On the Route Priority for Cognitive Radio Networks

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Abstract—To fully unleash the potentials of the cognitive radio (CR) paradigm, new challenges must be addressed. Specifically, as regards the network layer, the problem of the *route priority*, i.e., the problem of prioritizing the routes for the CR packet transmission, is crucial, since the communication opportunities provided by a route are deeply affected by the primary-user (PU) activity. Furthermore, whenever the CR network layer exploits proactively acquired information on the PU activity, update packets need to be exchanged among the CR users, inducing so a route overhead independently of the adopted routing protocol. Hence, in this paper, we analytically derive the *optimal route priority rule*, i.e., the route priority rule maximizing the achievable capacity, by jointly accounting for the PU activity and the route overhead. To this aim, at first, we formulate the *optimal route priority problem*, and we prove that its computational complexity through exhaustive search is exponential. Then, we provide the closed-form expressions of the achievable capacity. Stemming from these expressions, we derive the optimal route priority, and we design a computational-efficient search algorithm. All the theoretical results are derived by adopting two routing strategies and two PU activity models.

Index Terms—Cognitive radio, route, priority, routing.

I. Introduction

COGNITIVE RADIO (CR) paradigm has been recognized as a viable solution for the deployment of spectrum-efficient networks [2]. Specifically, the CR paradigm introduces the concept of *spectrum holes*, namely, portions of the radio spectrum temporarily vacated by licensed users, referred to as Primary Users (PUs), and exploited by unlicensed users, referred to as CR users, to establish communications in an opportunistic way [3], [4].

To fully unleash the potentials of the CR paradigm, new challenges must be addressed and solved at the network layer [5]. Let us consider a typical routing problem, i.e., a node must define a priority among the available routes to forward

data packets toward the destination. Since each route exhibits peculiar communication characteristics, e.g., capacity, failure probability, end-to-end delay, the criteria for establishing a priority among the routes deeply affect the routing performance, regardless of the adopted routing protocol [6], [7].

The aforementioned route priority issue is even more crucial in CR networks, due to the effects induced by the PU activities on the route availability [5], [8], [9]. In fact, the more persistent is the PU activity on a route, the shorter is the time interval during which the route is available to the CR network. Consequently, although a route, say route r_m, could exhibit more appealing communication characteristics (e.g., higher capacity) with respect to another route, say route r_l, in absence of PU activity, the presence of PU activity can make r_l a better choice than r_m. Hence, the route priority design must take into account the PU activity.

Furthermore, whenever the CR network layer exploits proactively acquired information on the PU activity, routing update packets need to be periodically exchanged among the CR users. The duration of the time interval between these exchanges, referred to as *routing update period*, deeply affects the overall routing performance, independently of the adopted routing protocol. In fact, whenever a CR user receives a routing update, it acquires some knowledge on the current PU activities over the different routes. Hence, the shorter are the update periods, the better the CR user can exploit such a knowledge to prioritize the routes [10]. On the other hand, the shorter are the periods, the higher is the overhead induced within the network.

In this paper, we address such an open problem [11] by analytically designing the *optimal route priority rule*, i.e., the route priority rule that maximizes the capacity available at the arbitrary CR user, by jointly accounting for the PU activities and the route overhead.

More in detail, at first, the problem of the optimal route priority is formulated by jointly accounting for the PU activities and the routing update period. Then, we prove that the computational complexity of the optimal route priority problem through exhaustive-search is exponential. Moreover, we provide the closed-form expressions of the average route capacity achievable by the arbitrary CR user. Stemming from these expressions, we derive the optimal route priority and we design a computational-efficient search algorithm. Specifically, the memory complexity of the designed algorithm is logarithmic with the number of routes, whereas the time complexity is linear with the number of routes. All the theoretical results are derived by adopting two different routing strategies and two widely-adopted PU activity models [12]: i) Bernoulli PU Activity Model, in which the PU activity is time independent; ii) *Markov Chain PU Activity Model*, in which the PU activity exhibits a time correlation according to a Markov Chain.

The rest of the paper is organized as follows. In Section II, we discuss the related work. In Section III, we describe the network model and we collect some definitions that will be used through the paper. In Section IV, we derive the *optimal route priority* and we design a computational-efficient search algorithm. We validate the analytical results derived in Section IV by simulations in Section V. Finally, in Section VI we conclude the paper.

II. RELATED WORK

With reference to the network layer functionalities, two are the key tasks: i) *path discovery*, whose aim is to discover within the network topology the available routes to forward data packets toward the destination; ii) *path selection*, whose aim is to select, among the discovered routes, the one assuring the highest communication opportunities.

Regarding the path discovery process, although several algorithms and protocols have been proposed for CR networks in the last years [8], they can be broadly classified in reactive and proactive schemes. In reactive discovery schemes, the discovery process is activated on-demand when a forwarding request is made at the CR source [9], [13]. Differently, when a proactive discovery scheme is employed, the discovery process is periodically activated and every CR node maintains updated path information, generally stored within routing tables [14], [15]. The advantage of the reactive schemes is the reduction of the overhead due to route maintenance at the price of longer path set-up times. By contrast, the proactive schemes avoid the path set-up delay, but the routing overhead does not scale with the network size and the route information can be outdated [5].

Nevertheless, independently from the adopted discovery scheme, realistic network topologies exhibit multiple paths between a source and a destination. Hence, a route priority is always needed to order the discovered routes in terms of communication opportunities. For this, in this manuscript, we focus on the optimal route priority design in CR networks. The advantage over the existing literature is that the proposed route priority jointly accounts for: i) the PU activity dynamics; ii) the quality (capacity) dynamics among the different routes; iii) the overhead induced by the routing process; iv) the routing process time parameter. To the best of our knowledge, this is the first work addressing such a key issue.

III. PRELIMINARIES

We consider an arbitrary CR source communicating with an arbitrary CR destination through M different routes as shown in Fig. 1, and we denote with $\{r_m\}_{m=1}^M$ the set of routes.

Cognitive User Time: As shown in Fig. 2, the CR user time is organized into fixed-sized slots of duration T, with KT denoting the routing update period, i.e., the duration of the time interval[1] between the reception of two route update packets.

Remark: The *routing update period* being a multiple of the time slot T allows us to account for the slotted nature of the CR time induced by the spectrum sensing functionality [16]. In fact,
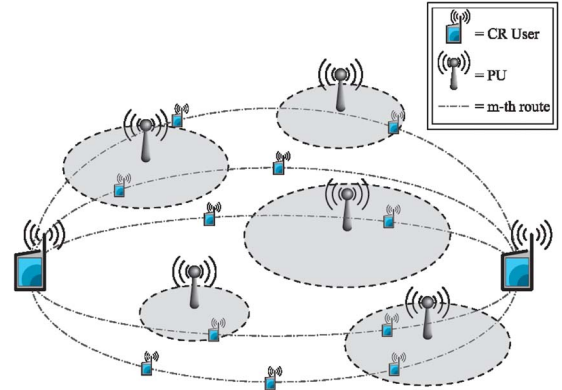

Fig. 1. Considered scenario: an arbitrary CR source communicating with an arbitrary CR destination through several different routes.
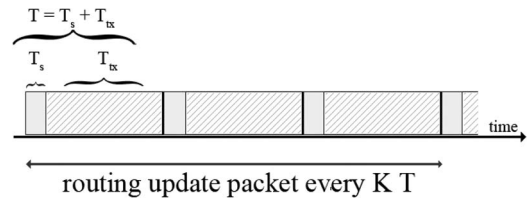

Fig. 2. CR user time.

without loss of generality, each time slot T can be assumed as organized in a sensing period T_s, which measures the portion of the time slot assigned to the spectrum sensing, and in a transmission period T_{tx}, which measures the portion of the time slot devoted to the CR user packet transmission.

PU Activity: The arbitrary route r_m is affected by the activity of PU v_m. When multiple PUs affect the same route r_m, v_m models the aggregate PU activity induced by multiple PUs, with v_m defined as active whenever at least one PU is active. The PU activities on different routes are assumed independent each other. The assumption of independent PU activities on different routes is not restrictive, as proved with Corollaries 3 and 5. In the following, two widely-adopted activity models [12] are considered.

i) **Bernoulli PU Activity Model**. The activity of the PU v_m is modeled as a Bernoulli process. Specifically, the PU activity is assumed independent and identically distributed among different time slots. In each time slot, v_m is inactive with probability p_m^{off} and active with probability $p_m^{\text{on}} = 1 - p_m^{\text{off}}$.

ii) **Markov Chain PU Activity Model**. The activity of the PU v_m during the arbitrary time slot is modeled as a two-state Markov process, hence the PU activity in subsequent time slots is correlated. In the arbitrary n-th time slot, v_m is inactive with probability $p_m^{\text{off}} \triangleq P(X_m(n) = 0)$ where $X_m(n)$ denotes the state of v_m, whereas v_m is active with probability $p_m^{\text{on}} \triangleq P(X_m(n) = 1) = 1 - p_m^{\text{off}}$. By denoting with $p_m^{0|1} \triangleq P(X_m(n + 1) = 0|X_m(n) = 1)$, $p_m^{1|1} = 1 - p_m^{0|1}$, $p_m^{0|0} \triangleq P(X_m(n + 1) = 0|X_m(n) = 0)$ and $p_m^{1|0} = 1 - p_m^{0|0}$ the transition probabilities, and by accounting for the Markov chain property [17], the following relations hold $p_m^{\text{off}} = p_m^{0|1}/(p_m^{0|1} + p_m^{1|0})$, $p_m^{\text{on}} = p_m^{1|0}/(p_m^{0|1} + p_m^{1|0})$.

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[1] For the sake of simplicity, we denote with T and KT also the corresponding time intervals. The potential ambiguity will be solved by the context of the proposition.

Definition 1 (Route Status): During an arbitrary routing update interval, the *route status* $s_m \in \{0, 1\}$ denotes the status of the route r_m as reported by the routing update packet, with $s_m = 0$ denoting the absence of PU activity on the route r_m during the time slot in which the update packet was received.

Remark: The assumption in Definition 1 is not restrictive: the results derived in the following can be easily extended to the case of a routing update received at time slot n and reporting the route availability during time slot k, with $k < n$. We note that, if the m-th route is a sequence of multiple links (hops), then $s_m = 0$ if each link is free from PU activity [18].

