A theoretical model for opportunistic routing in ad hoc networks

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***Abstract*—Traditional routing strategies for multi-hop wireless networks forward packets by selecting at the sender side the next hop for each packet. Recently, such a paradigm has been called into question by a new approach, namely the *Opportunistic Routing*. It exploits the broadcast nature of wireless transmissions to take advantage from spatial diversity by routing the packets according to the propagation conditions, i.e. by selecting the next hop at the receiver side. Although numerous opportunistic algorithms and protocols have been proposed in the last years, very few works have used an analytical approach to analyze the opportunistic routing behavior so as to provide a guideline for future protocol design. In this paper, we propose an analytical model to describe any routing procedures operating according to the opportunistic paradigm. It applies in a very general multi-hop scenario and is not restricted to any specific network topology or opportunistic protocol. The model requires the knowledge of both the delivery ratios and node priority, which is based on the adopted routing metric (Expected Transmission Count (ETX), geographic distance, etc). In this paper we exploit such a model to derive a closed-form expression of the average number of data-link transmissions needed to successfully deliver a packet.**

***Index Terms*—ad hoc networks, opportunistic routing, analytical model, average transmission number.**

I. Introduction

For more than a decade, multi-hop forwarding [1] has been considered a suitable strategy for networking in ad hoc networks, since it well fits in scenarios characterized by dynamic topology with no available infrastructure nor central management.

Multi-hop traditional routing tries to *fortify* the connectivity by adopting solutions which force the wireless network behavior to be similar to that of wired one. Specifically, multi-hop routing completely hides the broadcast nature of wireless communications in data forwarding, by imposing at the data-link layer that nodes have to discard data packets not directly sent to them, although they have correctly received such packets. Moreover, it usually counteracts the time-variant impairment of the wireless propagation by means of Automatic Repeat Request (ARQ) or Forward Error Control (FEC) data-link techniques or a combination of both of them.

As opposed to fortify the environment, a fairly approach, recently proposed, consists of exploiting the wireless communication good nature, namely the broadcasting, to compensate the bad one, i.e. the channel unreliability. This design philosophy, referred to as the *opportunistic routing* [2], relaxes the assumption that the wireless propagation conditions are stationary, namely it accounts for the time-variance of the channel and its instability due to node mobility to enhance the networking performances.

Unlike traditional routing protocols, in opportunistic routing the forwarder simply broadcasts the data packets without worrying about next hop selection, since such a choice is performed at the receiver side. Thus a routing progress is reached every time that a node closer to the destination than the forwarder correctly receives the packet, and such a node becomes the next hop.

Several opportunistic algorithms and protocols [2]–[7] have been proposed for a variety of multi-hop wireless networks. Few of them provide theoretical studies. Specifically, [8] focus on the characterization of the packet reception probability for shadowing and fading propagation models. It allows one to evaluate the average progress per transmission toward the destination under the assumption of location-based opportunistic routing uniform node distribution. In [9] a new and interesting metric, namely the Expected Any-path Transmissions (EAX), that computes the expected number of any-path transmissions required to deliver a packet has been proposed. The EAX is evaluated by resorting to a recursive algorithm since no closed-form expression is provided, and it refers to a specific opportunistic protocol.

In this paper, we propose a general framework to model any routing procedure operating according to the opportunistic paradigm. In our study, the unique assumption made is the knowledge of both the delivery ratios and the priority (order relationship) among the nodes, which is needed to single out which node, among the ones that have received the broadcasted packet, has to become in charge for the next packet forwarding. Our model is general since the features of both the particular routing protocol and connectivity scenario are fully summarized by the priority rule and the delivery ratios respectively.

Such a model allows us to analytically evaluate several

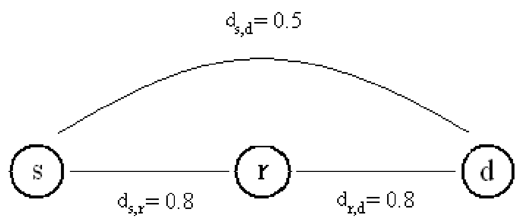

Fig. 1: An example of opportunistic routing. The number on each link represents the delivery ratios of the link, i.e. the success probability of a single transmission over the link.

performance parameters, such as the packet retransmission probability, the packet dropping probability, the packet average transmission number, the end-to-end delay, etc. In this paper, we focus on the evaluation of the average transmission number, for which a closed-form expression is also provided.
Briefly, the major contributions of this paper are: i) an analytical general model to describe an opportunistic routing procedure; ii) a closed-form expression for the average transmission number. Finally, the validity of the results are confirmed by numerical simulations.

