Optimal Strategy Design for Enabling the Coexistence of Heterogeneous Networks in TV White Space

Angela Sara Cacciapuoti, *Member, IEEE*, Marcello Caleffi, *Member, IEEE*, and Luigi Paura, *Member, IEEE*

***Abstract*—Very recently, regulatory bodies worldwide have approved dynamic access of unlicensed networks to the TV white space (TVWS) spectrum. Hence, in the near future, multiple heterogeneous and independently operated unlicensed networks will coexist within the same geographical area over shared TVWS. Although heterogeneity and coexistence are not unique to TVWS scenarios, their distinctive characteristics pose new and challenging issues. In this paper, the problem of the coexistence interference among multiple heterogeneous and independently operated secondary networks (SNs) in the absence of secondary cooperation is addressed. Specifically, the optimal coexistence strategy, which adaptively and autonomously selects the channel maximizing the expected throughput in the presence of coexistence interference, is designed. More in detail, at first, an analytical framework is developed to model the channel selection process for an arbitrary SN as a decision process. Then, the problem of the optimal channel selection, i.e., the channel maximizing the expected throughput, is proved to be computationally prohibitive (NP-hard). Finally, under the reasonable assumption of identically distributed interference on the available channels, the optimal channel selection problem is proved not to be NP-hard, and a computationally efficient (polynomial-time) algorithm for finding the optimal strategy is designed. Numerical simulations validate the theoretical analysis.**

***Index Terms*—Coexistence, cognitive radio (CR), interference, optimality, spectrum sharing, throughput, TV white space (TVWS).**

I. Introduction

VERY recently, regulatory bodies worldwide have approved the dynamic access of secondary networks[1] (SNs) to the TV white space (TVWS) spectrum. The existing rulings [1]–[3] obviate spectrum sensing as the mechanism for the SNs to determine the TVWS availability at their respective locations. Instead, they require the SNs to periodically access a geolocated database [4] for acquiring the list of TVWS channels free from incumbents.

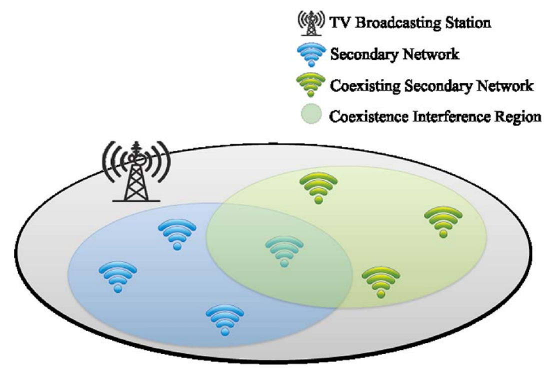

Fig. 1. Example of TVWS scenario. Two SNs coexist with a TV broadcast station within the same geographical area.

Clearly, the introduction of a database for incumbent protection significantly simplifies the secondary access to the TVWS spectrum, and the research community is actively working on defining several new standards aiming at enabling TVWS communications, such as IEEE 802.22 [5], IEEE 802.11af [6], IEEE 802.15m [7], ECMA 392 [8], and IEEE SCC41 [9]. Hence, in the near future, multiple heterogeneous and independently operated SNs will coexist within the same geographical area, as shown in Fig. 1.

Although heterogeneity and coexistence are not unique to TVWS scenarios, the distinctive characteristics of the TVWS scenarios pose new and challenging issues [10], [11]. At first, the excellent propagation characteristics of the TVWS spectrum cause severe interference among the coexisting SNs sharing the same spectrum band. In addition, the heterogeneity among the TVWS standards can prevent the adoption of interference-avoidance schemes based on cooperation. Finally, experimental studies have shown that the TVWS spectrum is significantly scarce in densely populated areas [12], [13]. As a consequence, it is likely to expect several SNs sharing the same spectrum band.

Hence, the research community is focusing on designing solutions for the secondary coexistence in TVWS [14], [15], as we shall discuss in detail in Section II-C.

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A. S. Cacciapuoti and M. Caleffi are with the Department of Electrical Engineering and Information Technologies, University of Naples Federico II, Naples 80138, Italy (e-mail: angelasara.cacciapuoti@unina.it; marcello.caleffi@unina.it).

L. Paura is with the Department of Electrical Engineering and Information Technologies, University of Naples Federico II, Naples 80138, Italy, and also with CeRICT, Naples 80125, Italy (e-mail: paura@unina.it).

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[1]In the following, we refer to the unlicensed networks aiming at opportunistically exploiting the TVWS spectrum when it is not used by the licensed users as *secondary networks*.

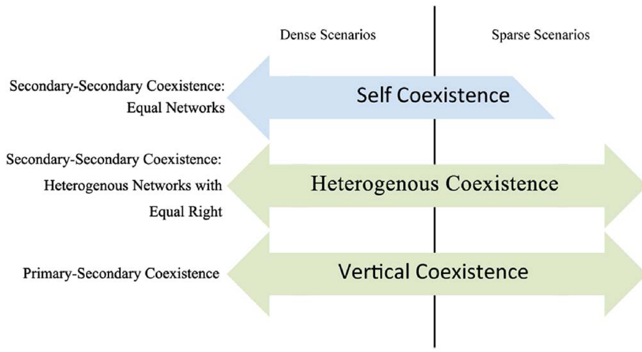

Fig. 2. Coexistence taxonomy in TVWS scenarios.

In this paper, we investigate the problem of interference avoidance among multiple heterogeneous and independently operated SNs coexisting within the same geographical area in the absence of secondary cooperation.

Specifically, we design the *optimal coexistence strategy*, which *adaptively* and *autonomously* selects the TVWS channel allowing the SN to maximize the expected throughput in the presence of coexistence interference.

More in detail, at first, we develop an analytical framework to model the TVWS channel selection process for an arbitrary SN as a decision process, where the reward models the data rate achievable on a channel, and the cost models the communication overhead for assessing the coexistence interference. Then, we prove that the problem of optimal channel selection, i.e., the channel maximizing the expected throughput, is computationally prohibitive (NP-hard). Finally, we prove that, under the reasonable assumption of identically distributed interference on the available TVWS channels, the problem of optimal channel selection is not anymore NP-hard. Specifically, we prove that the optimal strategy exhibits a threshold behavior, and we exploit this threshold structure to design a computationally efficient (polynomial-time) algorithm for finding the optimal strategy.

The rest of this paper is organized as follows. In Section II, we present the problem statement, and we highlight the contributions of this paper. In Section III, we describe the network model along with some preliminaries. In Section IV, we design the optimal strategy. In Section V, we validate the theoretical framework through a case study. In Section VI, we conclude this paper, and finally, some proofs are gathered in the Appendix.

II. Problem Statement

Let us consider the typical TVWS scenario shown in Fig. 1, in which an SN coexists with multiple heterogenous and independently operated SNs within the same geographical area by sharing a certain number of TVWS channels declared available from the TVWS database.

As shown in Fig. 2, there exist three different classes of coexistence interference: 1) the *self-coexistence interference*, which is caused by SNs operating according to the same standard and experienced mainly in dense scenarios; 2) the *heterogenous coexistence interference*, which is caused by SNs operating according to dissimilar standards or technologies; and 3) the *vertical coexistence interference*, which is caused by the incumbents.

While the self-coexistence interference is handled within the TVWS standards [5] and the vertical coexistence interference is handled by a centralized database as mentioned in Section I, the mitigation of heterogenous coexistence interference represents an open problem, as pointed out in Section II-A.

A. Challenges

The design of a strategy for the coexistence of heterogeneous and independently operated SNs poses several challenges.

- *Dynamic Interference*. In TVWS scenarios, the heterogenous coexistence interference is both time and spatial variant. In fact, as recently proved in [15], such dynamics depend on several factors, such as the number of SNs roaming within a certain geographical area, the number of secondary users (SUs) belonging to each SN, the interference range and the traffic/mobility patterns of each SN, as well as the SN interference ranges and the changes in wireless propagation conditions. Hence, any coexistence strategy should be adaptive to such dynamics.
- *Heterogeneity*. As mentioned in Section I, several TVWS standards have been proposed in the last years. Although significant work is currently ongoing [16], complete interoperability based on over-the-air communications among heterogeneous TVWS standards is still missing [10]. Hence, an appealing characteristic of any heterogenous coexistence strategy is to be *autonomous*, i.e., to be independent of any form of coordination with the coexisting SNs.
- *Harmless-to-Incumbents Interference*. In classical cognitive radio (CR) scenarios, any SN is required to adopt the *sense-before-talk* strategy as a mechanism to protect the incumbents from harmful interference. In TVWS scenarios, such a requirement does not hold necessarily. Specifically, since the vertical coexistence interference is managed through a database-based mechanism, any interference level on a channel granted by the database is harmless against the incumbents. As a consequence, an SN must handle only the *heterogenous coexistence interference* caused by the peer coexisting SNs. Hence, any strategy aiming at mitigating the coexistence interference is *discretionary*, i.e., it should be performed only when it is convenient. As an example, let us consider Fig. 3, in which only one TVWS channel is available for secondary communications. As we will prove in Section IV-B with Corollary 1, an SN aiming at maximizing the expected throughput should use the channel independently of any interference level. Hence, any coexistence strategy should allow *discretionary* interference-avoidance schemes.