Remark: We note that sensing inaccuracy can be easily incorporated in our model as detailed in the following. Let us adopt the Bernoulli PU activity model, and let us denote with p_{md} and p_{fa} the missing-detection and the false-alarm probability. Hence, by denoting with s_m and $\tilde{s}_m$ the true (error-free) and the sensed route status, we have:

$$\tilde{p}_{\text{off}}^m \triangleq P(\tilde{s}_m = 0) = P(s_m = 0)(1 - p_{fa}) + P(s_m = 1)p_{md}$$
$$= p_{\text{off}}^m \bar{p}_{fa} + p_{\text{on}}^m p_{md}$$
$$\tilde{p}_{\text{on}}^m \triangleq 1 - \tilde{p}_{\text{off}}^m = p_{\text{off}}^m p_{fa} + p_{\text{on}}^m \bar{p}_{md} \quad (1)$$

with $\bar{p}_{fa} \triangleq 1 - p_{fa}$ and $\bar{p}_{md} \triangleq 1 - p_{md}$. From (1), after some algebraic manipulations, we obtain:

$$p_{\text{off}}^m = \frac{\tilde{p}_{\text{off}}^m \bar{p}_{md} - \tilde{p}_{\text{on}}^m p_{md}}{p_{md} p_{fa} + \bar{p}_{md} \bar{p}_{fa}}, \quad p_{\text{on}}^m = \frac{\tilde{p}_{\text{on}}^m \bar{p}_{fa} - \tilde{p}_{\text{off}}^m p_{fa}}{p_{md} p_{fa} + \bar{p}_{md} \bar{p}_{fa}} \quad (2)$$

From (2), it follows that true values of the PU activity probabilities p_{off}^m and p_{on}^m can be easily obtained in presence of sensing errors from the sensed PU activity probabilities $\bar{p}_{\text{off}}^m$ and $\bar{p}_{\text{on}}^m$. As instance, by substituting (2) in (13), we are able to measure the actual average route capacity in presence of sensing errors. Similar results can be obtained when the Markov PU activity model is adopted. In the following we force the notation s_m in place of $\tilde{s}_m$, for the sake of simplicity.

Definition 2 (Route Status Vector): During an arbitrary routing update interval, the *route status vector* $s \triangleq (s_1, s_2, \ldots, s_M)$ denotes the vector constituted by the M route statuses $\{s_m\}_{m=1}^M$. Due to the assumption of independent PU activity, it results:

$$p(s) = \prod_{m=1}^{M} p(s_m) \quad (3)$$

where $p(s_m) = p_{\text{off}}^m$ if $s_m = 0$, $p(s_m) = p_{\text{on}}^m$ otherwise. In the following, we denote with Σ the set of the route status vectors, whose cardinality $|\Sigma|$ is equal to 2^M since each of the M routes can be in two different statuses.

Definition 3 (Route Priority Function): A *route priority function* is a function defined over the set Σ of the route status vectors that maps each route status vector $s \in \Sigma$ to a route r_m:

$$f : s \in \Sigma \rightarrow \{r_m\}_{m=1}^M \quad (4)$$

In the following, Φ denotes the set of route priority functions.

Remark: The concept of *route priority function* allows us to model in a compact form the decision process of an arbitrary CR routing protocol. In fact, given the route status vector s, the

route r_m that the CR user should use according to the adopted CR routing protocol is given by $f(s) = r_m$.

Routing Strategy: The route selected according to the route priority function is used for CR packet transmission until a new routing update packet is received. In the following, two different routing strategies are considered.

i) **Constrained Strategy**. During the first time slot of an arbitrary routing update interval, the CR source can select the route r_m to be used for packet transmission during the current routing update interval if and only if r_m has been reported as free from the PU activity within the first time slot:

$$f(s) = r_m \iff s_m = 0 \quad (5)$$

ii) **Unconstrained Strategy**. During the first time slot of an arbitrary routing update interval, the CR source can select the route r_m to be used for packet transmission during the current routing update interval even if r_m has been reported as affected by the PU activity within the first time slot:

$$f(s) = r_m \text{ with } s_m \in \{0, 1\} \quad (6)$$

Remark: Clearly, as clarified in Definition 6 and in the subsequent remark, if the PU v_m is active in an arbitrary time slot, the selected route $r_m = f(s)$ fails and no packet transmission can occur.

Definition 4 (Route Capacity): The *route capacity* C_m is the average bit-rate achievable during an arbitrary time slot through route r_m when PU v_m is not active.

Remark: In this paper, we consider the route capacity as routing metric. Nevertheless, the proposed analytical framework continues to hold if a different routing metric (e.g., route delay) is adopted.

Remark: We note that, if the m-th route is composed by multiple links (hops), then the route capacity C_m denotes, as well-known, the minimum of the link capacities. Hence, we can assume, without loss of generality, $C_m > 0$ for any r_m. Furthermore, we note that the value of the route capacity C_m can be acquired in absence of full topology information through the routing updates [19].

Definition 5 (Route Overhead): Given the update parameter K, the *route overhead* $\Omega_m(K)$ denotes the average bit-rate needed to propagate the routing information for route r_m.

Remark: The route overhead $\Omega_m(K)$ represents the reduction of the route capacity achievable through route r_m due to update packets propagation. Hence, without loss of generality, $\Omega_m(K)$ is given by:

$$\Omega_m(K) = \begin{cases} L_m/(KT) & \text{if } s_m = 0 \\ 0 & \text{if } s_m = 1 \end{cases} \quad (7)$$

where L_m is the bit cost associated with the reception of the routing update packet.

Definition 6 (Average Route Capacity): Given the routing update parameter K, the *average route capacity* $\overline{C}_{r_m|s_m}(K)$ denotes the route capacity achievable on average during an

arbitrary routing update interval when route r_m in status s_m is selected for packet transmission, by accounting for both PU activity and route overhead.

Remark: In the following, we assume that, whenever a route is affected by PU activity during a certain time slot, the route *fails* and no packet transmission can occur. Hence, the contribute to the average route capacity during such a time slot is null. We note that this assumption is not restrictive, since the results derived in the following can be easily extended to case of a not-null capacity in presence of PU activity, and the analytical framework continues to hold.

Remark: The *route capacity* measures the capacity achievable during a time slot in absence of PU activity, whereas the *average route capacity* measures the capacity achievable on average during a routing update interval by accounting for both PU activity and route overhead. In Section IV-B we derive closed-form expressions of the *average route capacity* for both the considered PU activity models.

Remark: In the following, for the sake of simplicity, we adopt the following notation when we focus on the route priority function $f(\cdot)$:

$$f(s) = r_m \Longrightarrow \overline{C}_{f(s)}(K) \triangleq \overline{C}_{r_m|s_m}(K) \qquad (8)$$

where s_m is the status of the m-th route selected through the priority function.

IV. OPTIMAL ROUTE PRIORITY

At first, in Section IV-A, we formulate the *optimal route priority problem* and we assess its memory and time complexity through exhaustive search (Propositions 1 and 2). Then, in Section IV-B, we derive closed-form expressions of the average route capacity $\overline{C}_{r_m|s_m}(K)$ for both the considered PU activity models (Propositions 3 and 4). Stemming from these results, in Section IV-C we derive the *optimal route priority function* for the constrained routing strategy (Theorem 1) along with a computational-efficient search algorithm (Theorem 2). Finally, in Section IV-D we derive the *optimal route priority function* for the unconstrained routing strategy (Theorem 3) along with a computational-efficient search algorithm (Theorem 4).

A. Optimal Route Priority Problem

Here, we formulate the *optimal route priority problem* in Definition 8 and we assess its complexity in terms of memory and running time in Propositions 1 and 2, respectively.

Definition 7 (Average Aggregate Capacity): Given the routing update parameter K and the route priority function $f(\cdot)$, the *average aggregate capacity* $\overline{C}_f(K)$ denotes the expected capacity achievable during an arbitrary routing update interval when $f(\cdot)$ is adopted:

$$\overline{C}_f(K) = \sum_{s \in \Sigma} p(s) \overline{C}_{f(s)}(K) \qquad (9)$$

where $p(s)$ is defined in (3), and $\overline{C}_{f(s)}(K)$ is defined in (8).

Remark: The *average aggregate capacity* depends on: i) the adopted PU activity model, through the probability $p(s)$; ii) the

selected routing strategy, i.e., constrained vs unconstrained, through the route priority function $f(\cdot)$.

Definition 8 (Optimal Route Priority Problem): Given the routing update parameter K, the statistics on the PU activity, and the average route capacities $\overline{C}_{r_m|s_m}(K)$, the goal is to choose the priority function $f^* \in \Phi$ that maximizes the *average aggregate capacity*:

$$\overline{C}_{f^*}(K) = \max_{f \in \Phi} \left\{ \overline{C}_f(K) \right\} \qquad (10)$$

and we refer to f^* as the *optimal route priority function*.

Remark: In the CR paradigm, the network layer functionalities must account for the dynamics of the PU activity. To this aim, we design in the following a rule to order the discovered routes that maximizes the expected capacity by explicitly:

— accounting for the PU activity dynamics through PU traffic statistics;
— accounting for the effects of the PU dynamics on the routes through the notion of route status;
— accounting for the delay in the route status dissemination through the concept of route update parameter K.

Proposition 1 (Memory Complexity): The memory complexity of the optimal route priority problem through exhaustive search is equal to $\mathcal{O}(2^M)$ for both constrained and unconstrained strategies.

    Proof: See Appendix A. ☐

Proposition 2 (Time Complexity): The time complexity of the optimal route priority problem through exhaustive search is equal to $\mathcal{O}(2^{M-1})$ when the constrained routing strategy is adopted, whereas it is equal to $\mathcal{O}(2^M)$ when the unconstrained strategy is adopted.

    Proof: See Appendix B. ☐

Remark: The exponential time and memory complexity of the *optimal route priority function* makes the exhaustive search computationally intractable. Nevertheless, in Section IV-C, we prove that, if the constrained strategy is adopted, then there exists a total relation order over the routes, i.e., it is possible to define the concept of *optimal route set* $\mathcal{R}^*$:

$$\mathcal{R}^* = \{r_{\sigma_1}, \ldots, r_{\sigma_M}\}, \sigma_m \in \{1, \ldots, M\} \wedge \sigma_m \neq \sigma_l \qquad (11)$$

so that:

$$f^*(s) = r_{\sigma_m} \Longleftrightarrow s_{\sigma_m} = 0 \wedge s_{\sigma_l} = 1 \; \forall l < m \qquad (12)$$

Hence, by exploiting this powerful result, we are able to design a computational-efficient search algorithm for the *optimal route priority function*. Similar results are derived in Section IV-D with reference to the unconstrained strategy.

B. Average Route Capacity

Here, we derive in Propositions 3 and 4 the closed-form expressions of the *average route capacity* for both the considered PU activity models.

Proposition 3 (Average Route Capacity: Bernoulli PU Activity Model): Given the routing update parameter K, when the Bernoulli PU activity model is adopted, the average route

capacity $\overline{C}^{\mathrm{B}}_{r_m|s_m}(K)$ achievable during an arbitrary routing update interval when route r_m in the status s_m is selected for packet transmission is equal to:

$$\overline{C}^{\mathrm{B}}_{r_m|s_m}(K) = \begin{cases} C_m\left[1 + (K-1)p_m^{\mathrm{off}}\right] - \Omega_m(K) & \text{if } s_m = 0 \\ C_m(K-1)p_m^{\mathrm{off}} - \Omega_m(K) & \text{if } s_m = 1 \end{cases} \tag{13}$$

where C_m and $\Omega_m(K)$ are given in Definition 4 and 5, respectively.