II. Preliminaries on Opportunistic Routing

Opportunistic Routing exploits the multi-user diversity, i.e. the availability of multiple neighbors whose links can be modeled as statistically independent channels [10], to face against the instability of the wireless propagation conditions. As example, let us consider the topology shown in Fig.1, where the node s sends a packet to d along one of the possible paths $\{(s, d); (s, r, d)\}$ and $d_{i,j}$ denotes the delivery ratio of the link between the nodes i and j.
Traditional multi-hop routing singles out the next hop at the sender side and it forwards the packet with unicast transmissions. So, if *r* is chosen as next hop, the link quality is good and no retransmission is required with probability $d_{s,r} = 0.8$, but the routing progress is small. Alternatively, if the final destination is chosen as next hop, the highest routing progress is achieved if the packet reaches the destination but, since the link quality is poor, the probability of single transmission is just $d_{s,d} = 0.5$.
In contrast, in opportunistic routing the sender broadcasts the packet, allowing so to pick up as relay the node closest to the destination among the ones that receive the packet. In this way, it is able to opportunistically leverage unexpected paths related with node mobility and/or changes in wireless propagation conditions, namely it exploits *spatial diversity*.
In order to achieve the potential benefits of opportunistic routing without causing duplicate transmissions or incurring significant coordination overhead, an effective opportunistic routing protocol has to implement a mechanism to single out, among all the neighbor nodes that have successfully received the packet, the one that maximize the routing progress of each data transmission toward the destination. At this end, it is necessary to define a routing progress metric which is able to prioritize the nodes. For example, the ExOR protocol [2] adopts as routing progress metric the Expected Transmission Count (ETX), [9] proposes the EAX and finally in [11] an utility based metric it is adopted.

III. Analytical Framework

A. Problem formulation and assumptions

We consider a set V of $n = |V|$ wireless nodes deployed in a given area. Each node has a unique identifier $v_i \in \{v_1, v_2, \ldots, v_n\}$ and has an omnidirectional antenna. We model the network with a probabilistic direct graph:

$$G = (V, L, D) \tag{1}$$

in which a vertex $v_i \in V$ denotes a node and an edge $l_{i,j} \in E$ represents a communication link from node v_i to node v_j. Each link $l_{i,j}$ is characterized by a delivery ratio $d_{i,j} \in D$, which measures the probability that a packet is correctly received in a single transmission along such a link. Clearly, we have that $d_{i,i} = 1$. We assume that during the packet delivery procedure the delivery ratios are constant [12], [13]. Moreover, we assume that the link failure events are statistically independent of each other [10].

B. Analytical Model

Let s, $d \in V$ be the source and the destination of a packet transmission respectively, and let $f : V \times V \to R$ be the priority function[1]. i.e. the function that measures the routing progress of a packet toward the destination.
We define the ordered set of the allowed relays $r_i \in V$ for the packet sent by s toward d as:

$$\mathcal{R}_{s,d} = \{(s = r_0, r_1, r_2, \ldots, r_N, r_{N+1} = d) : f(r_i, d) \leq f(r_{i+1}, d)\}. \tag{2}$$

More specifically, $\mathcal{R}_{s,d} \subseteq V$ represents the subset of V with $|R_{s,d}| = N+2$ constituted by all the possible forwarder for the packet plus the destination, ordered according to the priority function $f(\cdot, \cdot)$.
Let us define $L_{s,d} \subseteq L$ as the subset of links (among the ordered nodes of $\mathcal{R}_{s,d}$) that allow the packet to progress towards the destination:

$$L_{s,d} = \{l_{r_i,r_j} \in L : r_i, r_j \in \mathcal{R}_{s,d}, \ i \leq j\} \tag{3}$$

We underline that in (3) the condition $i \leq j$ simply means that a packet cannot be forwarded by a node with a certain priority toward nodes exhibiting lower priority.
Finally, we define with p_{r_i,r_j} the probability that the higher priority relay that receives the packet sent by $r_i \in \mathcal{R}_{s,d}$ is $r_j \in$

[1]The priority function depends on the particular opportunistic routing protocol adopted, so the choice of a real priority function is not mandatory. Our definition can be easily adapted to different routing progress metrics.