B. Optimal Autonomous Coexistent Strategy Design

By taking into account the aforementioned challenges, in this paper, we design a coexistence strategy for TVWS scenarios exhibiting the following attractive features.

1) The strategy is *optimal*, i.e., it allows the SN to maximize the expected throughput achievable by the SUs.

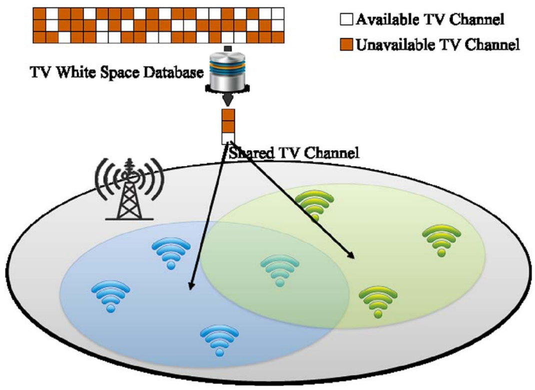

Fig. 3. Harmless-to-incumbents interference. The SNs do not need a *sense-before-talk* strategy as a mechanism to protect the incumbents from harmful interference.

2) The strategy is *feasible*, since it requires a reasonable amount of *a priori* knowledge, i.e., the first-order distribution of the interference levels.
3) The strategy is iterative and *adaptive* to interference dynamics.
4) The strategy is *autonomous*, since it allows the SN to make its decisions independently of any other coexisting network. Hence, it is low in complexity, and it can be easily integrated with centralized or distributed mechanisms to build hybrid strategies.
5) The strategy implements a *discretionary* interference-avoidance scheme, i.e., it accounts for the harmless-to-incumbents property of interference in TVWS.

More in detail, we model the problem of choosing the TVWS channel maximizing the expected throughput as a decision process [17], where the reward models the data rate provided by a channel, and the cost models the communication overhead (i.e., sensing times) for assessing the interference caused by coexisting SNs.

Then, we prove that the problem of optimal channel selection is NP-hard by reducing it to the widely known NP-hard traveling salesman problem (TSP) [18]. This is an important result since it follows that 1) the optimal strategy can be found only through exhaustive search, i.e., there is no *smart* (computationally efficient) way of searching for the optimal solution, and 2) the wide literature on exact and approximate algorithms for the TSP can be efficiently adopted in searching for the optimal solution.

Furthermore, we prove that, under the reasonable hypothesis of identically distributed interference, the problem of optimal channel selection is not anymore NP-hard. Specifically, we prove that the optimal strategy exhibits a threshold behavior. This result is valuable, since it allows us to design a computationally efficient algorithm for searching the optimal strategy by exploiting the threshold structure.

C. Related Work

Over the past ten years, the primary–secondary coexistence problem has been extensively studied in classical CR networks, and several solutions for mitigating the vertical coexistence interference have been proposed [19]–[26]. However, as pointed out in Section II-A, the existing results cannot be applied in TVWS scenarios, since they are based on the assumption that the *sense-before-talk* mechanism is mandatory. Hence, a more general model that accounts for the unique TVWS characteristics is required, and we address this issue in Sections III and IV.

Very recently, the problem of secondary–secondary coexistence has been gaining attention [14], and in [15], the intrinsic relationship between environmental and system parameters in affecting the secondary coexistence has been disclosed. Several TVWS standards, such as IEEE 802.22 [5], IEEE 802.11af [6], and ECMA 392 [8], define self-coexistence mechanisms to mitigate the mutual interference among similar networks. Nevertheless, none of these mechanisms can be applied to mitigate the interference among heterogenous networks.

Finally, few works address the heterogenous coexistence problem. Some works address the coexistence among low-power versus high-power [27] or contention-based versus reserved-based [28] networks. However, the proposed strategies are targeted to couples of specific technologies and, hence, are not suitable for heterogeneous scenarios such as the TVWS. Differently, in this paper, we design a general strategy allowing an arbitrary SN to make its decisions independently of the coexisting technologies. IEEE 802.16m [29] defines uncoordinated mechanisms for heterogenous coexistence. However, the standard focuses on the license-exempt spectrum, and the proposed strategy simply aims at selecting a channel with tolerable interference. Differently, in this paper, we focus on the licensed spectrum, and we propose an optimal strategy, i.e., a strategy maximizing the expected throughput achievable by the SN. IEEE 802.19.1. [16] aims at providing general solutions to the heterogenous coexistence by envisioning a coexistence manager, acting as a centralized resource allocator, and a coexistence enabler, aiming at maintaining interfaces between the coexistence enabler and coexisting CR networks. However, the proposed strategies either focus on selecting nonoverlapping channels or require a certain degree of collaboration among the coexisting networks. Differently, in this paper, we design an autonomous strategy allowing the SN to make its decisions independently of any other coexisting network.

III. Model and Preliminaries

Here, we first describe the system model in Section III-A. Then, in Section III-B, we collect several definitions that will be used throughout this paper.

A. System Model

We consider an SN operating within the TVWS spectrum according to the existing regulations and standards. Time is organized into fixed-size slots of duration T, and by accessing the TVWS database, the SN obtains the list of channels free from primary incumbents within an arbitrary time slot. In the following, we denote the set of incumbent-free channels in a given time slot with $\Omega = \{1, 2, \ldots, M\}$.

Since multiple SNs are allowed to operate within the same geographical region, a channel $m \in \Omega$ that is free from primary incumbents can be affected by coexistence interference caused by other heterogenous SNs coexisting within the same geographical area. Hence, an SU aiming at maximizing the available data rate assesses the strength of such an interference[2] by sensing the mth channel for a certain amount of time, e.g., τ_s. Depending on the measured strength, the SU can transmit over the mth channel with a certain data rate, whose value belongs to an ordered set of K discrete rates[3] $\{\tilde{r}_0, \tilde{r}_1, \ldots, \tilde{r}_K\}$, with $\tilde{r}_k$ increasing with k and $\tilde{r}_0 \triangleq 0$ denoting a channel sensed as unusable due to excessive interference. By denoting with R_m the random variable (r.v.) characterizing the data rate achievable on the mth channel during an arbitrary time slot,[4] $p_{m,k} = P(R_m = \tilde{r}_k)$ represents the probability of the data rate achievable on channel m at time slot n being $\tilde{r}_k$. The SUs can estimate the first-order distribution of the achievable data rates through the past-channel throughput history.

After assessing[5] the admissible data rate on channel m, e.g., rate r_m, the SU decides on whether to use or not to use channel m by comparing r_m with a certain threshold, e.g., y_m. Whenever $r_m \geq y_m$, the SU transmits over channel m for the remaining of the time slot, whereas whenever $r_m < y_m$, the SU skips channel m to sense another channel looking for better communication opportunities. Clearly, the SU can decide to use channel m without assessing the coexistence interference by setting the threshold as[6] $y_m = 0$.

Both the sequence of channels to be sensed and the corresponding rate thresholds, which are referred to in the following as *stopping rules*, deeply affect the performance of any SN. Hence, the goal can be summarized as *to find the channel providing the highest data rate as quickly as possible*. In the following section, we rigorously formulate the problem.

B. Problem Formulation

Here, we formulate the problem of choosing a coexistence strategy maximizing the expected data rate achievable by the SU during an arbitrary time slot n. Without loss of generality, in the following, we omit the time dependence to simplify the adopted notation, as shown in Table I.

TABLE I
ADOPTED NOTATION

Symbol	Definition
M	number of available channels
T	duration of a time slot
$\{\tilde{r}_k\}_{k=0}^{K}$	set of admissible data rates
$p_{m,k}$	probability of $\tilde{r}_k$ being the data rate achievable on channel m
x_m	m-th channel to be sensed
$\tilde{r}_{y_m}$	data rate threshold for channel x_m
$V_{\mathbf{x},\mathbf{y}}$	expected average throughput
$\mathbf{x}^*, \mathbf{y}^*$	sensing sequence and stopping rule maximizing $V_{\mathbf{x},\mathbf{y}}$
$p_{\mathbf{x},\mathbf{y}}(m)$	probability of using the m-th sensed channel
$\bar{r}_{\mathbf{x},\mathbf{y}}(m)$	expected data rate through channel x_m
$c_{\mathbf{y}}(m)$	portion of the time slot devoted to packet transmission by using channel x_m

Definition 1 (Sensing Sequence): The *sensing sequence* $\mathbf{x}$ is the ordered sequence of channels to be sensed, i.e.,

$$\mathbf{x} = (x_1, x_2, \ldots, x_M) \tag{1}$$

with $x_m \in \Omega$ for any m and $x_m \neq x_l$ for any $m \neq l$. In the following, we denote with $\mathbf{X}$ the set of all possible sensing sequences, and by recognizing that a sensing sequence is a *permutation without repetition* over the set Ω, we get that $|\mathbf{X}| = M!$.