Proof: See Appendix C. □

Remark: The *average route capacity* $\overline{C}^{\mathrm{B}}_{r_m|s_m}(K)$ is a function of four parameters: i) the PU activity statistics, namely, the off state probability p_m^{off} of PU v_m; ii) the route capacity C_m; iii) the routing update parameter K; iv) the route overhead $\Omega_m(K)$.

Corollary 1: Given the routing update parameter K, when the Bernoulli PU activity model is adopted, the average route capacity $\overline{C}^{\mathrm{B}}_{r_m|0}(K)$ provided by the route r_m in the status $s_m = 0$ is always greater than the average route capacity $\overline{C}^{\mathrm{B}}_{r_m|1}(K)$ provided by the same route r_m in the status $s_m = 1$:

$$\overline{C}^{\mathrm{B}}_{r_m|0}(K) > \overline{C}^{\mathrm{B}}_{r_m|1}(K) \tag{14}$$

Proof: See Appendix D. □

Proposition 4 (Average Route Capacity: Markov Chain PU Activity Model): Given the routing update parameter K, when the Markov chain PU activity model is adopted, the average route capacity $\overline{C}^{\mathrm{MC}}_{r_m|s_m}(K)$ achievable during an arbitrary routing update interval when route r_m in the status s_m is selected for packet transmission is equal to:

$$\overline{C}^{\mathrm{MC}}_{r_m|s_m}(K) = \begin{cases} \overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K) - \Omega_m(K) & \text{if } s_m = 0 \\ \overline{\Psi}^{\mathrm{MC}}_{r_m|1}(K) - \Omega_m(K) & \text{if } s_m = 1 \end{cases} \tag{15}$$

where $\Omega_m(K)$ is given in Definition 5, and $\overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K)$ and $\overline{\Psi}^{\mathrm{MC}}_{r_m|1}(K)$ are recursively defined as:

$$\overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K) = KC_m - p_m^{1|0}\left[\overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K-1) + \right.$$
$$\left. + \sum_{l=1}^{K-2}\left(p_m^{1|1}\right)^l \overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K-l-1)\right] \tag{16}$$

$$\overline{\Psi}^{\mathrm{MC}}_{r_m|1}(K) = p_m^{0|1}\left[\overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K-1) + \right.$$
$$\left. + \sum_{l=1}^{K-2}\left(p_m^{1|1}\right)^l \overline{\Psi}^{\mathrm{MC}}_{r_m|0}(K-l-1)\right] \tag{17}$$

with $\overline{\Psi}^{\mathrm{MC}}_{r_m|0}(0) = 0$.

Proof: See Appendix E. □

Remark: Similarly to (13), the *average route capacity* $\overline{C}^{\mathrm{MC}}_{r_m|s_m}(K)$ when the Markov Chain PU model is adopted is a function of: i) the off state probability p_m^{off} of PU v_m; ii) the route capacity C_m; iii) the routing update parameter K; iv) the route overhead $\Omega_m(K)$. Differently from (13), $\overline{C}^{\mathrm{MC}}_{r_m|s_m}(K)$ depends also on the PU transition probabilities $p_m^{1|1}$ and $p_m^{1|0}$.

Remark: When $K = 1$, from (17) it results $\overline{C}^{\mathrm{MC}}_{r_m|1}(1) = 0$, in agreement with Definition 6 and the subsequent remark. In fact, in such a case the routing update interval is constituted by a unique time slot in which the PU is active. Consequently, the CR source cannot use the route r_m and the corresponding capacity is therefore zero.

Corollary 2: Given the routing update parameter K, when the Markov Chain PU activity model is adopted, the average route capacity $\overline{C}^{\mathrm{MC}}_{r_m|0}(K)$ provided by the route r_m in status $s_m = 0$ is always greater than the average route capacity $\overline{C}^{\mathrm{MC}}_{r_m|1}(K)$ provided by the same route r_m in status $s_m = 1$:

$$\overline{C}^{\mathrm{MC}}_{r_m|0}(K) > \overline{C}^{\mathrm{MC}}_{r_m|1}(K) \tag{18}$$

Proof: See Appendix F. □

C. Constrained Routing Strategy

Here, we derive a computational-efficient algorithm for the *optimal route priority problem* when the constrained routing strategy is adopted (Algorithm 1), and we assess in Theorem 2 its computational and memory efficiency.

To this aim, we first introduce in Definition 9 the concept of *optimal route set*, and then we analytically derive in Theorem 1 the *optimal route priority function*.

Definition 9 (Optimal Route Set): Given the routing update parameter K, the *optimal route set* $\mathcal{R}^*$ is the ordered sequence without repetition of M routes in $\{r_m\}_{m=1}^M$ defined as follows:

$$\mathcal{R}^* = (r_{\sigma_1}, \dots, r_{\sigma_M}) :$$
$$\overline{C}_{r_{\sigma_m}|0}(K) \geq \overline{C}_{r_{\sigma_{m+1}}|0}(K) \ \forall m = 1, \dots, M-1 \tag{19}$$

where $\sigma_i, \sigma_j = 1, \dots, M$ with $\sigma_i \neq \sigma_j$ for any $i \neq j$, and where $\overline{C}_{r_{\sigma_m}|0}(K)$ is given in (13) when the Bernoulli PU activity model is adopted, whereas it is given in (15) when the Markov chain PU activity model is adopted.

Remark: From Definition (19), we note that only the 0-status average route capacities $\overline{C}_{r_{\sigma_m}|0}(K)$ contribute to the *optimal route set* construction, in agreement with (5).

Theorem 1 (Optimal Route Priority Function): Given the *optimal route set* $\mathcal{R}^*$ defined in (19), the route priority function $f_{\mathcal{R}^*}$ given by:

$$f_{\mathcal{R}^*}(\mathbf{s}) = r_{\sigma_m} \iff s_{\sigma_m} = 0 \wedge s_{\sigma_l} = 1 \ \forall l \leq m \tag{20}$$

is a solution for the *optimal route priority problem*, i.e., $f^*(\cdot) = f_{\mathcal{R}^*}(\cdot)$.

Proof: See Appendix G. □

Remark: (20) establishes a bijective correspondence between the *optimal route set* $\mathcal{R}^*$ and the *route priority function* $f_{\mathcal{R}^*}$. Hence, for any $\mathcal{R}^*$, there exists a unique $f_{\mathcal{R}^*} \in \Phi$.

The proof of Theorem 2 requires the following result.

Proposition 5 (Average Aggregate Capacity): Given the *optimal route priority function* $f_{\mathcal{R}^*}$, the *average aggregate capacity* $\overline{C}_{f_{\mathcal{R}^*}}(K)$ given in (9) can be rewritten as:

$$\overline{C}_{f_{\mathcal{R}^*}}(K) = \sum_{m=1}^M \left(\overline{C}_{r_{\sigma_m}|0}(K)p_m^{\mathrm{off}}\prod_{l=1}^{m-1}p_m^{\mathrm{on}}\right) \tag{21}$$

where $\overline{C}_{r_{\sigma_m}|0}(K)$ is given in (13) when the Bernoulli PU activity model is adopted, whereas it is given in (15) when the Markov chain PU activity model is adopted.

Proof: See Appendix H. □

Remark: According to (21), the *average route capacity* $\overline{C}_{r_{\sigma_m}|0}(K)$ of the σ_m-th route is weighted by the joint probability of route σ_m being available and routes σ_l being unavailable for any $l < m$. This is reasonable. In fact, since the concept of *optimal route set* allows us to define a total order relation over the route set, route r_{σ_m} is always preferable to route r_{σ_l} when $l > m$.

Corollary 3 (Optimal Route Priority in Presence of Correlated PU Activity): The route priority function $f_{\mathcal{R}^*}(\cdot)$ given in (20) is a solution for the *optimal route priority problem* also when the PU activities on different routes are correlated.

Proof: See Appendix I. □

Remark: We note that, when the PU activities on different routes are correlated, the expression of the route status vector probability $p(s)$, according to Definition 2, has to be calculated by exploiting the chain rule. Hence, by supposing as instance that the PU activities on the first ℓ routes $r_1, \ldots, r_\ell$ are correlated each others, $p(s)$ is given by:

$$p(s) = \prod_{m=1}^{\ell} p(s_m | s_1, \ldots, s_{m-1}) \prod_{m=\ell+1}^{M} p(s_m) \qquad (22)$$

with $p(s_m | s_1, \ldots, s_{m-1})$ denoting the conditional probability and $p(s_m | s_1, \ldots, s_{m-1}) = p(s_1)$ when $m = 1$. Hence, (21) can be generalized to the case of correlated PU activities on different routes through (22).

Theorem 2 (Optimal Route Priority Algorithm): Alg. 1 solves the *optimal route priority problem* with time complexity equal to $\mathcal{O}(M \log M)$ and memory complexity equal to $\mathcal{O}(M)$.

Proof: See Appendix J. □

Algorithm 1 Constrained Routing Strategy

1: // preliminaries:
2: // sorting $P\{r_m\}_{m=1}^M$ with increasing $C_{r_m|0}(K)$
3: $\mathcal{R}^* = \{r_{\sigma_m}\}_{m=1}^M = \text{sort}\left(\{r_m\}_{m=1}^M\right)$
4: // input:
5: // route vector status $s = (s_1, \ldots, s_m)$
6: **for** $m = 1 : M$ **do**
7:   **if** $s_{\sigma_m} = 0$ **then**
8:     // selecting r_{σ_m} for packet transmission
9:     **return** r_{σ_m}
10:   **end if**
11: **end for**

D. Unconstrained Routing Strategy

Here, we derive a computational-efficient algorithm for the *optimal route priority problem* when the unconstrained routing strategy is adopted (Algorithm 2), and we assess in Theorem 4 its computational efficiency. To this aim, we first define in Definition 10 the concept of *optimal route set*, and then we analytically derive in Theorem 3 the *optimal route priority function*.

Definition 10 (Optimal Route Set): Given the routing update parameter K, the *optimal route set* $\mathcal{R}^*$ is the ordered sequence of N routes in $\{r_m\}_{m=1}^M$ defined as in (23), shown at the bottom of the page, where $\sigma_i, \sigma_j \in \{1, \ldots, M\}$ with $\sigma_i \neq \sigma_j$ for any $i \neq j$ with $i, j < N$ and where there exists $i < N$ so that $\sigma_i = \sigma_N$. $\overline{C}_{r_{\sigma_m}|s_{\sigma_m}}(K)$ is given in (13) when the Bernoulli PU activity model is adopted, whereas it is given in (15) when the Markov chain PU activity model is adopted.

Remark: Similarly to (19), the *optimal route set* $\mathcal{R}^*$ given in (23) is an ordered sequence of routes. Differently from (19), also the 1-status average route capacities $\overline{C}_{r_{\sigma_m}|1}(K)$ contribute to the *optimal route set*, in agreement with (6).

Remark: The rationale for the definition (23) of the *optimal route set* $\mathcal{R}^*$ is as follows.

