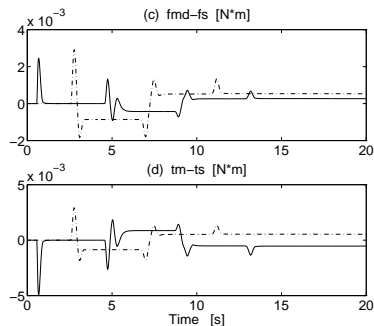- $G_{fr} = 1.0$ (solid),
- $G_{fr} = 1.5$ (dotted).

• The system is passive (even if not strictly passive) for any G_{fr} value.
• The introduction of the drift compensation schemes does not alter the passivity of the network.

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Experimental Verification

- Adapted passivity based teleoperation,
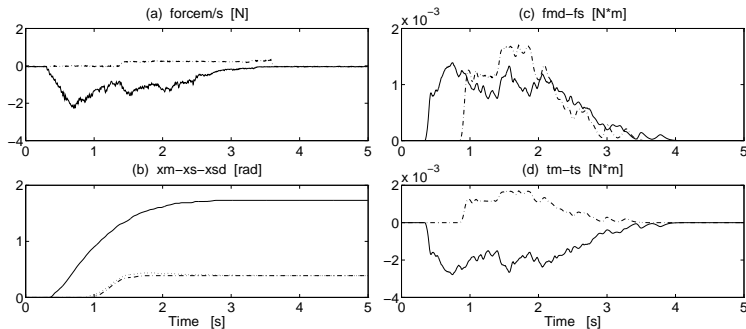- operator action (force pulse) on the master,
- time-delay $T = 1.0\ s$.
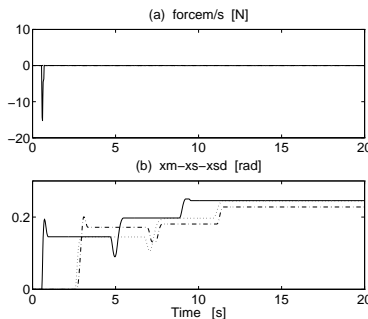
(a) operator/environment forces
(b) master/slave positions

(c) master/slave forces
(d) master/slave torques

Passivity Theory
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Traditional force reflection
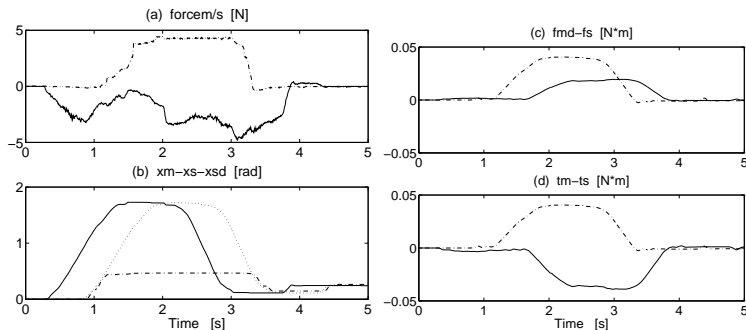Shared compliance control
Passive control
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Position Drift

- Adapted passivity based teleoperation,

- operator action on the master,

- interaction with the environment,

- position drift between master and slave (no use of drift compensation algorithm),
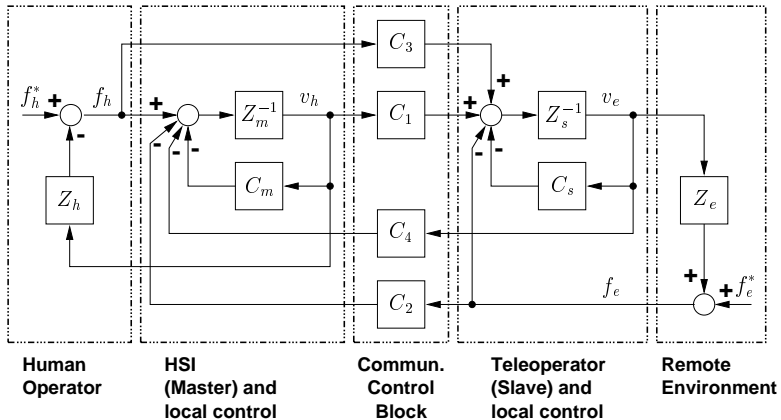
- time-delay $T = 0.5 \; s$.

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Position Drift

- Introduction of the position drift compensation algorithm:
- time-delay $T = 0.5\ s$.

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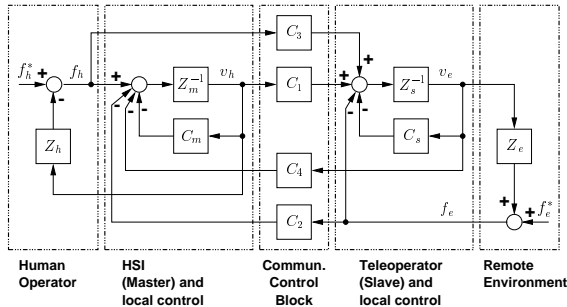Passive control
4-Channel control

4-Channel Control Scheme

The control schemes presented so far are based on a *Two Channel* scheme: a single signal is transmitted from master to slave a a single signal is transmitted from slave to master.
Another quite popular scheme is the so-called *Four Channel* control scheme.

| Human Operator | HSI (Master) and local control | Commun. Control Block | Teleoperator (Slave) and local control | Remote Environment |

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4-Channel Control Scheme

In this control scheme, the four channels C_1, C_2, C_3, and C_4 are designed in order to satisfy proper control criteria (transparency, . . .).

The two-channel schemes can be derived from this architecture by properly setting some of the parameters C_i to 0 or 1.