Definition 2 (Stopping Rule): The *stopping rule* $\mathbf{y}$ is the ordered sequence of data rate threshold indexes, i.e.,

$$\mathbf{y} = (y_1, y_2, \ldots, y_M), \quad y_m = 0, \ldots, K \tag{2}$$

where $\tilde{r}_{y_m} \in \{\tilde{r}_0, \ldots, \tilde{r}_K\}$ denotes the channel reward threshold for the mth sensed channel $x_m \in \Omega$. In the following, we denote with $\mathbf{Y}$ the set of stopping rules, and by recognizing that a stopping rule is a *permutation with repetition* over the set of admissible discrete rates indexes, it results $|\mathbf{Y}| = (K+1)^M$.

Remark: A stopping rule $\mathbf{y}$ is an M-tuple of integers in $[0, K]$, with the mth integer y_m denoting the minimum data rate, i.e., the data rate threshold, required to use the mth sensed channel x_m. As an example, by assuming $\mathbf{y} = (2, 1, 4)$ with $M = 3$, it results that 1) $\tilde{r}_2$ is the threshold for the first sensed channel x_1, 2) $\tilde{r}_1$ is the threshold for the second sensed channel x_2, and 3) $\tilde{r}_4$ is the threshold for the third sensed channel x_3. Hence, the first sensed channel x_1 will be used if and only if it admits a data rate that is equal to or greater than $\tilde{r}_2$. Clearly, channel x_2 will be sensed if and only if the first sensed channel admits a data rate that is lower than $\tilde{r}_2$, and it will be used if and only if it admits a data rate that is equal to or greater than $\tilde{r}_1$.

Remark: At the beginning of an arbitrary time slot, the SU can either 1) transmit over channel x_1, regardless of the coexistence interference, or 2) sense channel x_1 and, based on the sensed interference, decide on whether to use or not to use such a channel. According to the adopted notation, the former case is denoted with $y_1 = 0$, whereas the latter is denoted with $y_1 = i$ with $i \neq 0$. If the SU decides to skip the first sensed channel, then it can either transmit or sense channel x_2, based on the value of y_2. By iterating the aforementioned process, the

[2]We note that, although the transmitting SU should be concerned with the interference levels present at the receiver side, it is likely that both the transmitter and the receiver suffer similar interference, due to the excellent propagation conditions of TVWS [30]. In any case, the conducted analysis continues to hold since the interference can be estimated at the receiver based on the previous transmissions. In fact, since we develop a probabilistic model aiming at maximizing the expected reward, the instantaneous values of the interference are not of interest, and the average interference levels can be easily estimated based on the past-channel throughput interference history [31], [32].

[3]The data rate set depends on the communication standard of the arbitrary SN. As an example, IEEE 802.11af defines 11 different data rates for 6-MHz-wide TVWS channels, as detailed in Section V-A.

[4]It is well known [33] that 1) the achievable data rate over a channel is deeply affected by the signal-to-interference-plus-noise ratio (SINR) and 2) that the SINR range can be partitioned in different regions, each one associated with a certain data rate.

[5]The proposed framework can be easily extended to the case of imperfect sensing by setting the kth data rate for the mth channel to $p_{m,0}(1 - p_{fa})\tilde{r}_k$ as in [34], with p_{fa} denoting the false-alarm probability of the adopted sensing mechanism.

[6]Through the null threshold concept, we are able to design a *discretionary* coexistence strategy, as detailed in Section II-A.

SU eventually selects one of the M incumbent-free channels to be used for secondary communications.

Remark: To simplify the notation, we assume in Definition 2 that the thresholds belong to the set of admissible data rates $\{\tilde{r}_0, \dots, \tilde{r}_M\}$. It is easy to recognize that such an assumption is not restrictive by noting that for any threshold value $r \in (\tilde{r}_{m-1}, \tilde{r}_m]$, the SU uses channel x_m if $R_{x_m} \geq \tilde{r}_m$, whereas it skips such a channel if $R_{x_m} < \tilde{r}_m$. Hence, any $r \in (\tilde{r}_{m-1}, \tilde{r}_m]$ can be replaced by r_m without loss of generality.

Definition 3 (Expected Reward): The *expected reward* $V_{\mathbf{x},\mathbf{y}}$ denotes the expected throughput achievable by the SU in a given time slot by following the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$.

Remark: The *expected reward* represents a tradeoff between 1) the data rate achievable on the selected channel and 2) the time spent for selecting the channel. In general, longer search times assure higher data rates at the price of a shorter portion of time slot T devoted to packet transmission.

Problem 1 (Optimal Coexistence Strategy): The goal is to jointly choose the optimal sensing sequence $\mathbf{x}^*$ and the optimal stopping rule $\mathbf{y}^*$ maximizing the expected reward, i.e.,

$$V_{\mathbf{x}^*,\mathbf{y}^*} = \arg\max_{\mathbf{x}\in\mathbf{X},\mathbf{y}\in\mathbf{Y}} \{V_{\mathbf{x},\mathbf{y}}\}. \tag{3}$$

Insight 1: We note that jointly finding the *optimal sensing sequence* and the *optimal stopping rule* through brute-force searching is computationally unfeasible. In fact, for each of the $M!$ sensing sequences, $(K+1)^M$ stopping rules need to be evaluated.

Remark: Through the general notion of reward, we abstract the derived results from the particulars, making the conducted analysis general. In fact, it can be easily applied to a variety of real-world scenarios, by choosing the proper reward measure. Within the manuscript, we adopted as a performance metric the data rate; hence, the reward $\tilde{r}_{y_m}$ models the data rate achievable on channel y_m. Clearly, depending on the scenario, a different performance metric can be more suitable [35], [36], e.g., channel reliability. In such a case, by simply modeling with the reward $\tilde{r}_{y_m}$ the reliability of channel y_m, all the results derived within this paper continue to hold.

IV. Optimal Coexistence Interference-Avoidance Strategy

At first, in Section IV-A, we derive in Theorem 1 the closed-form expression of the *expected reward*. Then, in Section IV-B, we efficiently (polynomial-time complexity) compute the stopping rule maximizing the expected reward for a given sensing sequence with Algorithm 1. Stemming from this, we first prove that the problem of computing the optimal sensing sequence is NP-hard in Theorem 3, and then, we compute both the optimal sensing sequence and the optimal stopping rule with Algorithm 2. Finally, in Section IV-C, we efficiently (polynomial-time complexity) compute both the optimal sensing sequence and the optimal stopping rule with Algorithm 3 under the reasonable hypothesis of identically distributed coexistence interference levels.

Algorithm 1 Optimal Stopping Rule Given Sensing Sequence

// input: $\mathbf{x} = \{x_1, \dots, x_M\}$
// output: $v, \breve{\mathbf{y}} = \{\breve{y}_1, \dots, \breve{y}_M\}$
// base step
$\breve{y}_M = 0$
$v = (1 - (M-1)\tau_s/T) \sum_{k=1}^{K} p_{x_M,k} r_k$
// recursive step
for $m = M-1 : 1$ **do**
  $\tilde{y}_m = \min_k \{\tilde{r}_k (1 - m\tau_s/T) \geq v$
  $t_0 = (1 - (m-1)\tau_s/T) \sum_{k=1}^{K} p_{x_m,k} r_k$
  $t_1 = (1 - m\tau_s/T) \sum_{k=\tilde{y}_m}^{K} p_{x_m,k} r_k + \sum_{k=0}^{\tilde{y}_m - 1} v$
  if $t_0 > t_1$ **then**
    $\breve{y}_m = 0, v = t_0$
  else
    $\breve{y}_m = \tilde{y}_m, v = t_1$
  end if
end for

Algorithm 2 Optimal Sensing Strategy

// input: $\mathbf{X} =$ permutations$(\{1, \dots, M\})$
// output: $\mathbf{x}^*, \mathbf{y}^*$
// base step
$v^* = 0, \mathbf{x}^* = \{0, \dots, 0\}, \mathbf{y}^* = \{0, \dots, 0\}$
// iterative step
for $\mathbf{x} \in \mathbf{X}$ **do**
  $v, \mathbf{y}$ computed with Algorithm 1
  if $v^* > v$ **then**
    $v^* = v, \mathbf{x}^* = \mathbf{x}, \mathbf{y}^* = \mathbf{y}^*$
  end if
end for

Algorithm 3 Optimal Sensing Strategy for Identically Distributed Channel Interference

// output: $\mathbf{y}^*$
// base step
$y_M = 0$
$v = (1 - (M-1)\tau_s/T) \sum_{k=1}^{K} p_k r_k$
// recursive step
for $m = M-1 : 1$ **do**
  $\tilde{y}_m = \min_k \{\tilde{r}_k (1 - m\tau_s/T) \geq v$
  $t_0 = (1 - (m-1)\tau_s/T) \sum_{k=1}^{K} p_k r_k$
  $t_1 = (1 - m\tau_s/T) \sum_{k=\tilde{y}_m}^{K} p_k r_k + \sum_{k=0}^{\tilde{y}_m - 1} v$
  if $t_0 > t_1$ **do**
    $y_m = 0, v = t_0$
  else
    $y_m = \tilde{y}_m, v = t_1$
end if
end for

A. Preliminaries

Here, we derive in Theorem 1 the closed-form expression of the *expected reward*. The proof of Theorem 1 requires the following preliminary lemmas.