— Similarly to (19) in Section IV-C, $\mathcal{R}^*$ is a sequence of routes.
— Differently from (19), a route, say route r_m, can be selected for $\mathcal{R}^*$ when its status is either 0 or 1, and the corresponding average route capacities are $\overline{C}_{r_m|0}(K)$ and $\overline{C}_{r_m|1}(K)$.
— $\mathcal{R}^*$ is constituted by $N - 1$ routes in status 0, denoted as $r_{\sigma_1}, \ldots, r_{\sigma_{N-1}}$, and a route in status 1, denoted as r_{σ_N}.
— Route r_{σ_N} is the route exhibiting the highest average capacity in status 1, i.e., $\overline{C}_{r_{\sigma_N}|1}(K) \geq \overline{C}_{r_m|1}(K)$ for any $m = 1, \ldots, M$.
— Routes $r_{\sigma_1}, \ldots, r_{\sigma_{N-1}}$ are all and only the routes exhibiting an average capacity in status 0 greater than the average capacity $\overline{C}_{r_{\sigma_N}|1}(K)$ of route r_{σ_N} in the status 1, i.e., $\overline{C}_{r_{\sigma_m}|0}(K) \geq \overline{C}_{r_{\sigma_N}|1}(K)$ for any $m = 1, \ldots, N-1$.
— Routes $r_{\sigma_1}, \ldots, r_{\sigma_{N-1}}$ are ordered according to their average capacities in status 0, i.e., $\overline{C}_{r_{\sigma_m}|0}(K) \geq \overline{C}_{r_{\sigma_{m+1}}|0}(K)$ for any $m = 1, \ldots, N-2$.

Corollary 4 (Optimal Route Set Cardinality): The *optimal route set* $\mathcal{R}^*$ given in (23) is composed by N routes, with $N \in [2, M+1]$.

Proof: See Appendix K. □

Theorem 3 (Optimal Route Priority Function): Given the *optimal route set* $\mathcal{R}^*$ defined in (23), the route priority function $f_{\mathcal{R}^*}$ given by:

$$f_{\mathcal{R}^*}(s) = \begin{cases} r_{\sigma_N} & \Longleftrightarrow s_{\sigma_m} = 1 \ \forall \sigma_m \neq \sigma_N \\ r_{\sigma_m}, m < N & \Longleftrightarrow s_{\sigma_m} = 0 \wedge s_{\sigma_l} = 1 \ \forall l < m \end{cases} \qquad (24)$$

$$\mathcal{R}^* = (r_{\sigma_1}, \ldots, r_{\sigma_N}) : \begin{cases} \overline{C}_{r_{\sigma_N}|1}(K) \geq \overline{C}_{r_m|1}(K) & \forall m = 1, \ldots, M \\ \overline{C}_{r_{\sigma_m}|0}(K) \geq \overline{C}_{r_{\sigma_{m+1}}|0}(K) & \forall m = 1, \ldots, N-2 \\ \overline{C}_{r_{\sigma_{N-1}}|0}(K) \geq \overline{C}_{r_{\sigma_N}|1}(K) \\ \overline{C}_{r_{\sigma_N}|1}(K) \geq \overline{C}_{r_m|0}(K) & \forall m \neq \sigma_l, \text{ with } l = 1, \ldots N-1 \end{cases} \qquad (23)$$

is a solution for the *optimal route priority problem*, i.e., $f^*(\cdot) = f_{\mathcal{R}^*}(\cdot)$.

Proof: See Appendix L. $\square$

Remark: Similarly to Section IV-C, (24) establishes a bijective correspondence between the *optimal route set* $\mathcal{R}^*$ and the *route priority function* $f_{\mathcal{R}^*}$. Hence, for any $\mathcal{R}^*$, there exists a unique $f_{\mathcal{R}^*} \in \Phi$.

The proof of Theorem 4 requires the following result.

Proposition 6 (Average Aggregate Capacity): Given the *optimal route priority function* $f_{\mathcal{R}^*}$, the *average aggregate capacity* $\overline{C}_{f_{\mathcal{R}^*}}(K)$ given in (9) can be rewritten as:

$$\overline{C}_{f_{\mathcal{R}^*}}(K) = \sum_{m=1}^{N-1}\left(\overline{C}_{r_{\sigma_m}|0}(K)p_m^{\text{off}}\prod_{l=1}^{m-1}p_m^{\text{on}}\right)$$
$$+ \overline{C}_{r_{\sigma_N}|1}(K)\prod_{l=1}^{N-1}p_m^{\text{on}} \quad (25)$$

where $\overline{C}_{r_{\sigma_m}|s_{\sigma_m}}(K)$ is given in (13) when the Bernoulli PU activity model is adopted, whereas it is given in (15) when the Markov chain PU activity model is adopted.

Proof: See Appendix M. $\square$

Corollary 5 (Optimal Route Priority in Presence of Correlated PU Activity): The route priority function $f_{\mathcal{R}^*}(\cdot)$ given in (24) is a solution for the *optimal route priority problem* also when the PU activities on different routes are correlated.

Proof: The proof follows by adopting the same reasoning of Corollary 3. $\square$

The same considerations made in the remark following Corollary 3 hold when the unconstrained routing strategy is considered.

Theorem 4 (Optimal Route Priority Algorithm): Alg. 2 solves the *optimal route priority problem* with time complexity equal to $\mathcal{O}(M\log M)$ and memory complexity equal to $\mathcal{O}(M)$.

Proof: See Appendix N. $\square$

Algorithm 2 Unconstrained Routing Strategy

1: // preliminaries:
2: // searching the route with the highest $C_{r_m|1}(K)$
3: $r_{\sigma_N} = \text{highestCap}\left(\{C_{r_m|1}(K)\}_{m=1}^{M}\right)$
4: // searching the routes with $C_{r_m|0}(K) > C_{\sigma_N|1}(K)$
5: $\{r_{\sigma_m}\}_{m=1}^{N-1} = \text{higherCap}\left(C_{r_{\sigma_N}|1}(K), \{C_{r_m|0}(K)\}_{m=1}^{M}\right)$
6: // sorting $\{r_{\sigma_m}\}_{m=1}^{N-1}$ with increasing $C_{r_{\sigma_m}|0}(K)$
7: $\mathcal{R}^* = \text{sort}(\{r_{\sigma_m}\}_{m=1}^{N-1})$
8: $\mathcal{R}^* = \mathcal{R}^* \cup \{r_{\sigma_N}\}$
9: // input:
10: // route vector state $\boldsymbol{s} = (s_1, \ldots, s_m)$
11: **for** $m = 1 : N-1$ **do**
12:   **if** $s_{\sigma_m} = 0$ **then**
13:     // selecting r_{σ_m} in state 0 for packet transmission
14:     **return** r_{σ_m}
15:   **end if**
16: **end for**
17: // selecting r_{σ_N} in state 1 for packet transmission
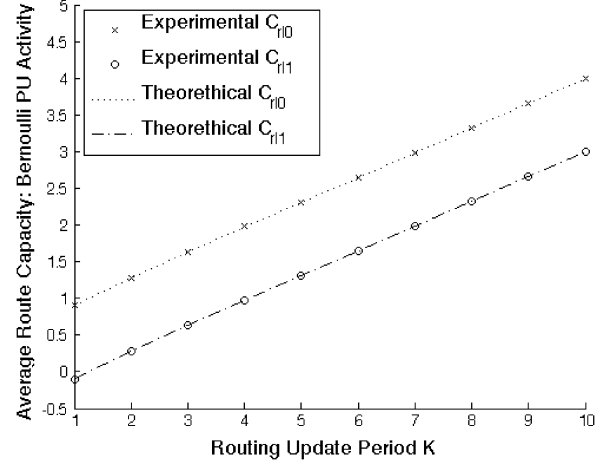18: **return** r_{σ_N}

Fig. 3. Average Route Capacity $\overline{C}_{r_m|s_m}^{\text{B}}(K)$ Validation: theoretical vs experimental average route capacity when Bernoulli PU activity model is adopted for different values of the routing update parameter K.

V. PERFORMANCE EVALUATION

In this section, we validate the theoretical framework presented in Section IV. More in detail, we first validate the theoretical results derived in Section IV-B, i.e., the closed-form expressions of the average route capacity given in (13) and (15). Then, we validate the optimality of the route priority function stated in Theorems 1 and 3 for both the considered routing strategies, and we assess the scalability of the proposed algorithms as the number M of available routes increases. Furthermore, we analyze the impact of the routing update parameter K on the average aggregate capacities as the PU activity probability decreases. Finally, we assess the impact of the route priority on the overall network layer performance by adopting as routing substrate a distance-vector routing scheme.

A. Average Route Capacity

In this subsection, we validate the closed-form expressions of the *average route capacity* derived in Propositions 3 and 4. More in detail, we compare the *average route capacities* $\overline{C}_{r_m|s_m}^{\text{B}}(K)$ and $\overline{C}_{r_m|s_m}^{\text{MC}}(K)$ given in (13) and (15), respectively, with those obtained through Monte Carlo simulations as the update parameter K increases. The adopted simulation set is as follows: $M = 1$, $C_m = 1$, $L_m = 0.1$, and $p_m^{\text{off}} = 0.3$.

Fig. 3 shows the *average route capacities* $\overline{C}_{r_m|0}^{\text{B}}(K)$ and $\overline{C}_{r_m|1}^{\text{B}}(K)$ when the Bernoulli PU activity model is adopted. First, we note that there is a very good agreement between the theoretical and the experimental results. Then, we note that the route capacities increase with K, in agreement with (13). Furthermore, for any value of K, it results $\overline{C}_{r_m|0}^{\text{B}}(K) > \overline{C}_{r_m|1}^{\text{B}}(K)$, validating so Corollary 1.

Fig. 4 shows the *average route capacities* $\overline{C}_{r_m|0}^{\text{MC}}(K)$ and $\overline{C}_{r_m|1}^{\text{MC}}(K)$ when the Markov Chain PU activity model is adopted with $p_m^{0|1} = 1/6$ and $p_m^{1|0} = 1/3$. Similarly to Fig. 3, we note that: i) there is a very good agreement between the theoretical
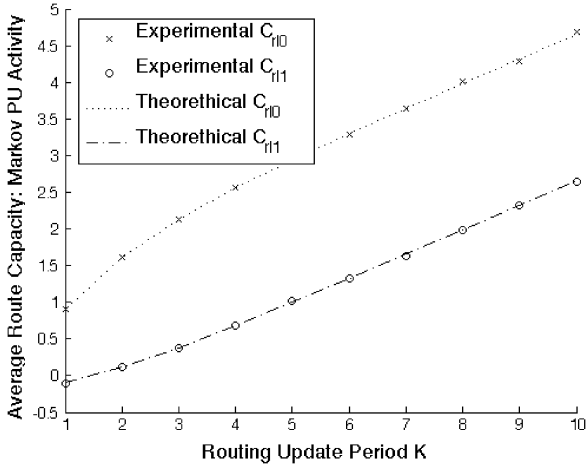
Fig. 4. Average Route Capacity $\overline{C}_{r_m|s_m}^{\text{MC}}(K)$ Validation: theoretical vs experimental average route capacity when Markov Chain PU activity model is adopted for different values of the routing update parameter K.

and the experimental results; ii) for any value of K, it results $\overline{C}_{r_m|0}^{\text{B}}(K) > \overline{C}_{r_m|1}^{\text{B}}(K)$, validating so Corollary 2.