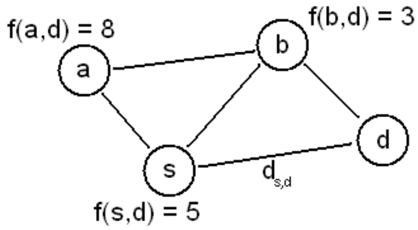

Fig. 2: An example of physical graph, where $d_{\cdot,\cdot}$ is the delivery ratio and $f(\cdot,\cdot)$ is the priority function.

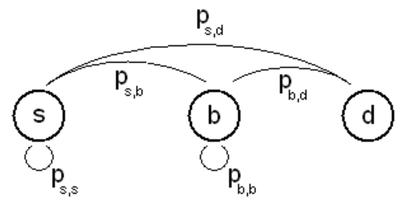

Fig. 3: An example of an overlay graph for the flow (s, d).

$\mathcal{R}_{s,d}$. Therefore, accounting for the statistical independence among the link failure events, we have that:

$$p_{r_i,r_j} = \begin{cases} d_{r_i,r_j} \sum_{k=j+1}^{N+1} (1 - d_{r_i,r_k}) & \forall i \leq j \\ 0 & \forall i > j \end{cases} \tag{4}$$

Basing on the above definition, we construct the matrix $P_{s,d} \in [0,1]^{(N+2)\times(N+2)}$ as:

$$P_{s,d} = \begin{bmatrix} p_{r_0,r_0} & p_{r_0,r_1} & \cdots & p_{r_0,r_{N+1}} \\ 0 & p_{r_1,r_1} & \cdots & p_{r_1,r_{N+1}} \\ & \cdots & \cdots & \\ & \cdots & \cdots & \\ 0 & 0 & \cdots & p_{r_{N+1},r_{N+1}} \end{bmatrix} \tag{5}$$

Given the probabilistic direct graph G and the previous definitions, we are able to introduce our analytical model, namely the overlay graph $G_{s,d}$, as:

$$G_{s,d} = (\mathcal{R}_{s,d}, L_{s,d}, P_{s,d}) \tag{6}$$

In the following, we will adopt (6) to model any opportunistic routing protocol, since it describes the protocol procedure for whatever network topology with a very limited set of parameters, i.e. the delivery ratios and the order relationship $f(\cdot,\cdot)$. As example, Fig.2 shows the physical graph (1) of a simple network, while in Fig.3 the related overlay graph (6) associated with the pair (s, d) is depicted.
We underline that the proposed model allows us to analytically evaluate several performance parameters, as the packet re-transmission probability, the packet dropping probability, the packet average transmission number, the average end-to-end delay, etc. In the following subsection, we utilize our model to estimate the average transmission number and we derive a closed-form expression for such a parameter.

C. Analytical derivation of the average transmission number

In this subsection we first present a simple example to gain insight about the the main idea behind our analytical derivation of the average data-link transmission number, and then we derive the closed-form expression for its computation.
With reference again the topology shown in Fig.1 and with the priority function introduced in Sec. III-B, let us denote with $e_{i,j}$ the event "node j is the node with the highest priority that has received a packet from node i directed toward node d" and with $e_{i,i}$ the event "no node has received the packet sent by i". Clearly, since the event $e_{i,j}$ is related with the amount of progress toward the destination reached by the packet, we refer to it as a *progress event*.
At the first transmission, we have three possible mutually exclusive progress events: $e_{s,d}$, $e_{s,r}$ and $e_{s,s}$, and the related probabilities are, according to (4):

$$\begin{aligned} p_{s,d} &= P(e_{s,d}) = \frac{1}{2} \\ p_{s,r} &= P(e_{s,r}) = \frac{1}{2}\frac{4}{5} \\ p_{s,s} &= P(e_{s,s}) = \frac{1}{2}\frac{1}{5} \end{aligned} \tag{7}$$