Lemma 1 (Conditional Stopping Probability): The conditional stopping probability $p_{\mathbf{x},\mathbf{y}}(m)$, i.e., the probability of the SU using the mth sensed channel given that it skipped the first $m-1$ sensed channels, is equal to

$$p_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^{K} p_{x_m,k} \triangleq 1 - \bar{p}_{\mathbf{x},\mathbf{y}}(m) \tag{4}$$

with $\mathbf{x} \in \mathbf{X}$ denoting the adopted sensing sequence and $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: See Appendix A. ■

Remark: The SU uses channel x_1 with probability $p_{\mathbf{x},\mathbf{y}}(1)$, whereas it skips x_1 with probability $\bar{p}_{\mathbf{x},\mathbf{y}}(1)$. Given that it skipped channel x_1, it uses channel x_2 with probability $p_{\mathbf{x},\mathbf{y}}(2)$. Clearly, the lower y_1 is, the more likely the SU uses channel x_1, and from Definition 2, we have $y_1 = 0 \Longrightarrow p_{\mathbf{x},\mathbf{y}}(1) = 1$.

Lemma 2 (Rate Expectation): The rate expectation $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$, i.e., the expected data rate achievable by the SU through the mth sensed channel, is equal to

$$\bar{r}_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^{K} \frac{p_{x_m,k}}{p_{\mathbf{x},\mathbf{y}}(m)} \tilde{r}_k \tag{5}$$

with $\mathbf{x} \in \mathbf{X}$ denoting the adopted sensing sequence and $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: See Appendix B. ■

Remark: The lower is the threshold index y_m, the lower is the data rate threshold $\tilde{r}_{y_m}$, the more likely the data rate on channel x_m exceeds the threshold. Hence, the more likely the SU uses channel x_m, but the lower is the expected data rate $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$ through channel x_m.

Theorem 1 (Expected Reward): The expected reward $V_{\mathbf{x},\mathbf{y}}$ achievable by the SU following the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$ is equal to

$$V_{\mathbf{x},\mathbf{y}} = \sum_{m=1}^{M} p_{\mathbf{x},\mathbf{y}}(m) q_{\mathbf{x},\mathbf{y}}(m) \bar{r}_{\mathbf{x},\mathbf{y}}(m) c_{\mathbf{y}}(m) \tag{6}$$

where the probability $q_{\mathbf{x},\mathbf{y}}(m)$ of skipping the first $m-1$ sensed channels is given by

$$q_{\mathbf{x},\mathbf{y}}(m) = \begin{cases} 1, & \text{if } m = 1 \\ \prod_{l=1}^{m-1} \bar{p}_{\mathbf{x},\mathbf{y}}(l), & \text{otherwise} \end{cases} \tag{7}$$

and the scaling factor $c_{\mathbf{y}}(m)$ is given by

$$c_{\mathbf{y}}(m) = \begin{cases} (1 - (m-1)\tau_s/T), & \text{if } y_m = \tilde{r}_0 \\ (1 - m\tau_s/T), & \text{otherwise.} \end{cases} \tag{8}$$

Proof: See Appendix C. ■

Remark: The *expected reward* $V_{\mathbf{x},\mathbf{y}}$ allows us to estimate the expected throughput achievable by the SU during the time slot. Specifically, $V_{\mathbf{x},\mathbf{y}}$ is the sum of the rate expectation $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$ on channel x_m, weighted by the probability of using such a channel. Since the more channels are sensed by the SU, the less time can be devoted to packet transmission, the rate expectation for channel x_m is weighted by the scaling factor $c_{y_m}(m)$, which accounts for the portion of time slot T devoted to packet transmission.

B. Optimal Coexistence Strategy

Here, we derive in Theorem 2 the optimal stopping rule for a given sensing sequence. Stemming from this, we prove with Corollary 2 that Problem 1 can be polynomial-time reduced to another problem, which is referred to in the following as Problem 2. Intuitively, a polynomial-time reduction proves that the first problem is no more difficult than the second problem, because whenever an efficient algorithm exists for the second problem, one exists for the first problem as well. Furthermore, in Theorem 3, we prove that Problem 2 is NP-hard. Hence, due to the reduction property, we can conclude that an efficient algorithm does not exist for the considered problem, i.e., Problem 1.

The proof of Theorem 2 requires the following preliminary lemmas.

Lemma 3 (Expected Remaining Reward): The expected remaining reward $v_{\mathbf{x},\mathbf{y}}(m)$, i.e., the expected reward achievable by the SU through channels $x_{m+1}, \ldots, x_M$, given that it skipped the first m sensed channels, is given by

$$v_{\mathbf{x},\mathbf{y}}(m) = \sum_{l=m+1}^{M} p_{\mathbf{x},\mathbf{y}}(l) \prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{x},\mathbf{y}}(i) \bar{r}_{\mathbf{x},\mathbf{y}}(l) c_{\mathbf{y}}(l). \tag{9}$$

Proof: See Appendix D. ■

Remark: The *expected remaining reward* $v_{\mathbf{x},\mathbf{y}}(m)$ allows us to estimate the expected throughput given that the first m sensed channels are skipped, i.e., $q_{\mathbf{x},\mathbf{y}}(m+1) = 1$.

Lemma 4: Given the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$ with $y_m > 0$, we get

$$V_{\mathbf{x},\mathbf{y}} \leq V_{\mathbf{x},\tilde{\mathbf{y}}} \tag{10}$$

with

$$\tilde{y}_l = y_l \quad \forall\, l \neq m \wedge \tilde{y}_m = \min_k \{\tilde{r}_k (1 - m\tau_s/T) \geq v_{\mathbf{x},\mathbf{y}}(m)\}. \tag{11}$$

Proof: See Appendix E. ■

Remark: Lemma 4 allows us to establish, for an arbitrary sensing sequence, the mth stopping rule maximizing the expected reward *given that channel x_m is sensed*. Stemming from this, in Theorem 2, we derive the optimal mth stopping rule for an arbitrary sensing sequence.

Theorem 2 (Stopping Rule Given Sensing Sequence): Given the sensing sequence $\mathbf{x} \in \mathbf{X}$ and the stopping rule $\mathbf{y} \in \mathbf{Y}$, we get

$$V_{\mathbf{x},\mathbf{y}} \leq V_{\mathbf{x},\breve{\mathbf{y}}} \tag{12}$$

with

$$\breve{y}_l = y_l \quad \forall\, l \neq m \tag{13}$$

$$\breve{y}_m = \begin{cases} 0, & \text{if } E\left[R_{x_m}\right] \geq \frac{v_{\mathbf{x},\mathbf{y}}(m-1)}{1-(m-1)\tau_s/T} \\ \tilde{y}_m, & \text{otherwise} \end{cases} \tag{14}$$

with $\tilde{y}_m$ given in (11).

Proof: See Appendix F. ■

Corollary 1 (Mth Stopping Rule Given Sensing Sequence): For any sensing sequence $\mathbf{x} \in \mathbf{X}$, the Mth component of the stopping rule $\mathbf{y} \in \mathbf{Y}$ maximizing the expected reward $V_{\mathbf{x},\mathbf{y}}$ is given by

$$y_M = 0. \tag{15}$$

Proof: The proof follows directly from Theorem 2 since $v_{\mathbf{x},\mathbf{y}}(M) = 0$ for any $\mathbf{x}$ and $\mathbf{y}$. ■

Remark: From Corollary 1, it follows that when only one channel is left, the SU reduces the achievable expected reward by sensing such a channel instead of simply using it. This is reasonable since, even if the SU senses the channel as unavailable, i.e., $R_M = r_0$, there is no other option (channel) left.

Remark: By iteratively applying Theorem 1, it follows that Algorithm 1 effectively finds the stopping rule $\check{\mathbf{y}} \in \mathbf{Y}$ maximizing the expected reward for any given sensing sequence $\mathbf{x} \in \mathbf{X}$. We note that the time complexity of Algorithm 1 is linear with respect to both M and K, i.e., $\mathcal{O}(NK)$.

Stemming from Theorem 2, we can now reformulate Problem 1 as follows.

Problem 2 (Optimal Sensing Sequence): The goal is to choose the optimal sensing sequence $\mathbf{x}^*$ maximizing the expected reward, i.e.,

$$V_{\mathbf{x}^*,\mathbf{y}(\mathbf{x}^*)} = \arg\max_{\mathbf{x}\in\mathbf{X}} \left\{V_{\mathbf{x},\mathbf{y}(\mathbf{x})}\right\} \tag{16}$$

where $\mathbf{y}(\mathbf{x})$ is given by Algorithm 1 for any $\mathbf{x}$.