B. Optimality

In this subsection, we validate the optimality of the route priority function stated in Theorems 1 and 3 for both the considered routing strategies. More in detail, we compare through Monte Carlo simulations the *average aggregate capacity* $\overline{C}_{f_{\mathcal{R}^*}}(K)$ computed with Algorithms 1 and 2 with those obtained by exhaustive search of the priority function f^* maximizing the *average aggregate capacity* as in (10).

The adopted simulation set is as follows: $K = 7$, $M = 10$, $\{C_m\}_{m=1}^M$ are uniformly distributed in $[0; 1]$, each L_m is uniformly distributed[2] in $[0; C_m]$, and $\{p_m^{\text{off}}\}_{m=1}^M$ are uniformly distributed in $[0; 1]$.

In the first set of experiments, we consider the constrained routing strategy. Specifically, Fig. 5 presents the difference between the results obtained with Algorithm 1 and the results obtained through exhaustive search for both the considered PU activity models. The (x) coordinate of the dot represents the aggregate route capacity $\overline{C}_{f_{\mathcal{R}^*}}(K)$ computed with Algorithm 1, whereas the (y) coordinate represents the *average aggregate capacity* $\overline{C}_{f^*}(K)$ computed with (10). Clearly, if $y = x$, then the two capacities are exactly the same, meaning that Algorithm 1 actually finds the *optimal route priority function*. Since Fig. 5 clearly shows that, for each realization and for both the considered PU activity models, $y = x$, then Algorithm 1 is optimal according to Definition 8. This result validates the optimality property stated by Theorems 1.

With reference to the scalability of the proposed algorithms, Fig. 6 shows the running times of Alg. 1 vs the exhaustive search algorithm as the number M of routes increases. We adopt a base 10 logarithmic scale for the y-axis to focus on the growth rate. First, we note that the running time of Alg. 1 increases

[2] The bit cost L_m can not be greater than the average bit-rate C_m when PU v_m is not active.
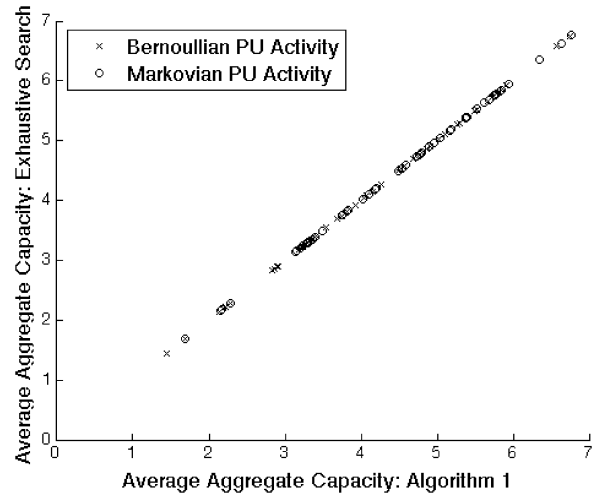

Fig. 5. Constrained Routing Strategy Optimality: Algorithm 1 vs Exhaustive Search. For each dot, coordinate (x) represents the average aggregate capacity computed with Algorithm 1, whereas coordinate (y) represents the average aggregate capacity computed with Eq. (10).
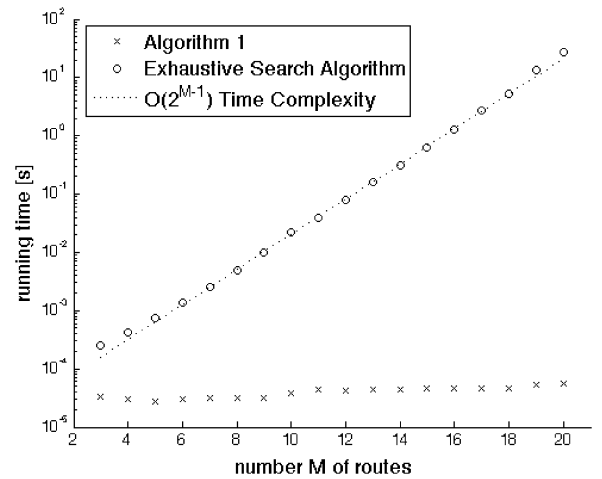

Fig. 6. Constrained Routing Strategy Running Time: Algorithm 1 vs Exhaustive Search. Logarithmic scale for axis y.

very slow with M, with a roughly linearithmic growth rate. This result validates the time complexity stated by Theo. 2. Moreover, we note that the running times of the exhaustive search algorithm exponentially increase with M. This result validates the time complexity stated by Prop. 2.

In the second set of experiments, we consider the unconstrained routing strategy. Specifically, Fig. 7 shows the difference between the results obtained with Algorithm 2 and the results obtained through exhaustive search for both the considered PU activity models. The same considerations made with reference to Fig. 5 hold also in this case, and the optimality property stated by Theorems 3 is validated as well. Regarding the time complexity, Fig. 8 shows the running time of Algorithm 2 vs the exhaustive search algorithm as the number M of routes increases. The same considerations made for the constrained strategy hold, and the time complexity stated by Theorem 4 is validated.
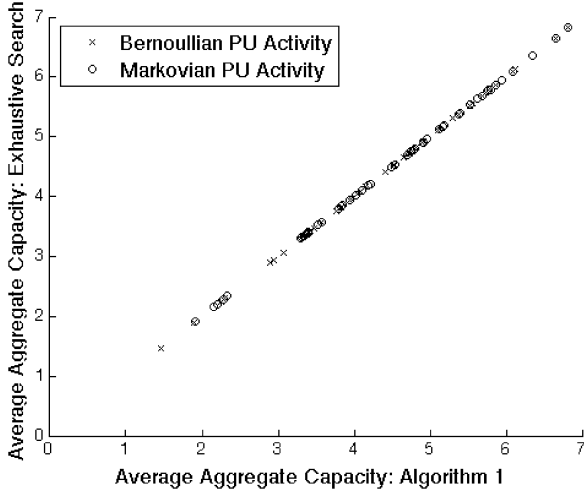
Fig. 7. Unconstrained Routing Strategy Optimality: Algorithm 2 vs Exhaustive Search. For each dot, coordinate (*x*) represents the average aggregate capacity computed with Algorithm 2, whereas coordinate (*y*) represents the average aggregate capacity computed with Eq. (10).
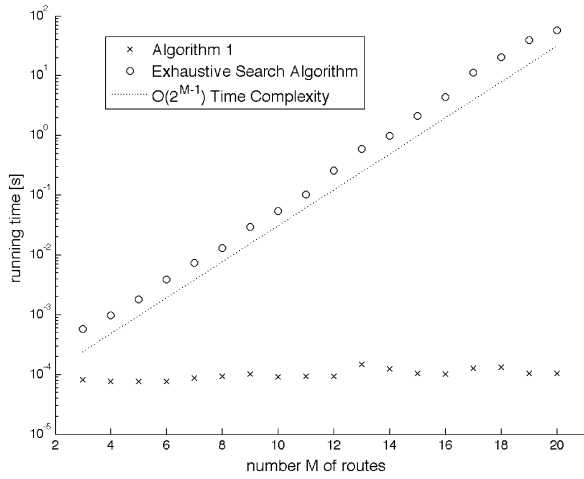

Fig. 8. Unconstrained Routing Strategy Running Time: Algorithm 2 vs Exhaustive Search. Logarithmic scale for axis *y*.

Fig. 9. Constrained Routing Strategy: average aggregate capacity $\overline{C}_{f*}(K)$ vs PU inactivity probability p_m^{off} for different values of the route update parameter K.

Fig. 10. Unconstrained Routing Strategy: average aggregate capacity $\overline{C}_{f*}(K)$ vs PU inactivity probability p_m^{off} for different values of the route update parameter K.

C. Average Aggregate Capacity

In this subsection, we analyze the impact of the routing update parameter K on the average aggregate capacity as the PU activity probability decreases.

The simulation set is as follows: $M = 10$, $\{C_m\}_{m=1}^M$ are uniformly distributed in $[0; 1]$, each L_m is uniformly distributed in $[0; C_m]$, and $\{p_m^{\text{off}}\}_{m=1}^M$ are uniformly distributed in $[0; 1]$. Finally, when the Markov Chain PU Activity model is adopted, we have[3] $p_m^{0|1} = \frac{1}{3} \min \left\{ 1, \frac{p_m^0}{p_m^1} \right\}$, $p_m^{1|0} = \frac{p_m^1}{p_m^0} p_m^{01}$.

Fig. 9 shows the *aggregate average capacity* $\overline{C}_{f*}(K)$ as the PU inactivity probability p_m^{off} increases for both the PU activity models, when the constrained routing strategy is adopted. We have considered three different values of the routing update parameter K, i.e., $K = 5$, $K = 7$, $K = 9$. The results of Fig. 9 con-
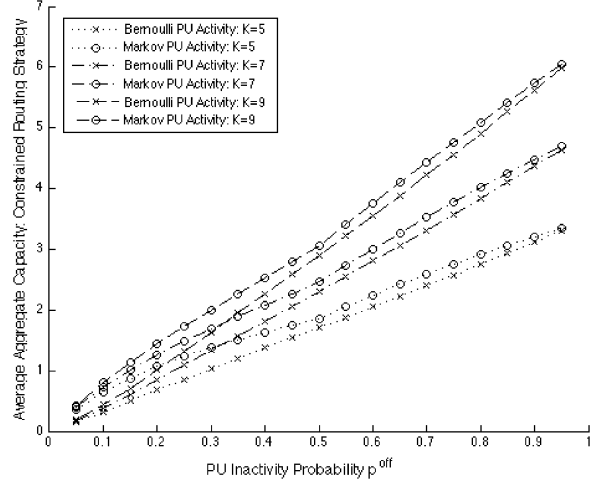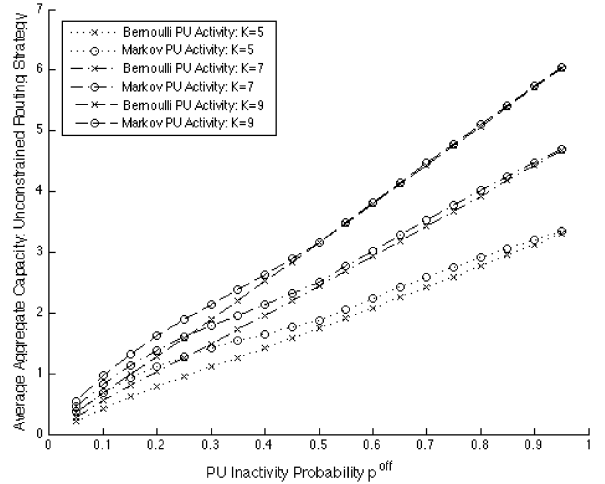
firm the theoretical analysis developed in Section IV. More in detail, we observe that the *average aggregate capacity* $\overline{C}_{f*}(K)$ is deeply affected by the routing update parameter K. In fact, the higher is K, the higher is $\overline{C}_{f*}(K)$, independently of the adopted PU activity model.