If $e_{s,d}$ occurs, the packet has reached the destination and no additional transmissions are required. Otherwise, a second transmission is needed and, if $e_{s,r}$ occurs, the possible events are $e_{r,d}$ and $e_{r,r}$ with probabilities respectively $p_{r,d} = \frac{4}{5}$ and $p_{r,r} = \frac{1}{5}$. Differently if $e_{s,s}$ occurs, the events and the probabilities are the same of the first transmission. Fig. 4 shows all the possible sequences of events, where the number of links from the root to a leaf accounts for the number of transmissions.
By exploring all the branches of the tree in Fig. 4, after simple algebraic manipulations, the average number of transmissions $\overline{n}_{s,d}$ is given by:

$$\overline{n}_{s,d} = \sum_{i=1}^{\infty} (p_{s,s})^{i-1} \left[i\, p_{s,d} + p_{s,r} \sum_{j=1}^{\infty} (i+j) p_{r,d} (p_{r,r})^{j-1} \right] \tag{8}$$

By using the notable relations $\sum_{n=0}^{+\infty} n\, x^n = \frac{x}{(x-1)^2}$ and $\sum_{n=0}^{+\infty} x^n = \frac{1}{1-x}$ if $|x| < 1$ and, after algebraic manipulations, the closed-form expression for the (8) is given by:

$$\overline{n}_{s,d} = \frac{1}{1-p_{s,s}} \left[\frac{p_{s,d}}{1-p_{s,s}} + \frac{p_{s,r} p_{r,d}}{1-p_{r,r}} \left(\frac{1}{1-p_{s,s}} + \frac{1}{1-p_{r,r}} \right) \right] \tag{9}$$

In particular, by substituting in (9) the values of the progress-event probabilities above indicated, we have that $\overline{n}_{s,d} = 1.8381$.

The above discussion is extended to an arbitrary number of available relays, we have the following proposition.

Proposition 1: Let s be the source, d the destination of a packet, and let $\mathcal{R}_{s,d}$ be the ordered set of the allowed relays

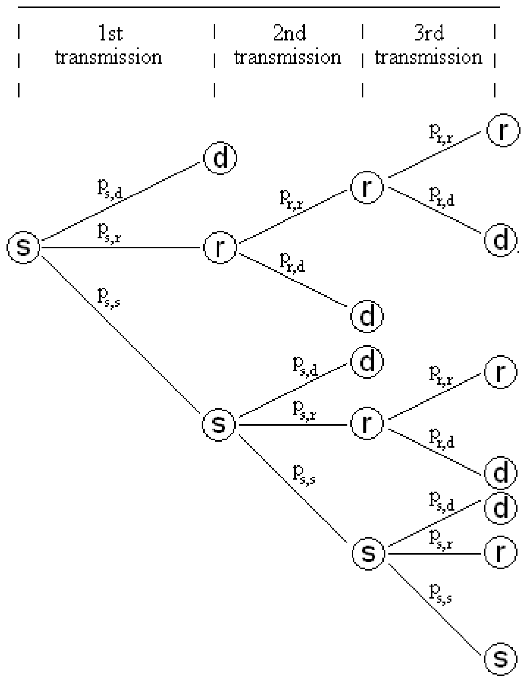

Fig. 4: Opportunistic Routing Tree

for the packet defined in (2). Given the matrix $P_{s,d}$ defined in (5), the average number of data-link transmissions $\overline{n}_{s,d}$ is equal to the equation (10) (reported at the top of the next page).

Proof: See Appendix A. ■

We underline that the closed-form expression of the average transmission number depends only on the progress-event probabilities, given the priority rule. Therefore, the expression is general and it is not restricted to any specific network topology or opportunistic protocol. Of course, equation (10) agrees with (9) when there is a unique available relay ($N = 1$).
In our analysis, we assume a perfect coordination among the nodes to avoid duplicate transmissions, i.e. to avoid that a packet received by a node with higher priority is forwarded also by a node with lower priority. In case of imperfect coordination, our result is a lower bound for the average number of opportunistic transmissions.