Corollary 2 (Problem Equivalence): Problem 1 can be polynomial-time reduced to Problem 2.

Proof: The proof follows from Theorem 2 by accounting for the time complexity of Algorithm 1. ■

Theorem 3 (Problem Complexity): Problem 2 is NP-hard.

Proof: See Appendix G. ■

Remark: As proved in Appendix G, the *optimal coexistence strategy* problem can be polynomial-time reduced to the TSP. Hence, the existing literature on exact/approximate algorithms for solving the TSP can be efficiently adopted for searching the optimal/suboptimal strategy.

Insight 2: In scenarios of practical interest such as the urban scenarios, it has been shown that the TVWS spectrum is scarce with roughly five channels available for secondary access [12]. Hence, as we show in Section V-B, the optimal solution can be found in almost real time with commercial hardware through Algorithm 2. More in detail, with Algorithm 2, 1) the sensing sequence maximizing the expected reward is found through exhaustive search, and 2) the stopping rule maximizing the expected reward for any given sensing sequence is found through Algorithm 1. Furthermore, in Section IV-C, by considering identically distributed coexistence interference levels, we derive an efficient (polynomial-time) algorithm for searching the optimal strategy.

C. Optimal Coexistence Strategy Under Identically Distributed Interference

Here, we derive in Theorem 4 the optimal sensing strategy when the coexistence interference levels are identically distributed among the available channels. Hence, in the following, we denote with p_k the probability of $\tilde{r}_k$ being the admissible data rate for channel x_m for any $m \in \Omega$.

The proof of Theorem 4 requires the following intermediate results.

Lemma 5 (Conditional Stopping Probability): For any sensing sequence $\mathbf{x} \in \mathbf{X}$, we get

$$p_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^{K} p_k \triangleq p_{\mathbf{y}}(m) \triangleq 1 - \bar{p}_{\mathbf{y}}(m) \tag{17}$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: See Appendix H. ■

Remark: In the following, we adopt the notation $p_{\mathbf{y}}(m)$ to highlight the independence of the conditional stopping probability from the sensing sequence due to the identical distribution hypothesis.

Lemma 6 (Rate Expectation): For any sensing sequence $\mathbf{x} \in \mathbf{X}$, we get

$$\bar{r}_{\mathbf{x},\mathbf{y}}(m) = \sum_{k=y_m}^{K} \frac{p_k}{p_{\mathbf{y}}(m)} \tilde{r}_k \triangleq \bar{r}_{\mathbf{y}}(m) \tag{18}$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule.

Proof: The proof follows by reasoning, as in Appendix H. ■

Corollary 3 (Expected Reward): For any sensing sequence $\mathbf{x} \in \mathbf{X}$, we get

$$V_{\mathbf{x},\mathbf{y}} = \sum_{m=1}^{M} p_{\mathbf{y}}(m) q_{\mathbf{y}}(m) \bar{r}_{\mathbf{y}}(m) c_{\mathbf{y}}(m) \triangleq V_{\mathbf{y}} \tag{19}$$

with $\mathbf{y} \in \mathbf{Y}$ denoting the adopted stopping rule, $c_{\mathbf{y}}(m)$ given in (8), and $q_{\mathbf{y}}(m)$ equal to

$$q_{\mathbf{y}}(m) = \begin{cases} 1, & \text{if } m = 1 \\ \prod_{l=1}^{m-1} \bar{p}_{\mathbf{y}}(l) \triangleq \prod_{l=1}^{m-1} \left(1 - p_{\mathbf{y}}(l)\right), & \text{otherwise.} \end{cases} \tag{20}$$

Proof: The proof follows from Lemmas 5 and 6. ■

Remark: The result of Corollary 3 is reasonable: Since the coexistence interference is identically distributed over the available channels, the optimal sensing strategy 1) depends on the stopping rule $\mathbf{y}$ and 2) does not depend on the sensing sequence $\mathbf{x}$. Stemming from this, we can now derive in Theorem 4 the optimal sensing strategy.

Theorem 4 (Optimal Sensing Strategy): The optimal stopping rule $\mathbf{y}^* \in \mathbf{Y}$ is recursively defined as

$$y_m^* = \begin{cases} 0, & m = M \\ = \begin{cases} 0, & \text{if } \sum_{k=0}^{K} p_k \tilde{r}_k \geq \frac{v_{\mathbf{y}}(m)}{1-(m-1)\tau_s/T}, \\ \tilde{y}_m, & \text{otherwise} \end{cases} & m < M \end{cases} \tag{21}$$

with $\tilde{y}_m = \min_k\{\tilde{r}_k(1 - m\tau_s/T) \geq v_{\mathbf{y}}(m)\}$ and $v_{\mathbf{y}}(m)$ equal to

$$v_{\mathbf{y}}(m) = \sum_{l=m+1}^{M} p_{\mathbf{y}}(l) \prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{y}}(i) \bar{r}_{\mathbf{y}}(l) c_{\mathbf{y}}(l). \tag{22}$$

Proof: See Appendix I. ■

TABLE II
PERFORMANCE EVALUATION PARAMETER SETTING

Symbol	Definition	Fig. 4	Fig. 5	Fig. 6	Fig. 7	Fig. 8
M	number of TVWS channels	4	4	2-8	4	2-8
$\tilde{r}_k$	admissible data rate	{0,1.8,3.6,5.4,7.2,10.8,14.4,16.2,18,21.6,24}Mbit/s				
τ	normalized sensing time	0.01	0.01	0.01	0.01	0.01-0.5
$p_{m,k}$	probability of $\tilde{r}_k$ being the data rate achievable on channel m	uniformly distributed in $[0, 1]$				

Remark: From Theorem 4, it follows that Algorithm 3 efficiently (polynomial-time) finds the optimal sensing strategy in the presence of identically distributed interference levels. Furthermore, in Section V-B, we evaluate the feasibility of Algorithm 3 in the presence of nonidentically distributed interference levels.

Remark: By assuming a negligible sensing overhead, i.e., $\tau_s \ll T$, it is straightforward to prove that the optimal stopping rule y^*_m is equal to $\tilde{y}_m$ for any $m < M$.

V. PERFORMANCE EVALUATION

Here, we evaluate the performance of the proposed coexistence strategy by adopting, as a case study, an IEEE 802.11af network operating in the TVWS spectrum.

A. Optimality

Here, we validate the optimality property of the proposed coexistence strategy by showing that the sensing rule derived in Algorithm 1 assures, for any sensing sequence, the highest expected reward.

The simulation set, which is summarized in Table II, is as follows: $M = 4$ channels are available for secondary access, and by adopting 6-MHz-wide channels, the admissible data rates in IEEE 802.11af are {0, 1.8, 3.6, 5.4, 7.2, 10.8, 14.4, 16.2, 18, 21.6, 24} Mb/s. The channel interference levels are independent of each other and uniformly distributed within the corresponding SINR regions, and the sensing process is characterized by a normalized sensing time[7] $\tau_s/T = 0.01$.

In Fig. 4, for each of the $M! = 24$ sensing sequences, we report 1) the expected reward achievable by using the stopping rule $\check{\mathbf{y}}$ derived in Algorithm 1 and 2) the expected rewards achievable by using any other stopping rule $\mathbf{y} \in \mathbf{Y}$, with $|\mathbf{Y}| = (K+1)^M = 14641$, found through exhaustive enumeration. First, we note that Algorithm 1 effectively finds the stopping rule maximizing the expected reward for any given sensing sequence. Hence, the proposed coexistence strategy is *optimal*. We further observe that there is a significant variability among the different stopping rules in terms of expected reward, ranging from roughly 8 Mb/s to over 16 Mb/s. This result highlights that, even when the TVWS spectrum is scarce ($M = 4$), there exists significant diversity among the strategies in terms of expected throughput. Hence, an optimal strategy design is crucial to take advantage of such diversity.

To better understand the effects of the strategy design in terms of expected reward, in Fig. 5, we report on the expected reward as a function of the couple *sensing sequence–stopping rule. Specifically, each three-dimensional point refers to a pair* $(\mathbf{x}, \mathbf{y})$, *where coordinate* (x) *denotes the sensing sequence index, coordinate* (y) *denotes the stopping rule index, and coordinate* (z) *denotes the expected reward* $V_{\mathbf{x},\mathbf{y}}$. *We observe that the expected reward achievable significantly changes not only with the stopping rule but also with the sensing sequence, i.e., there exist two degrees of freedom that must be explored to design an optimal strategy*. This agrees with the theoretical results derived in Section IV. Furthermore, we note that, for a given stopping rule, by changing the sensing sequence, the expected reward can vary up to a notable 40%, confirming the considerations made for Fig. 4.

[7]By abstracting from the sensing particulars, the notion of normalized sensing time allows us to focus on the effects of the coexistence strategy in terms of expected reward.