Fig. 10 presents the *aggregate average route capacity* $\overline{C}_{f*}(K)$ as the PU inactivity probability p_m^{off} increases for both the PU activity models, when the unconstrained routing strategy is adopted and three different values of the routing update parameter K have been considered, i.e., $K = 5$, $K = 7$, $K = 9$. The considerations made for the previous experiment continue to hold.

D. Integration With the Network Layer

In this subsection, we assess the impact of the route priority on the overall network layer performance. To this aim, we have considered the network topology shown in Fig. 11, similar to

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[3]It follows from p_m^{off} setting, by accounting for the Markov chain property.
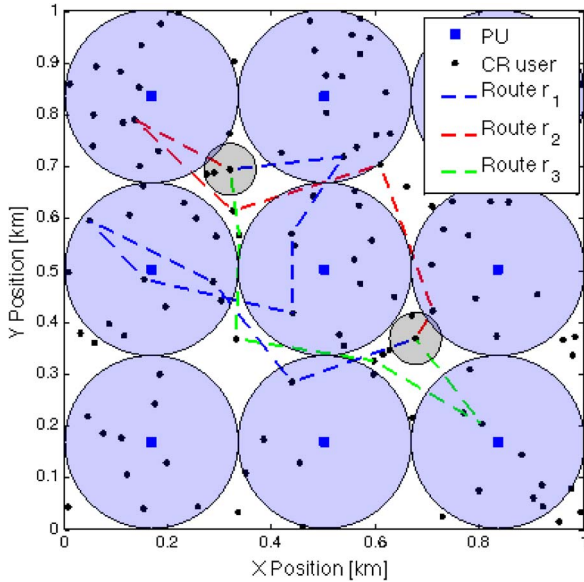
Fig. 11. Considered topology: the figure shows the three largest-capacity routes discovered by the path discovery process between the same couple source-destination.

the ones used in [9], [18]. Furthermore, we adopted as routing substrate a distance-vector path discovery scheme. Hence, each CR user shares its routing information with its neighbors through routing updates as in [19], without the need of a-priori knowledge about the full network topology or the PU activities. Finally, we adopted the constrained routing strategy.

The simulation set is as follows: 128 CR users are spread in a squared region of side 1 Km. The CR user transmission standard is IEEE 802.11af, and the link capacities are randomly distributed within the admissible data rates.[4] L_m is equal to 10 Kb, the normalized CR transmission range is set equal to 0.3, the PU interference range is shown in Fig. 11, and the on probability of each PU is uniformly distributed in [0, 1].

As shown by Fig. 11, three different routes are singled out by the path discovery process for the considered couple source-destination. Specifically, since the adopted routing metric is the route capacity, the discovery process singled out the three largest capacity routes, say routes $\{r_1, r_2, r_3\}$, ordered according to the decreasing route capacities, i.e., {24, 21.6, 16.2} Mbit/s. Fig. 12 shows the *average route capacities* $\overline{C}^{B}_{r_m|0}(K)$ of the three routes as a function of the routing update parameter K, when the Bernoulli PU activity model is adopted. We note that the average route capacities $\overline{C}^{B}_{r_m|0}(K)$ increase as K increases with different slopes due to the impact of the PU activities on the achievable capacity, in agreement with the theoretical results derived in Section IV.

Fig. 13 shows the *aggregate average capacity* $\overline{C}_{\mathcal{R}}(K)$ as function of the routing update parameter K when the Bernoulli PU activity model is considered. Specifically, we consider three different route sets: the optimal route set $\mathcal{R}^* = \{r_1, r_2, r_3\}$, i.e., the set constituted by the routes in descending order of
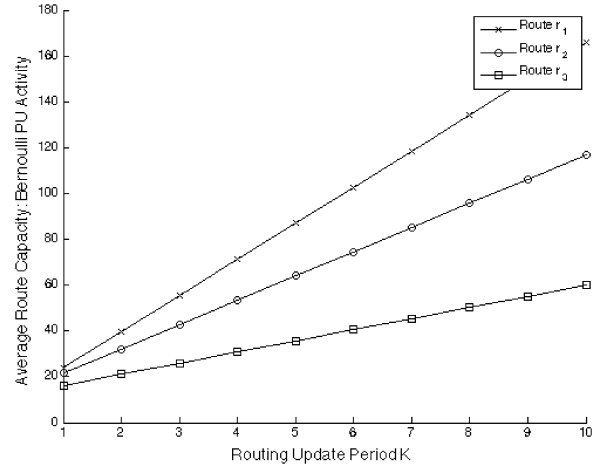
Fig. 12. Bernoulli PU Model: average route capacity $\overline{C}^{B}_{r_m|0}(K)$ vs routing update parameter K for the three routes $\{r_1, r_2, r_3\}$ shown in Fig. 11.
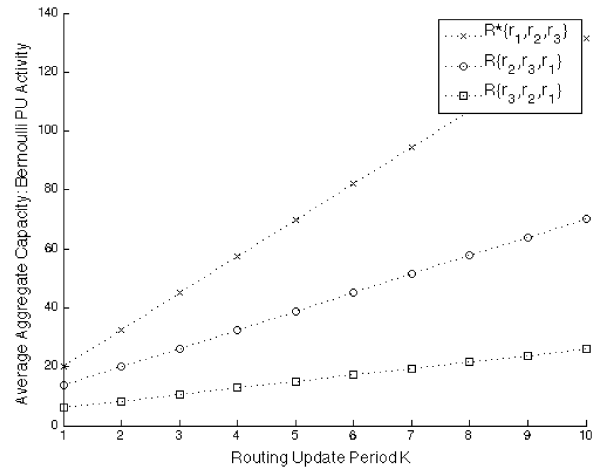

Fig. 13. Bernoulli PU Model: average aggregate capacity $\overline{C}_{\mathcal{R}}(K)$ vs route update parameter K for three different route sets.

average route capacity according to (19), and two sub-optimal sets, i.e., $\mathcal{R} = \{r_2, r_3, r_1\}$ and $\mathcal{R} = \{r_3, r_2, r_1\}$. The results of Fig. 14 confirm the importance of the route priority rule in terms of performance. Specifically, we observe that the *average aggregate capacity* $\overline{C}_{\mathcal{R}}(K)$ is deeply affected by the route priority rule, with $\overline{C}_{\mathcal{R}^*}(K)$ significantly outperforming the other average aggregate capacities.

Fig. 14 shows the *aggregate average capacity* $\overline{C}_{\mathcal{R}}(K)$ as function of the routing update parameter K when the Markov PU activity model is considered. We consider the same route sets of the previous experiment, and the same considerations hold also in this case.

VI. CONCLUSION

In cognitive radio (CR) networks the problem of prioritizing the routes for the CR packet transmission is particularly challenging, since the communication opportunities provided by a route are deeply affected by the primary-user (PU) activity. Furthermore, whenever the CR network layer exploits proactively acquired information on the PU activity, update
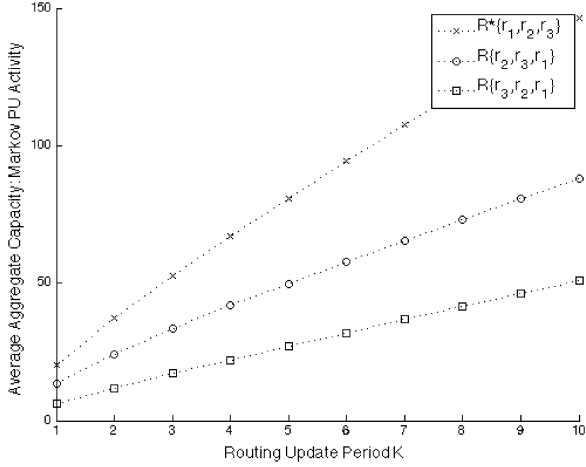
Fig. 14. Markov PU Model: average aggregate capacity $\overline{C}_{\mathcal{R}}(K)$ vs route update parameter K for three different route sets.

packets need to be exchanged among the CR users, inducing so a route overhead independently of the adopted routing protocol. In this paper, we analytically derived the *optimal route priority rule*, i.e., the route priority maximizing the achievable capacity, by accounting for both PU activity and route overhead. In particular, we derived closed-form expressions of the achievable capacity through which we design computational-efficient search algorithms for the optimal priority function. The theoretical analysis has been conducted by adopting two routing strategies and two PU activity models for conferring generality to the analysis. Extensive numerical simulations proved the optimality of the proposed route priority function, as well as the computational efficiency of the designed search algorithms. As future work, we will explore the impact of the CR user activities on the route priority rule.

APPENDIX

A. Proof of Proposition 1

It follows from $|\Sigma| = 2^M$ since, for any $s \in \Sigma$, $|f(s)| = 1$.

B. Proof of Proposition 2

We first note that:

$$\overline{C}_{f^*}(K) = \max_{f \in \Phi} \left\{ \overline{C}_f(K) \right\} = \sum_{s \in \Sigma} \left(p(s) \max_{f \in \Phi} \left\{ \overline{C}_{f(s)}(K) \right\} \right)$$
$$= \sum_{s \in \Sigma} \left(p(s) \max_m \left\{ \overline{C}_{r_m|s_m}(K) \right\} \right) \quad (26)$$

Thus, maximizing $f(\cdot)$ is equivalent to maximizing $\overline{C}_{f(s)}(K)$ for any $s \in \Sigma$.

Case 1: Constrained Strategy. It results $f(s) = r_m$ if and only if $s_m = 0$. Thus, if the route status vector s is constituted by m null components, with $m > 1$, then the route status vector can be associated to one of m different routes, and the number of operations for the max operator over a set of cardinality m is equal to $m - 1$. Since the number of distinct route status vectors

constituted by m null components can modeled as the number of permutations of M elements with repetition of m 0-elements and $M - m$ 1-elements, we have that such a number is equal to $\binom{M}{m}$. Thus, by noting that:

$$\sum_{m=1}^{M} \binom{M}{m}(m - 1) = (M - 2)2^{M-1} + 1 \quad (27)$$

we have the thesis.

Case 2: Unconstrained Strategy. Since each of the 2^M route status vectors can be associated to one of M different routes, and since the time complexity of the max operator over a set of cardinality M is equal to $\mathcal{O}(M)$, we have the thesis.