D. Recursive closed-form expression for the average transmission number

The closed-form expression for the average number of opportunistic data-link transmissions (10) can be expressed in the following recursive form:

$$\overline{n}_{s,d} = \frac{1}{1-p_{s,s}}\left[1+\sum_{k=1}^{N} p_{s,r_k}\overline{n}_{r_k,d}\right] \tag{11}$$

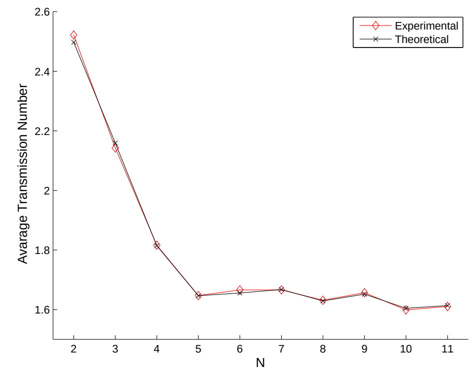

Fig. 5: Average Transmission Number vs. the number of nodes N for a LoS scenario.

where

$$\overline{n}_{r_k,d} = \frac{1}{1-p_{r_k,r_k}}\left[1+\sum_{l=k+1}^{N} p_{r_k,r_l}\overline{n}_{r_l,d}\right] \tag{12}$$

The derivation of (11) is not reported here for lack of space and will be provided in [14]. Such a derivation is based on the evaluation of the average number of transmissions performed by each node which can be expressed in terms of the average number of transmission of the nodes with higher priority. The expression (11) shows how the average data-link transmission number of a source can be expressed in terms of a weighted sum of the average transmission numbers of the neighbor relays with higher priority. It suggests a simple estimation procedure which any node can performe to implement a routing algorithm.

IV. Numerical Results

In this section, we validate our analytical framework with Monte Carlo simulations for both Line of Sight (LOS) and Non Line of Sigh (NLoS) scenarios. More in detail, we compare the theoretical average number of data-link transmissions obtained by means of the proposed closed-form expression (10) with the one obtained by means of numerical simulations.
At this end we compute both the average transmission numbers (the theoretical and the one obtained by simulation) as a function of the number of allowed relays n, randomly placed between the source and the destination. The propagation model is the *Shadowing* one, which accounts for the long-term fading effects by means of a zero-mean Gaussian random variable $X(0, \sigma)$ measured in dB. According to it, the received mean power $P_{dB}(d)$ at distance d is:

$$P_{dB}(d) = P_{dB}(d_0) - 10\beta\log\left(\frac{d}{d_0}\right) + X_{dB}, \tag{13}$$

where $P_{dB}(d_0)$ is the received mean power,β is the path loss exponent and σ is the shadow deviation. We set $\beta = 3.8$, $\sigma = 8$dB, the transmitted power $p_{tx} = -37$dBW and the receiver sensibility threshold $p_{th} = -126$dBW. Such

$$
\begin{aligned}
\overline{n}_{s,d} = &\frac{1}{1-p_{s,s}}\Big\{\frac{p_{s,d}}{1-p_{s,s}} + \sum_{k=1}^{N}\frac{p_{s,r_k}p_{r_k,d}}{1-p_{r_k,r_k}}\left[\frac{1}{1-p_{s,s}}+\frac{1}{1-p_{r_k,r_k}}\right]+\\
&\sum_{k_1=1}^{N-1}\sum_{k_2=k_1+1}^{N}\frac{p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},d}}{(1-p_{r_{k_1},r_{k_1}})(1-p_{r_{k_2},r_{k_2}})}\left[\frac{1}{1-p_{s,s}}+\frac{1}{1-p_{r_{k_1},r_{k_1}}}+\frac{1}{1-p_{r_{k_2},r_{k_2}}}\right]+\\
&\sum_{k_1=1}^{N-2}\sum_{k_2=k_1+1}^{N-1}\sum_{k_3=k_2+1}^{N}\frac{p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}p_{r_{k_3},d}}{(1-p_{r_{k_1},r_{k_1}})(1-p_{r_{k_2},r_{k_2}})(1-p_{r_{k_3},r_{k_3}})}\left[\frac{1}{1-p_{s,s}}+\frac{1}{1-p_{r_{k_1},r_{k_1}}}+\frac{1}{1-p_{r_{k_2},r_{k_2}}}+\frac{1}{1-p_{r_{k_3},r_{k_3}}}\right]+\\
&\ldots+\sum_{k_1=1}^{N-m+1}\sum_{k_2=k_1+1}^{N-m+2}\cdots\sum_{k_m=k_{m-1}+1}^{N}\frac{p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}\cdots p_{r_{k_{m-1}},r_{k_m}}p_{r_{k_m},d}}{\prod_{j=1}^{m}(1-p_{r_{k_j},r_{k_j}})}\left[\frac{1}{1-p_{s,s}}+\sum_{j=1}^{m}\frac{1}{1-p_{r_{k_j},r_{k_j}}}\right]+\ldots+\\
&\frac{p_{s,r_1}p_{r_1,r_2}p_{r_2,r_3}\cdots p_{r_{N-1},r_N}p_{r_N,d}}{\prod_{j=1}^{N}(1-p_{r_j,r_j})}\left[\frac{1}{1-p_{s,s}}+\sum_{j=1}^{N}\frac{1}{1-p_{r_j,r_j}}\right]\Big\} \qquad (10)
\end{aligned}
$$