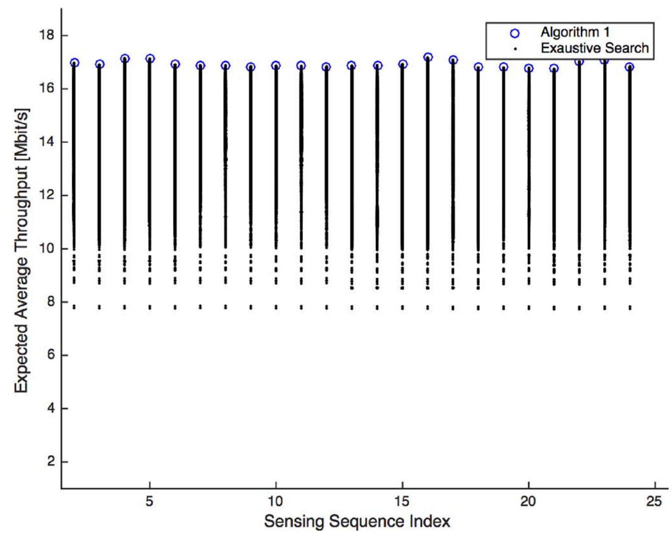

Fig. 4. Optimality: Algorithm 1 versus exhaustive search. Each dot refers to a pair $(\mathbf{x}, \mathbf{y})$, where coordinate (x) denotes the sensing sequence index, and coordinate (y) denotes the expected reward $V_{\mathbf{x},\mathbf{y}}$. Each circle refers to a pair $(\mathbf{x}, \check{\mathbf{y}})$ with $\check{\mathbf{y}}$ given by Algorithm 1, where coordinate (x) denotes the sensing sequence index, and coordinate (y) denotes the expected reward $V_{\mathbf{x},\check{\mathbf{y}}}$.

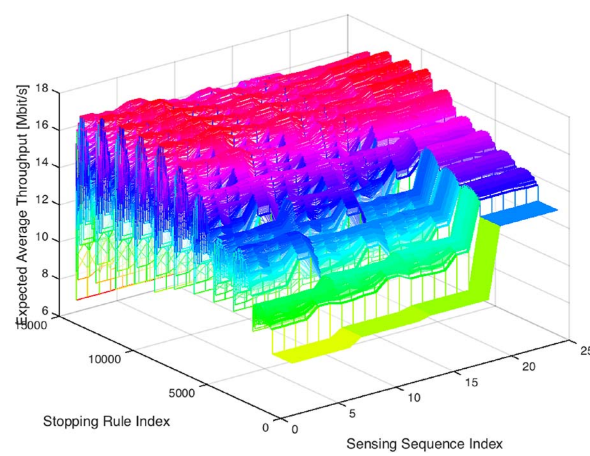

Fig. 5. Optimality: Expected reward versus *sensing sequence–stopping rule*.

B. Feasibility

Here, we assess the feasibility of the proposed coexistence strategy in terms of computational complexity. Specifically, we

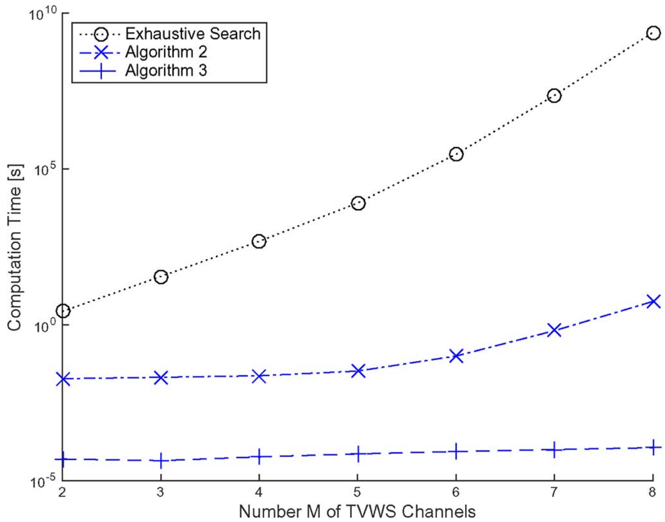

Fig. 6. Time complexity: Running times versus number M of available TVWS channels. Logarithmic scale for (y) axis.

compare the running times for computing the optimal coexistence strategy with three different algorithms: 1) Algorithm 2; 2) Algorithm 3; and 3) exhaustive search.

Fig. 6 presents the running times[8] of the considered algorithms as the number M of available TVWS channels increases, with a logarithmic scale for the (y) axis. The simulation set is as in Section V-A. First, we observe that the time complexity of the exhaustive search makes this choice unfeasible even in urban scenarios, with running times within the order of magnitude of minutes and hours for $M = 3$ and $M = 4$, respectively. On the other hand, we observe that Algorithm 2 performs satisfactory in urban scenarios, with running times within the order of magnitude of seconds or less up to $M = 8$. Finally, we note that Algorithm 3 performs considerably well in both urban and rural scenarios, with running times on the order of 10^{-4} seconds, also for the larger values of M.

One question arises spontaneously: What if the considered scenario is characterized by TVWS abundance in the presence of independent but nonidentically distributed interference? In other words, what if we resort to Algorithm 3 when the interference is not identically distributed?

We focus on such a scenario in Fig. 7. Specifically, the figure presents the expected reward as a function of time for both Algorithms 2 and 3, along with the corresponding time averages of the expected rewards. The simulation set is as in Section V-A, with Algorithm 3 rate probability p_k set to the average value of the channel data rate probabilities $p_{m,k}$, i.e., $p_k = 1/M \sum_{m=1}^{M} p_{m,k}$. Clearly, Algorithm 2, i.e., the algorithm designed for the considered scenario, outperforms Algorithm 3 both instantaneously and in average. Nevertheless, the differences in terms of expected reward between the optimal and the approximate algorithms are moderate. Hence, Algorithm 3 represents a suboptimal but efficient solution when the running times of Algorithm 2 are unfeasible.

[8]We note that the times have been obtained by running the algorithms on a general-purpose architecture (MacBook Pro). Hence, it is reasonable to believe that a reduction of one or two orders of magnitude can be easily obtained by adopting dedicated hardware.

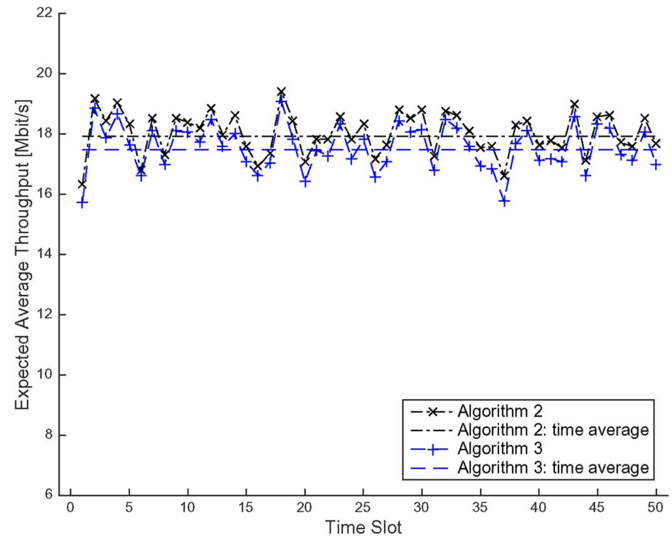

Fig. 7. Fast computation: Expected reward versus time in the presence of independent interference.

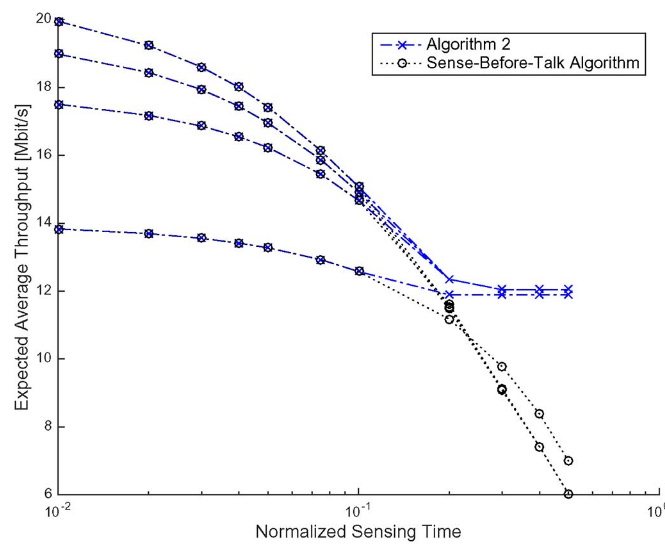

Fig. 8. Discretionary interference sensing: Expected reward versus normalized sensing time τ_s/T for different values of the number M of available TVWS channels. Logarithmic scale for (x) axis.