C. Proof of Proposition 3

Case 1: $s_m = 0$. Since $s_m = 0$, then PU v_m is not active during the first time slot and the available route capacity at the CR source is C_m. Consequently, by exploiting the Binomial Theorem [20], it results:

$$\overline{C}^{\mathrm{B}}_{r_m|0}(K)$$
$$= C_m \sum_{i=0}^{K-1} (K - i) \binom{K-1}{i} \left(p_m^{\mathrm{on}} \right)^i \left(p_m^{\mathrm{off}} \right)^{K-1-i} - \Omega_m(K)$$
$$= C_m \left(K - (K-1)p_m^{\mathrm{on}} \right) - \Omega_m(K) \quad (28)$$

and the proof follows.

Case 2: $s_m = 1$. Since $s_m = 1$, then PU v_m is active during the first time slot and the available route capacity at the CR source is zero. Consequently, by following the same reasoning of *Case 1*, it results:

$$\overline{C}^{\mathrm{B}}_{r_m|1}(K)$$
$$= C_m \sum_{i=0}^{K-1} (K - 1 - i) \binom{K-1}{i} \left(p_m^{\mathrm{on}} \right)^i \left(p_m^{\mathrm{off}} \right)^{K-1-i} - \Omega_m(K)$$
$$= C_m \left((K-1) - (K-1)p_m^{\mathrm{on}} \right) - \Omega_m(K) \quad (29)$$

and the proof follows.

D. Proof of Corollary 1

We prove the corollary with a *reductio ad absurdum* by supposing that it results $\overline{C}^{\mathrm{B}}_{r_m|0}(K) \leq \overline{C}^{\mathrm{B}}_{r_m|1}(K)$.

From (13), we have $\overline{C}^{\mathrm{B}}_{r_m|0}(K) = C_m + \overline{C}^{\mathrm{B}}_{r_m|1}(K) - \Omega_m(K)$ and, by substituting it in the *reduction hypothesis*, we obtain $C_m - \Omega_m(K) \leq 0$, which constitutes a *reductio ad absurdum* since the route overhead, i.e., the reduction of the route capacity due to update packets propagation, cannot exceed the route capacity by definition.

E. Proof of Proposition 4

We have two cases.

Case 1: $s_m = 0$. We adopt the mathematical induction method to prove the thesis.

Basis: Show that the statement holds for $K = 1$. When $K = 1$ and $s_m = 0$, one has $\overline{C}^{\mathrm{MC}}_{r_m|0}(1) = C_m - \Omega_m(1) \triangleq \overline{\Psi}^{\mathrm{MC}}_{r_m|0}(1) - \Omega_m(1)$

where $\overline{\Psi}_{r_m|0}^{MC}(1)$ can be rewritten using the following recursive expression, since $\overline{\Psi}_{r_m|0}^{MC}(i) = 0$ with $i < 0$:

$$\overline{\Psi}_{r_m|0}^{MC}(1) = 1 \cdot C_m - $$
$$- p_m^{1|0}\left[\overline{\Psi}_{r_m|0}^{MC}(K-1) + \sum_{l=1}^{K-2}\left(p_m^{1|1}\right)^l \overline{\Psi}_{r_m|0}^{MC}(K-l-1)\right] \quad (30)$$

Hence the statement is true for $K = 1$.

Inductive Step: Show that if $\overline{\Psi}_{r_m|0}^{MC}(K-1)$ is given by the recursive expression (16), then also $\overline{\Psi}_{r_m|0}^{MC}(K)$ is given by (16). When the routing update parameter is K and the route r_m is in the status $s_m = 0$, the average route capacity is equal to (31) [10], shown at the bottom of the page. To prove the thesis we need to prove that $\overline{\Psi}_{r_m|0}^{MC}(K)$ reported in (31) can be rewritten as in (16). To this aim, we recognize that $\overline{\Psi}_{r_m|0}^{MC}(K)$ reported in (31) can be rewritten as (32), shown at the bottom of the page. Since for $l \in \{1, \ldots, K-1\}$, we have:

$$\overline{\Psi}_{r_m|0}^{MC}(K-l) = C_m \sum_{n_{K-l-1}=0}^{1} p_m^{n_{K-l-1}|0} \ldots$$
$$\ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-l-1}|1-n_j|\right) \quad (33)$$

after some algebraic manipulations, it results:

$$\overline{\Psi}_{r_m|0}^{MC}(K) = p_m^{0|0}\left(C_m + \overline{\Psi}_{r_m|0}^{MC}(K-1)\right) +$$
$$+ p_m^{1|0}\left(p_m^{0|1}\left(C_m + \overline{\Psi}_{r_m|0}^{MC}(K-2)\right) + \right.$$
$$+ p_m^{1|1}\left(p_m^{0|1}\left(C_m + \overline{\Psi}_{r_m|0}^{MC}(K-3)\right) + \right.$$
$$+ p_m^{1|1}C_m \sum_{n_{K-4}=0}^{1} p_m^{n_{K-4}|1} \ldots$$
$$\left.\left.\left.\ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-4}|1-n_j|\right)\right)\right)\right) \quad (34)$$

By accounting for $p_m^{1|1} = 1 - p_m^{0|1}$ and $p_m^{0|0} = 1 - p_m^{1|0}$ and by exploiting the inductive hypothesis $\overline{\Psi}_{r_m|0}^{MC}(K-1) = (K-1)C_m - p_m^{1|0}\left[\overline{\Psi}_{r_m|0}^{MC}(K-2) + \sum_{l=1}^{K-3}(p_m^{1|1})^l\overline{\Psi}_{r_m|0}^{MC}(K-l-2)\right]$, one can rewrite (34) as (35), shown at the bottom of the page.

Since $\overline{\Psi}_{r_m|0}^{MC}(1) = C_m$, it is easy to recognize that the equality (36) holds, as shown at the bottom of the page, and by substituting (36) in (35), the proof follows.

$$\overline{C}_{r_m|0}^{MC}(K) \triangleq \overline{\Psi}_{r_m|0}^{MC}(K) - \Omega_m(K) = C_m\left(\sum_{n_{K-1}=0}^{1} p_m^{n_{K-1}|0} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|n_{K-1}} \ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-1}|1-n_j|\right)\right) - \Omega_m(K) \quad (31)$$

$$\overline{\Psi}_{r_m|0}^{MC}(K) = C_m\left(p_m^{0|0} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|0} \ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(2 + \sum_{j=1}^{K-2}|1-n_j|\right) + p_m^{1|0} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|1} \ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-2}|1-n_j|\right)\right) \quad (32)$$

$$\overline{\Psi}_{r_m|0}^{MC}(K) = KC_m - p_m^{1|0}\left(\overline{\Psi}_{r_m|0}^{MC}(K-1) + p_m^{1|1}\overline{\Psi}_{r_m|0}^{MC}(K-2) + \sum_{l=1}^{K-3}\left(p_m^{1|1}\right)^l \overline{\Psi}_{r_m|0}^{MC}(K-l-2) + p_m^{1|1}C_m - \right.$$
$$\left. - p_m^{1|1}\left(p_m^{0|1}\left(C_m + \overline{\Psi}_{r_m|0}^{MC}(K-3)\right) + p_m^{1|1}C_m \sum_{n_{K-4}=0}^{1} p_m^{n_{K-4}|1} \ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-4}|1-n_j|\right)\right)\right) \quad (35)$$

$$\sum_{l=2}^{K-2}\left(p_m^{1|1}\right)^l \overline{\Psi}_{r_m|0}^{MC}(K-l-1) = \sum_{l=1}^{K-3}\left(p_m^{1|1}\right)^l \overline{\Psi}_{r_m|0}^{MC}(K-l-2) + p_m^{1|1}C_m - p_m^{1|1}\left(p_m^{0|1}\left(C_m + \overline{\Psi}_{r_m|0}^{MC}(K-3)\right) + \right.$$
$$\left. + p_m^{1|1}C_m \sum_{n_{K-4}=0}^{1} p_m^{n_{K-4}|1} \ldots \sum_{n_1=0}^{1} p_m^{n_1|n_2}\left(1 + \sum_{j=1}^{K-4}|1-n_j|\right)\right) \quad (36)$$

Case 2: $s_m = 1$. According to Def. 6 and the subsequent remark, the average route capacity can be expressed as:

$$
\begin{aligned}
\overline{C}_{r_m|1}^{\mathrm{MC}}(K) &\triangleq \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K) - \Omega_m(K) = \\
&= C_m \sum_{n_{K-1}=0}^{1} p_m^{n_{K-1}|1} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|n_{K-1}} \cdots \\
&\quad \cdots \sum_{n_1=0}^{1} p_m^{n_1|n_2} \sum_{j=1}^{K-1} |1 - n_j| - \Omega_m(K) \quad (37)
\end{aligned}
$$

By accounting for $\overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - l) = C_m \sum_{n_{K-l-1}=0}^{1} p_m^{n_{K-l-1}|0} \cdots \sum_{n_1=0}^{1} p_m^{n_1|n_2} \left(1 + \sum_{j=1}^{K-l-1} |1 - n_j|\right)$ it results in (38), shown at the bottom of the page. By further manipulating $\overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 1)$ and by adopting similar reasonings as described above, the proof follows.

F. Proof of Corollary 2

From (31), after some algebraic manipulations, it results:

$$
\begin{aligned}
\overline{C}_{r_m|0}^{\mathrm{MC}}(K) &= C_m - \Omega_m(K) + p_m^{0|0} \overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 1) + \\
&\quad + p_m^{1|0} \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 1) \quad (39)
\end{aligned}
$$

We adopt the mathematical induction method.

Basis: Show that the statement holds for $K = 1$. From Proposition 4, it results:

$$
\overline{C}_{r_m|s_m}^{\mathrm{MC}}(K) = \begin{cases} C_m - \Omega_m(1), & s_m = 0 \\ 0, & s_m = 1 \end{cases} \quad (40)
$$

and the statement $\overline{C}_{r_m|0}^{\mathrm{MC}}(K) > \overline{C}_{r_m|1}^{\mathrm{MC}}(K)$ is true for $K = 1$.

Inductive step: Show that if $\overline{C}_{r_m|0}^{\mathrm{MC}}(K - 1) > \overline{C}_{r_m|1}^{\mathrm{MC}}(K - 1)$, then also $\overline{C}_{r_m|0}^{\mathrm{MC}}(K) > \overline{C}_{r_m|1}^{\mathrm{MC}}(K)$ is true. To this aim, we conduct a *reductio ad absurdum* by supposing:

$$
\overline{C}_{r_m|0}^{\mathrm{MC}}(K) \leq \overline{C}_{r_m|1}^{\mathrm{MC}}(K) \quad (41)
$$

By substituting (39) in (41) and by accounting for (38), one has:

$$
\begin{aligned}
&\overline{C}_{r_m|0}^{\mathrm{MC}}(K) \leq \overline{C}_{r_m|1}^{\mathrm{MC}}(K) \Longleftrightarrow \\
&\Longleftrightarrow C_m - \frac{L_m}{KT} + \left(p_m^{1|1} - p_m^{1|0}\right) \overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 1) \leq \\
&\leq \left(p_m^{1|1} - p_m^{1|0}\right) \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 1) \quad (42)
\end{aligned}
$$

By substituting in (42) the expressions of $\overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 1)$ and $\overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 1)$, one has:

$$
\begin{aligned}
&\overline{C}_{r_m|0}^{\mathrm{MC}}(K) \leq \overline{C}_{r_m|1}^{\mathrm{MC}}(K) \Longleftrightarrow C_m - \frac{L_m}{KT} + \left(p_m^{1|1} - p_m^{1|0}\right) C_m + \\
&+ \left(p_m^{1|1} - p_m^{1|0}\right)^2 \left(\overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 2) - \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 2)\right) \leq 0 \quad (43)
\end{aligned}
$$

This constitutes an absurdum since, by exploiting the inductive hypothesis, it results $C_m - \frac{L_m}{KT} + (p_m^{1|1} - p_m^{1|0})C_m + (p_m^{1|1} - p_m^{1|0})^2 \left(\overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 2) - \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 2)\right) > 0$.