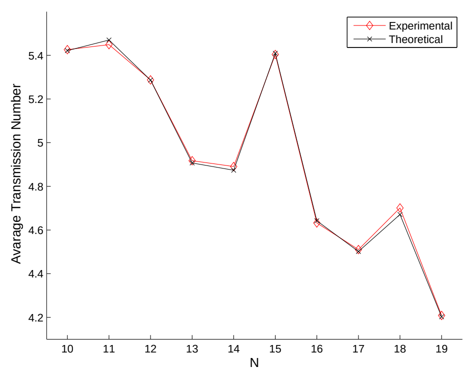

Fig. 6: Average Transmission Number vs. the number of nodes N for a NLoS scenario.

parameters allow us to simulate an IEEE 802.11b network interface with a transmission range of roughly 250 meters. Therefore, in the case of LoS scenario, the distance between s and d is chosen equal to 250 m, whereas in the NLOS one the distance is chosen equal to 1000 m.

The routing protocol is a modified version of ExOR protocol [2], in which the relays are prioritized according to the ETX metric and in which there is a perfect coordination among nodes, i.e. no duplicate transmissions occur.

In order to compute the closed-form expression (10), we estimate the progress-event probabilities with 10^4 independent Monte Carlo runs. The same number of runs is used to estimate the simulation-based transmission number with a purpose-made network simulator.

Fig. 5 presents the mentioned comparison as function of the number of allowed relays N, included both the source and the destination, for the LoS case, whereas the Fig. 6 shows the comparison for the NLoS case. In both the cases, there is a very good agreement between the theoretical and the experimental values, confirming so the accuracy of proposed closed-form expression (10). We note that the irregular behavior of the curves is due to the random node placement.

V. Conclusion

The paper proposes an analytical model to describe any routing procedures operating according to the opportunistic paradigm. It applies in a very general multi-hop scenario.The model requires the knowledge of both the delivery ratios and node priority, which is based on the adopted routing metric. To show the effectiveness of the model we utilize it to derive a closed-form expression for the average transmission number. Numerical simulation results confirm the goodness of such closed-form expression.

Appendix

A. *Proof of Proposition 1*

To obtain the closed-form expression (10), we first express the average transmission number by exploring all the branches of the tree obtained by generalizing the one shown in Fig. 4 to the case with N relays. In such a way, we obtain the expression (14) reported at the top of next page.

It is easy to recognize that the equation (14) is the sum of $N+1$ terms $\{c_{s,d}(i,N)\}_{i=0}^{N}$, where the first one

$$c_{s,d}(0,N) = p_{s,d}\sum_{i=1}^{+\infty} i(p_{s,s})^{i-1} \qquad (15)$$

represents the contribution to $\overline{n}_{s,d}$ due to the direct transmissions between s and d, the second one

$$c_{s,d}(1,N) = \sum_{k=1}^{N} p_{s,r_k}p_{r_k,d}\sum_{i=1}^{+\infty}(p_{s,s})^{i-1}\sum_{j=1}^{+\infty}(i+j)(p_{r_k,r_k})^{j-1} \qquad (16)$$

is the contribution due to the use of only one relay in forwarding the packet, and the $c_{s,d}(m,N)$ is the contribution due to the use of m relays (17) reported in the next page.