C. Discretionary Interference Sensing

Here, we analyze the benefits provided by discretionary interference sensing in terms of expected throughput. More in detail, we compare the expected throughputs achievable with the proposed coexistence strategy (see Algorithm 2) with those achievable by an algorithm that implements mandatory interference sensing, which is referred to as the *sense-before-talk algorithm*. As pointed out in Section II-A, the mandatory interference sensing represents a requirement of the existing literature on channel selection for CR networks. Hence, by comparing the proposed coexistence strategy with the *sense-before-talk algorithm*, we aim at assessing the performance improvement over the state of the art.

Fig. 8 presents the expected reward as the normalized sensing time τ_s/T increases for different values of the number M of available TVWS channels. The simulation set is as in Section V-A.

At first, we observe that the higher the normalized sensing time is, the lower is the expected reward. This agrees with both the intuition and Theorem 1. Furthermore, we observe that the differences between the optimal and the suboptimal strategies in terms of rewards increase as τ_s/T. This result is reasonable since the larger the sensing times are, the higher are the sensing overheads, and hence, the higher is the positive impact of the discretionary interference sensing in terms of reward. Finally, we observe that the differences between the optimal and the suboptimal strategies in terms of rewards increase as M decreases for a fixed normalized sensing time. This is reasonable: The lower the available channels, the less the sensing sequences, i.e., the less significant the effects of channel diversity on the expected reward. Consequently, the lower the available channels, the more significant the stopping rules in terms of expected reward.

VI. Conclusion

In this paper, we have addressed the problem of the coexistence interference among multiple heterogeneous and independently operated SNs coexisting within the same geographical area over shared TVWS in the absence of secondary cooperation. Specifically, we designed the optimal coexistence strategy, i.e., the strategy that maximizes the throughput achievable by an arbitrary SN. Such a strategy exhibits several attractive features: 1) feasibility, since it requires a reasonable amount of *a priori* knowledge; 2) adaptivity to interference dynamics; 3) autonomy, since it allows the SN to make its decisions independently of any other coexisting networks; and 4) discretionary interference avoidance, since it accounts for the harmless-to-incumbents property of interference in TVWS. More in detail, we proved that the problem of optimal channel selection, i.e., the channel maximizing the expected throughput, is computationally prohibitive (NP-hard). Nevertheless, under the reasonable assumption of identically distributed interference on the available TVWS channels, we prove that the optimal channel selection problem is not anymore NP-hard. Specifically, we proved that the optimal strategy exhibits a threshold behavior, and by exploiting this threshold structure, we designed a computationaly efficient (polynomial-time) algorithm. The performance evaluation validated the proposed theoretical analysis.

Appendix A
Proof of Lemma 1

By observing that the SU uses the mth sensed channel, given that it skipped the first $m-1$ sensed channels, if and only if $R_{x_m} \geq \tilde{r}_{y_m}$, it follows that

$$p_{\mathbf{x},\mathbf{y}}(m) = P\left(R_{x_m} \geq \tilde{r}_{y_m}\right) = \sum_{k=y_m}^{K} P\left(R_{x_m} = \tilde{r}_k\right) = \sum_{k=y_m}^{K} p_{x_m,k}. \tag{23}$$

■

Appendix B
Proof of Lemma 2

By noting that the rate expectation $\bar{r}_{\mathbf{x},\mathbf{y}}(m)$ represents the expectation of R_{x_m} given that channel x_m is used, i.e., given that $R_{x_m} \geq \tilde{r}_{y_m}$, it follows that

$$\bar{r}_{\mathbf{x},\mathbf{y}}(m) = E\left[R_{x_m} | R_{x_m} \geq \tilde{r}_{y_m}\right] = \frac{\sum_{k=y_m}^{K} p_{x_m,k}\tilde{r}_k}{p_{\mathbf{x},\mathbf{y}}(m)} \tag{24}$$

where the last equality accounts for Lemma 1 and for the definition of expectation of a truncated r.v. ■

Appendix C
Proof of Theorem 1

First, we observe that when the mth sensed channel is used, the portion of the time slot devoted to packet transmission is not greater than $1-(m-1)\tau_s/T$. In fact, any channel in $\{x_1, \ldots, x_{m-1}\}$ has been sensed, i.e., $y_k > 0$ for any $k < m$. Specifically, such a time slot fraction is equal to $1-(m-1)\tau_s/T$ when $y_m = 0$, whereas it is equal to $1 - m\tau_s/T$ when $y_m > 0$. Hence, the expected reward achievable on channel x_m is equal to $\bar{r}_{\mathbf{x},\mathbf{y}}(m)c_{\mathbf{y}}(m)$. Since channel x_m is used with probability $p_{\mathbf{x},\mathbf{y}}(m)$ if and only if the previous $m-1$ sensed channels were skipped and since such a probability is equal to $q_{\mathbf{x},\mathbf{y}}(m)$ given in (7), the thesis follows. ■

Appendix D
Proof of Lemma 3

Similar to the proof of Theorem 1, the expected reward achievable through channel x_l in $x_{m+1}, \ldots, x_K$ is equal to $\bar{r}_{\mathbf{x},\mathbf{y}}(l)c_{\mathbf{y}}(l)$. Since channel x_l is used with probability $p_{\mathbf{x},\mathbf{y}}(l)$ if and only if the channels $x_{m+1}, \ldots, x_{l-1}$ were skipped and since such a probability is equal to $\prod_{i=m+1}^{l-1} \bar{p}_{\mathbf{x},\mathbf{y}}(i)$, the thesis follows. ■

Appendix E
Proof of Lemma 4

We prove the thesis with a *reductio ad absurdum* by supposing that there exist $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ such that

$$V_{\mathbf{x},\mathbf{y}} > V_{\mathbf{x},\tilde{\mathbf{y}}} \tag{25}$$

with $\tilde{y}$ given in (11). We have two cases.

i) Case $y_m < \tilde{y}_m$. Let us assume, without loss of generality, that $y_m = \tilde{y}_m - 1$. Hence, by accounting for (11), we have

$$\begin{aligned} V_{\mathbf{x},\tilde{\mathbf{y}}} &= \sum_{l=1}^{M} p_{\mathbf{x},\tilde{\mathbf{y}}}(l) q_{\mathbf{x},\tilde{\mathbf{y}}}(l) \bar{r}_{\mathbf{x},\tilde{\mathbf{y}}}(l) c_{\tilde{\mathbf{y}}}(l) \\ &= \sum_{l=1}^{m-1} p_{\mathbf{x},\mathbf{y}}(l) q_{\mathbf{x},\mathbf{y}}(l) \bar{r}_{\mathbf{x},\mathbf{y}}(l) c_{\mathbf{y}}(l) \\ &\quad + p_{\mathbf{x},\tilde{\mathbf{y}}}(m) q_{\mathbf{x},\mathbf{y}}(m) \bar{r}_{\mathbf{x},\tilde{\mathbf{y}}}(m) c_{\mathbf{y}}(m) \\ &\quad + \bar{p}_{\mathbf{x},\tilde{\mathbf{y}}}(m) q_{\mathbf{x},\mathbf{y}}(m) v_{\mathbf{x},\mathbf{y}}(m). \end{aligned} \tag{26}$$

By substituting (26) in (25) and by accounting for (4) and (5), we obtain

$$\begin{aligned}\sum_{k=\tilde{y}_m}^{K} p_{x_m,k}\tilde{r}_k c_{\mathbf{y}}(m) + p_{x_m,\tilde{y}_m-1}\tilde{r}_{\tilde{y}_m-1}c_{\mathbf{y}}(m)\\ + \sum_{k=0}^{\tilde{y}_m-2} p_{x_m,k}v_{\mathbf{x},\mathbf{y}}(m) > \sum_{k=\tilde{y}_m}^{K} p_{x_m,k}\tilde{r}_k c_{\mathbf{y}}(m)\\ + p_{x_m,\tilde{y}_m-1}v_{\mathbf{x},\mathbf{y}}(m) + \sum_{k=0}^{\tilde{y}_m-2} p_{x_m,k}v_{\mathbf{x},\mathbf{y}}(m)\end{aligned} \tag{27}$$

and, since $\tilde{r}_{\tilde{y}_m-1}c_{\mathbf{y}}(m)=\tilde{r}_{\tilde{y}_m-1}(1-m\tau_s/T) < v_{\mathbf{x},\mathbf{y}}(m)$ from (11), (27) constitutes a *reductio ad absurdum*.