G. Proof of Theorem 1

We prove the theorem with a *reductio ad absurdum* by supposing that $\exists\, s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_m} \neq f^*(s) = r_{\sigma_k}$. Thus, we have:

$$
p(s)C_{r_{\sigma_k}|s_{\sigma_k}} > p(s)C_{r_{\sigma_m}|r_{\sigma_m}} \Longleftrightarrow C_{r_{\sigma_k}|s_{\sigma_k}} > C_{r_{\sigma_m}|s_{\sigma_m}} \quad (44)
$$

From (20) it results $k > m$ since $s_{\sigma_l} = 1$ for any $l < m$, and from (5) it results $f^*(s) = r_{\sigma_k}$ if and only if $s_{\sigma_k} = 0$. By accounting for (19), it results $\overline{C}_{r_{\sigma_m}|0}(K) \geq \overline{C}_{r_{\sigma_k}|0}(K)$ for any $k > m$. Hence, (44) constitutes a *reductio ab absurdum*.

H. Proof of Proposition 5

We have:

$$
\begin{aligned}
\overline{C}_{f_{\mathcal{R}^*}}(K) &= \sum_{s \in \Sigma} p(s)\overline{C}_{f_{\mathcal{R}^*}(s)}(K) \\
&= \sum_{m=1}^{M} \overline{C}_{r_{\sigma_m}|0}(K) \left(\sum_{s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_m}} p(s)\right) \quad (45)
\end{aligned}
$$

Let us consider, without loss of generality, the right side of (45) for $m = M - 1$. From (20), it results:

$$
f_{\mathcal{R}^*}(s) = r_{\sigma_{M-1}} \, \forall s \in \Sigma : s_{\sigma_m} = 0 \wedge s_{\sigma_l} = 1 \, \forall l \leq M - 1 \quad (46)
$$

The set of route state vectors satisfying the constraints in (46) has cardinality equal to $2^{M-(M-1)} = 2$, since $s_{\sigma_M} \in \{0, 1\}$. Hence, it results:

$$
\begin{aligned}
\sum_{s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_m}} p(s) &= p_{\sigma_M}^{\mathrm{off}} p_{\sigma_{M-1}}^{\mathrm{off}} \prod_{l=1}^{M-2} p_{\sigma_l}^{\mathrm{on}} + \\
&+ p_{\sigma_M}^{\mathrm{on}} p_{\sigma_{M-1}}^{\mathrm{off}} \prod_{l=1}^{M-2} p_{\sigma_l}^{\mathrm{on}} = p_{\sigma_{M-1}}^{\mathrm{off}} \prod_{l=1}^{M-2} p_{\sigma_l}^{\mathrm{on}} \quad (47)
\end{aligned}
$$

By adopting the same reasoning for any value of m in the right side of (45), the thesis follows.

$$
\begin{aligned}
\overline{C}_{r_m|1}^{\mathrm{MC}}(K) &= C_m \left(p_m^{0|1} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|0} \cdots \sum_{n_1=0}^{1} p_m^{n_1|n_2} \left(1 + \sum_{j=1}^{K-2} |1 - n_j|\right) + p_m^{1|1} \sum_{n_{K-2}=0}^{1} p_m^{n_{K-2}|1} \cdots \right. \\
&\quad \left. \cdots \sum_{n_1=0}^{1} p_m^{n_1|n_2} \sum_{j=1}^{K-2} |1 - n_j|\right) - \Omega_m(K) = p_m^{0|1} \overline{\Psi}_{r_m|0}^{\mathrm{MC}}(K - 1) + p_m^{1|1} \overline{\Psi}_{r_m|1}^{\mathrm{MC}}(K - 1) - \Omega_m(K) \quad (38)
\end{aligned}
$$

I. Proof of Corollary 3

For the sake of simplicity let us consider two existing routes, say routes r_1 and r_2, whose PU activities are correlated. We assume that the average route capacity of the first route is larger than the corresponding capacity of the second route, i.e., $\overline{C}_{r_1|0}(K) > \overline{C}_{r_2|0}(K)$ with $\overline{C}_{r_1|0}(K)$ given either in (13) or (15) depending on the adopted PU activity model. Hence, according to (20), the optimal route set is $\mathcal{R}^* = (r_1, r_2)$. We prove the corollary with a *reductio ab absurdum* by supposing that:

$$\overline{C}_{f_\mathcal{R}}(K) > \overline{C}_{f_{\mathcal{R}^*}}(K) \tag{48}$$

with $\mathcal{R} = (r_2, r_1)$. As discussed in the remark following Corollary 3, the average aggregate capacity (9) in presence of correlated PU activity among different routes can be calculated by exploiting the chain rule. Consequently:

$$\overline{C}_{f_{\mathcal{R}^*}}(K) = P(X_1 = 0)\overline{C}_{r_1|0}(K) + P(X_1 = 1 \wedge X_2 = 0)$$
$$\overline{C}_{r_2|0}(K) = P(X_1 = 0)\overline{C}_{r_1|0}(K) + P(X_1 = 1)$$
$$P(X_2 = 0|X_1 = 1)\overline{C}_{r_2|0}(K) \tag{49}$$
$$\overline{C}_{f_\mathcal{R}}(K) = P(X_2 = 0)\overline{C}_{r_2|0}(K) + P(X_2 = 1 \wedge X_1 = 0)$$
$$\overline{C}_{r_1|0}(K) = P(X_2 = 0)\overline{C}_{r_2|0}(K) + P(X_2 = 1)$$
$$P(X_1 = 0|X_2 = 1)\overline{C}_{r_1|0}(K) \tag{50}$$

By substituting (49) and (50) in (48), we have:

$$\overline{C}_{r_2|0}(K) \left(P(X_2 = 0) - P(X_2 = 0|X_1 = 1)P(X_1 = 1) \right) =$$
$$= \overline{C}_{r_2|0}(K)P(X_2 = 0|X_1 = 0)P(X_1 = 0) =$$
$$= \overline{C}_{r_2|0}(K)P(X_2 = 0 \wedge X_1 = 0) >$$
$$> \overline{C}_{r_1|0}(K) \left(P(X_1 = 0) - P(X_1 = 0|X_2 = 1)P(X_2 = 1) \right)$$
$$= \overline{C}_{r_1|0}(K)P(X_1 = 0|X_2 = 0)P(X_2 = 0) =$$
$$= \overline{C}_{r_1|0}(K)P(X_1 = 0 \wedge X_2 = 0) \tag{51}$$

Clearly, (51) constitutes a *reductio ab absurdum* since by hypothesis $\overline{C}_{r_1|0}(K) > \overline{C}_{r_2|0}(K)$.

J. Proof of Theorem 2

As regards the optimality property, it follows directly from Theorem 1. As regards the time complexity, it follows directly from lines 1–3 of Algorithm 1, by accounting for the computational complexity $\mathcal{O}(M \log M)$ of the sorting procedure. Finally, as regards the memory complexity, it follows directly from lines 1–3 of Algorithm 1, by accounting for the cardinality M of the optimal route set $\mathcal{R}^*$.

K. Proof of Corollary 4

Since the number of distinct routes in the status 0, whose average capacity $\left\{\overline{C}_{r_{\sigma_m}|0}\right\}$ is greater than $\overline{C}_{r_{\sigma_N}|1}$, is at most M, it results $N \leq M + 1$, with $N = M + 1$ if and only if $\overline{C}_{r_{\sigma_m}|0} \geq \overline{C}_{r_{\sigma_N}|1} \; \forall m = 1, \ldots, M$. Furthermore, from Cor. 1 and 2, we have that $\overline{C}_{r_{\sigma_1}|0} \geq \overline{C}_{r_{\sigma_1}|1}$ for any σ_1. Hence, $N \geq 2$, with $N = 2$ if and only if $\overline{C}_{r_{\sigma_1}|1} \geq \overline{C}_{r_m|0} \; \forall m \neq \sigma_1$.

L. Proof of Theorem 3

We prove the theorem with a *reductio ad absurdum* by supposing that $\exists s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_m} \neq f^*(s) = r_{\sigma_k}$. Thus, we have:

$$p(s)C_{r_{\sigma_k}|s_{\sigma_k}} > p(s)C_{r_{\sigma_m}|r_{\sigma_m}} \iff C_{r_{\sigma_k}|s_{\sigma_k}} > C_{r_{\sigma_m}|s_{\sigma_m}} \tag{52}$$

Case 1: $s_{\sigma_k} = 0$. From (24), it results $k > m$ since $s_{\sigma_l} = 1$ for any $l < m$. By accounting for (23), it results $\overline{C}_{r_{\sigma_m}|r_{\sigma_m}}(K) \geq \overline{C}_{r_{\sigma_k}|0}(K)$ for any $k > m$. Hence, (52) constitutes a *reductio ab absurdum*.

Case 2: $s_{\sigma_k} = 1$. From (23), it results $\overline{C}_{r_{\sigma_m}|r_{\sigma_m}}(K) \geq \overline{C}_{r_{\sigma_k}|1}(K)$ for any k. Hence, (52) constitutes a *reductio ab absurdum*.

M. Proof of Proposition 6

We have:

$$\overline{C}_{f_{\mathcal{R}^*}}(K) = \sum_{s \in \Sigma} p(s)\overline{C}_{f_{\mathcal{R}^*}(s)}(K)$$
$$= \sum_{m=1}^{N-1} \overline{C}_{r_{\sigma_m}|0}(K) \left(\sum_{s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_m}} p(s) \right) +$$
$$+ \overline{C}_{r_{\sigma_N}|1}(K) \left(\sum_{s \in \Sigma : f_{\mathcal{R}^*}(s) = r_{\sigma_N}} p(s) \right) \tag{53}$$

and, since $p_{\sigma_m}^{\text{off}} + p_{\sigma_m}^{\text{on}} = 1$ for any σ_m, by following the same reasoning of Appendix H, the thesis follows.

N. Proof of Theorem 4

As regards the optimality property, it follows directly from Theorem 3. As regards the time complexity, it follows directly from lines 1–8 of Algorithm 2, since the number of operations is equal to M for line 3, M for line 5, and $M \log M$ for lines 7–8. Finally, as regards the memory complexity, it follows directly from Corollary 4.

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