$$\overline{n}_{s,d} = \sum_{i=1}^{+\infty}(p_{s,s})^{i-1}\{i\,p_{s,d} + \sum_{k=1}^{N} p_{s,r_k}p_{r_k,d}\sum_{j=1}^{+\infty}(i+j)(p_{r_k,r_k})^{j-1} +$$
$$\sum_{k_1=1}^{N-1}\sum_{k_2=k_1+1}^{N} p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},d}\sum_{j_1=1}^{+\infty}\sum_{j_2=1}^{+\infty}(i+j_1+j_2)(p_{r_{k_1},r_{k_1}})^{j_1-1}(p_{r_{k_2},r_{k_2}})^{j_2-1}+$$
$$\sum_{k_1=1}^{N-2}\sum_{k_2=k_1+1}^{N-1}\sum_{k_3=k_2+1}^{N} p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}p_{r_{k_3},d}\sum_{j_1=1}^{+\infty}\sum_{j_2=1}^{+\infty}\sum_{j_3=1}^{+\infty}(i+j_1+j_2+j_3)(p_{r_{k_1},r_{k_1}})^{j_1-1}(p_{r_{k_2},r_{k_2}})^{j_2-1}(p_{r_{k_1},r_{k_3}})^{j_3-1}+$$
$$\ldots + \sum_{k_1=1}^{N-m+1}\sum_{k_2=k_1+1}^{N-m+2}\cdots\sum_{k_m=k_{m-1}+1}^{N} p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}\ldots p_{r_{k_{m-1}},r_{k_m}}p_{r_{k_m},d}$$
$$\sum_{j_1=1}^{+\infty}\sum_{j_2=1}^{+\infty}\sum_{j_3=1}^{+\infty}\cdots\sum_{j_m=1}^{+\infty}(i+j_1+j_2+j_3+\ldots+j_m)(p_{r_{k_1},r_{k_1}})^{j_1-1}(p_{r_{k_2},r_{k_2}})^{j_2-1}(p_{r_{k_3},r_{k_3}})^{j_3-1}\ldots(p_{r_{k_m},r_{k_m}})^{j_m-1}+\ldots+$$
$$p_{s,r_1}p_{r_1,r_2}p_{r_2,r_3}\ldots p_{r_{N-1},r_N}p_{r_N,d}\sum_{j_1=1}^{+\infty}\sum_{j_2=1}^{+\infty}\sum_{j_3=1}^{+\infty}\cdots\sum_{j_N=1}^{+\infty}(i+\sum_{q=1}^{N}j_q)\prod_{q=1}^{N}(p_{r_q,r_q})^{j_q-1}\} \tag{14}$$

$$c_{s,d}(m,N) = \sum_{k_1=1}^{N-m+1}\sum_{k_2=k_1+1}^{N-m+2}\cdots\sum_{k_m=k_{m-1}+1}^{N} p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}\ldots p_{r_{k_{m-1}},r_{k_m}}p_{r_{k_m},d}$$
$$\sum_{i=1}^{+\infty}(p_{s,s})^{i-1}\sum_{j_1=1}^{+\infty}\sum_{j_2=1}^{+\infty}\sum_{j_3=1}^{+\infty}\cdots\sum_{j_m=1}^{+\infty}(i+j_1+j_2+j_3+\ldots+j_m)(p_{r_{k_1},r_{k_1}})^{j_1-1}(p_{r_{k_2},r_{k_2}})^{j_2-1}(p_{r_{k_3},r_{k_3}})^{j_3-1}\ldots(p_{r_{k_m},r_{k_m}})^{j_m-1} \tag{17}$$

