Games You Cannot Win

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Abstract. We consider games played on finite graphs for an infinite number of turns, whose goal is to obtain a trace belonging to a given set of winning traces. We focus on those states from which Player 1 cannot force a win. For such losing states, we discuss different notions of best-effort strategies.
In particular, we propose to employ the game-theoretic notion of dominance, and we compare it with the notion of optimal strategy in a corresponding stochastic game.

1 Introduction

Games played on finite graphs have been widely investigated in Computer Science, with applications including controller synthesis [MPS95], protocol verification [KR01], logic and automata theory [EJ91], and compositional software verification [dAH01].

In such games, we are given a finite graph, whose set of states is partitioned into Player 1 and Player 2 states, and a *goal*, which is a set of infinite sequences of states. The game consists in the two players taking turns at picking a successor state, giving rise to an ever increasing and eventually infinite sequence of states.

A (deterministic) *strategy* for a player is a function that, given the current history of the game (a finite sequence of states), chooses the next state. We say that a strategy is *winning* if, for all strategies of the adversary, the resulting infinite path belongs to the goal. Additionally, we say that a state s is winning if there exists a strategy that is winning from s.

The main algorithmical concern of the classical theory of these games is determining the set of winning states. In this paper, we shift the focus to *losing* states, and we put forward the claim that many applications would benefit from a theory of best-effort strategies which allowed Player 1 to play in a rational way even from losing states.

For instance, many games models correspond to real-world problems which are not really competitive: the game is just a tool which enables to distinguish internal from external non-determinism. In practice, the behavior of the adversary may turn out to be random, or even cooperative. A strategy of Player 1 which does not "give up", but rather tries its best at winning, may in fact end up winning, even starting from states that are theoretically losing.

In other cases, the game is an over-approximation of reality, giving to Player 2 a wider set of capabilities (i.e., moves in the game) than what most adversaries actually have in practice. Again, a best-effort strategy for Player 1 can thus often lead to victory, even against an adversary which is strictly competitive.

2 Best-effort Strategies for Losing States

One notion of best-effort strategy which is well known in the literature is the notion of optimal strategy in a particular type of game called a *Markov Decision Process* (MDP). In such a game (also called 1.5-player game), Player 2 plays according to a fixed distribution over successor states. Thus, each strategy of Player 1 gives rise to a stochastic process, i.e. to a distribution over infinite sequences of states. Given a goal, one may then ask which is the strategy of Player 1 which maximizes the probability of satisfying the goal. There are known algorithms for solving this problem for various classes of goals [BdA95].

This notion of optimal strategy suggests a first solution to our original problem of dealing with losing states in a 2-player game. We can assume that Player 2 plays uniformly at random and compute an optimal strategy for Player 1. Although this approach may be of interest to some cases, it is worth considering alternative notions, which do not make as strong an assumption on the behavior of Player 2. One such notion comes from classical Game Theory and is termed *dominance* [OR94]. Given two strategies f and g of Player 1, we say that f *dominates* g (written $f \sqsubset g$) if f is always at least as good as g, and better than g in at least one case. Dominance induces a strict partial order over strategies. We claim that undominated strategies (i.e. strategies that are maximal w.r.t. $\sqsubset$) represent a convincing alternative notion of best-effort strategy. Similarly to optimality in a 1.5-player game, this notion is goal-independent. Moreover, it does not make any assumption on the behavior of Player 2. Simple examples show that being undominated in a 2-player game and being optimal in the corresponding MDP are incomparable notions.

We are currently investigating the properties of undominated strategies, in terms of the amount of memory that they require and of the algorithmical complexity of finding such strategies w.r.t. different types of goals. Preliminary results indicate that undominated strategies for common game classes can be efficiently determined.

Aside from the mentioned goal-independent best-effort criteria, it is worth noticing that goal-specific criteria offer an entirely different range of possibilities, many of which remain unexplored in the literature.

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