Therefore, the equation (14) can be rewrite as

$$\overline{n}_{s,d} = \sum_{m=0}^{N} c_{s,d}(m,N) \tag{18}$$

After some algebraic manipulation and using the notable relations $\sum_{n=0}^{+\infty} n\,x^n = \frac{x}{(x-1)^2}$ e $\sum_{n=0}^{+\infty} x^n = \frac{1}{1-x}$ if $|x|<1$, it is possible to obtain a closed form for $c_{s,d}(m,N)$ as in the following:

$$c_{s,d}(m,N) = \frac{1}{1-p_{s,s}}\sum_{k_1=1}^{N-m+1}\sum_{k_2=k_1+1}^{N-m+2}\cdots\sum_{k_m=k_{m-1}+1}^{N}$$
$$\frac{p_{s,r_{k_1}}p_{r_{k_1},r_{k_2}}p_{r_{k_2},r_{k_3}}\ldots p_{r_{k_{m-1}},r_{k_m}}p_{r_{k_m},d}}{\prod_{j=1}^{m}(1-p_{r_{k_j},r_{k_j}})}$$
$$\left[\frac{1}{1-p_{s,s}}+\sum_{j=1}^{m}\frac{1}{1-p_{r_{k_j},r_{k_j}}}\right]. \tag{19}$$

Accounting for $c_{s,d}(0,N) = \frac{p_{s,d}}{(1-p_{s,s})^2}$ and by substituting (19) in (14), we obtain (10). It is worthwhile to note that the condition $P_{r_{k_j},r_{k_j}} \neq 1$ avoids that a packet is noosed by the node r_{k_j}.

REFERENCES

[1] E. M. Royer and C.-K. Toh, "A review of current routing protocols for ad hoc mobile wireless networks," *IEEE Personal Communications*, vol. 6, pp. 46–55, 1999.

[2] S. Biswas and R. Morris, "Exor: opportunistic multi-hop routing for wireless networks," *SIGCOMM Comput. Commun. Rev.*, vol. 35, no. 4, pp. 133–144, 2005.

[3] E. Rozner, J. Seshadri, Y. Mehta, and L. Qiu, "Simple opportunistic routing protocol for wireless mesh networks," in *Wireless Mesh Networks, 2006. WiMesh 2006. 2nd IEEE Workshop on*, Sept. 2006, pp. 48–54.

[4] S. Katti and D. Katabi, "Mixit: The network meets the wireless channel," in *HotNets-VI: Proc. of the Sixth ACM Workshop on Hot Topics in Networks*, 2006.

[5] S. Chachulski, M. Jennings, S. Katti, and D. Katabi, "Trading structure for randomness in wireless opportunistic routing," *SIGCOMM Comput. Commun. Rev.*, vol. 37, no. 4, pp. 169–180, 2007.

[6] A. Zubow, M. Kurth, and J.-P. Redlich, "Multi-channel opportunistic routing," in *European Wireless*, 2007.

[7] M. Caleffi and L. Paura, "Opportunistic routing for disruption tolerant networks," in *AINA '09: the IEEE 23rd International Conference on Advanced Information Networking and Applications*, May 2009.

[8] C.-P. Luk, W.-C. Lau, and O.-C. Yue, "An analysis of opportunistic routing in wireless mesh network," in *Communications, 2008. ICC '08. IEEE International Conference on*, May 2008, pp. 2877–2883.

[9] Z. Zhong and S. Nelakuditi, "On the efficacy of opportunistic routing," in *Sensor, Mesh and Ad Hoc Communications and Networks, 2007. SECON '07. 4th Annual IEEE Communications Society Conference on*, June 2007, pp. 441–450.

[10] A. Zubow, M. Kurth, and J.-P. Redlich, "Considerations on forwarder selection for opportunistic protocols in wireless networks," in *Wireless Conference, 2008. EW 2008. 14th European*, June 2008, pp. 1–7.

[11] J. Wu, M. Lu, and F. Li, "Utility-based opportunistic routing in multi-hop wireless networks," in *Distributed Computing Systems, 2008. ICDCS '08. The 28th International Conference on*, June 2008, pp. 470–477.

[12] F. Bai, N. Sadagopan, B. Krishnamachari, and A. Helmy, "Modeling path duration distributions in manets and their impact on reactive routing protocols," *Selected Areas in Communications, IEEE Journal on*, vol. 22, no. 7, pp. 1357–1373, September 2004.

[13] M. Caleffi, G. Ferraiuolo, and L. Paura, “A reliability-based framework for multi-path routing analysis in mobile ad-hoc networks,” *International Journal of Communication Networks and Distributed Systems*, vol. 1, no. 4-5-6, pp. 507–523, 2008.
[14] A. S. Cacciapuoti, M. Caleffi, and L. Paura, “On the misbehavior of routing metrics for opportunistic routing,” in *To be submitted*.