ii) Case $y_m > \tilde{y}_m$. Let us assume, without loss of generality, that $y_m = \tilde{y}_m + 1$. By substituting (26) in (25) and by accounting for (4) and (5), we obtain

$$\begin{aligned}\sum_{k=\tilde{y}_m+1}^{K} p_{x_m,k}\tilde{r}_k c_{\mathbf{y}}(m) + p_{x_m,\tilde{y}_m}v_{\mathbf{x},\mathbf{y}}(m)\\ + \sum_{k=0}^{\tilde{y}_m-1} p_{x_m,k}v_{\mathbf{x},\mathbf{y}}(m) > \sum_{k=\tilde{y}_m+1}^{K} p_{x_m,k}\tilde{r}_k c_{\mathbf{y}}(m)\\ + p_{x_m,\tilde{y}_m}\tilde{r}_{\tilde{y}_m}c_{\mathbf{y}}(m) + \sum_{k=0}^{\tilde{y}_m-1} p_{x_m,k}v_{\mathbf{x},\mathbf{y}}(m)\end{aligned} \tag{28}$$

and, since $\tilde{r}_{\tilde{y}_m}c_{\mathbf{y}}(m)=\tilde{r}_{\tilde{y}_m}(1-m\tau_s/T)\geq v_{\mathbf{x},\mathbf{y}}(m)$ from (11), (28) constitutes a *reductio ad absurdum*. ■

APPENDIX F
PROOF OF THEOREM 2

i) Case $E[R_{x_m}]\geq v_{\mathbf{x},\mathbf{y}}(m-1)/(1-(m-1)\tau_s/T)$. We prove the thesis with a *reductio ad absurdum* by supposing that there exist $\mathbf{x}\in\mathbf{X}$ and $\mathbf{y}\in\mathbf{Y}$ such that

$$V_{\mathbf{x},\mathbf{y}} > V_{\mathbf{x},\check{\mathbf{y}}} \tag{29}$$

with $\check{\mathbf{y}}$ given in (13). Hence, we have

$$\begin{aligned}V_{\mathbf{x},\check{\mathbf{y}}} &= \sum_{l=1}^{M} p_{\mathbf{x},\check{\mathbf{y}}}(l)q_{\mathbf{x},\check{\mathbf{y}}}(l)\bar{r}_{\mathbf{x},\check{\mathbf{y}}}(l)c_{\check{\mathbf{y}}}(l)\\ &= \sum_{l=1}^{m-1} p_{\mathbf{x},\mathbf{y}}(l)q_{\mathbf{x},\mathbf{y}}(l)\bar{r}_{\mathbf{x},\mathbf{y}}(l)c_{\mathbf{y}}(l)\\ &\quad + \bar{p}_{\mathbf{x},\mathbf{y}}(m-1)q_{\mathbf{x},\mathbf{y}}(m-1)v_{\mathbf{x},\check{\mathbf{y}}}(m-1).\end{aligned} \tag{30}$$

By substituting (30) in (29), we obtain

$$\begin{aligned}v_{\mathbf{x},\mathbf{y}}(m-1) = \sum_{k=y_m}^{K} p_{x_m,k}\tilde{r}_k c_{\mathbf{y}}(m) + \sum_{k=0}^{y_m-1} p_{x_m,k}v_{\mathbf{x},\mathbf{y}}(m)\\ > \sum_{k=0}^{K} p_{x_m,k}\tilde{r}_k c_{\check{\mathbf{y}}}(m).\end{aligned} \tag{31}$$

By accounting for the hypothesis and by noting that $c_{\check{\mathbf{y}}}(m) = 1-(m-1)\tau_s/T$, (31) constitutes a *reductio ad absurdum*.

ii) Case $E[R_{x_m}] < v_{\mathbf{x},\mathbf{y}}(m-1)/(1-(m-1)\tau_s/T)$. By following the same reasoning of *Case i)*, the thesis follows. ■

APPENDIX G
PROOF OF THEOREM 3

We prove the theorem by adopting a typical tool of computational complexity theory, i.e., reduction [18]. A reduction is a procedure for transforming one problem into another problem, and it can be used to show that the second problem is at least as difficult as the first. Specifically, we reduce Problem 2 to a notable NP-hard problem, i.e., the TSP, by showing that there exists a one-to-one mapping between Problem 2 and the single-machine job-scheduling problem with sequence-dependent setup times. Since such a problem can be polynomial-time reduced to the TSP, we have the thesis.

Let us focus on the contribution of the mth sensed channel to the expected reward $V_{\mathbf{x},\mathbf{y}(\mathbf{x})}$, i.e.,

$$p_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m)q_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m)\bar{r}_{\mathbf{x},\mathbf{y}(\mathbf{x})}(m)c_{\mathbf{y}(\mathbf{x})}(m). \tag{32}$$

From Algorithm 1, it follows that the mth component $y_m(\mathbf{x})\in\mathbf{y}(\mathbf{x})$ is a function of the last $M-m+1$ components of $\mathbf{x}$. Hence, by accounting for (4), (5), and (8), it follows that (32) depends on $(x_m,\ldots,x_M)$. Furthermore, by accounting for (7), it follows that (32) depends on $(x_1,\ldots,x_{m-1})$. Hence, by denoting $y_m(\mathbf{x})$ as $f(x_m,\ldots,x_M)$, it is easy to recognize that the expected reward $V_{\mathbf{x},\mathbf{y}(\mathbf{x})}$ achievable by using the sensing sequence $\mathbf{x}=(x_1,\ldots,x_M)$ is equivalent to

$$V_{\mathbf{x},\mathbf{y}(\mathbf{x})} = \sum_{m=1}^{M} g_{(x_1,\ldots,x_M)}(m) \tag{33}$$

with $g_{(x_1,\ldots,x_M)}(m)$ recursively defined as in (34), shown at the bottom of the page.

$$g_{(x_1,\ldots,x_M)}(m) = \begin{cases} \left(\sum\limits_{l=1}^{M-1}\prod\limits_{k<f(x_l,\ldots,x_M)} p_{x_l,k}\right)\sum\limits_{k\geq 0} p_{x_M,k}\bar{r}_k\left(1-(M-1)\tau_s/T\right), & \text{if } m=M\\ \left(\sum\limits_{l=1}^{m-1}\prod\limits_{k<f(x_l,\ldots,x_M)} p_{x_l,k}\right)\max\left\{\begin{array}{l}\sum\limits_{k\geq 0}^{K} p_{x_m,k}\bar{r}_k\left(1-(m-1)\tau_s/T\right)\\ \sum\limits_{k\geq f(x_m,\ldots,x_M)} p_{x_m,k}\bar{r}_k(1-m\tau_s/T)\end{array}\right\}, & \text{if } 1<m<M\\ \max\left\{\begin{array}{l}\sum\limits_{k\geq 0}^{K} p_{x_1,k}\bar{r}_k T\\ \sum\limits_{k\geq f(x_1,\ldots,x_M)} p_{x_1,k}\bar{r}_k(1-\tau_s/T)\end{array}\right\}, & \text{if } m=1\end{cases} \tag{34}$$

By denoting with $s_{(x_1,\ldots,x_M)}(m) = -g_{(x_1,\ldots,x_M)}(m)$, we have

$$\max_{\mathbf{x}\in\mathbf{X}} \left\{V_{\mathbf{x},\mathbf{y}(\mathbf{x})}\right\} = \min_{\mathbf{x}\in\mathbf{X}} \left\{\sum_{m=1}^{M} s_{(x_1,\ldots,x_M)}(m)\right\} = \min_{\mathbf{x}\in\mathbf{X}} \left\{\sum_{m=1}^{M} p_m + \sum_{m=1}^{M} s_{(x_1,\ldots,x_M)}(m)\right\}. \tag{35}$$

Hence, solving Problem 2 is equivalent to solving the single-machine job-scheduling problem with 1) equal-release times $p_m = 0$ and 2) sequence-dependent setup times $s_{(x_1,\ldots,x_M)}(m)$. Since such a problem can be polynomial-time reduced [37], [38] to the TSP, we have the thesis. ■

APPENDIX H
PROOF OF LEMMA 5

By hypothesis, we get

$$p_{x_m,k} = P\left(R_{x_m} = \tilde{r}_k\right) = p_k \quad \forall x_m \in \Omega. \tag{36}$$

Hence, by substituting (36) in (23), we have the thesis. ■

APPENDIX I
PROOF OF THEOREM 4

We prove the thesis through backward induction.

i) Case $m = M$. The thesis follows by noting that, for any $y_m \neq 0$ and for any $\tau_s > 0$, we get

$$\sum_{k=0}^{K} p_k \tilde{r}_k \left(1-(m-1)\tau_s/T\right) > \sum_{k=y_m}^{K} p_k \tilde{r}_k (1-m\tau_s/T). \tag{37}$$

ii) Case $m < M$. It is straightforward to prove the thesis by accounting for the results derived in Lemma 4 and Theorem 2. ■

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Angela Sara Cacciapuoti (M'10) received the Dr.Eng. degree (*summa cum laude*) in telecommunications engineering and the Ph.D. degree (with score "excellent") in electronic and telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2005 and 2009, respectively.

She is currently an Assistant Professor with the Department of Electrical Engineering and Information Technologies, University of Naples Federico II. From 2010 to 2011, she was a Visiting Researcher with the Broadband Wireless Networking Laboratory, Georgia Institute of Technology, Atlanta, GA, USA. In 2011, she was a Visiting Researcher with the NaNoNetworking Center in Catalunya (N3Cat), Universitat Politècnica de Catalunya, Barcelona, Spain. Her current research interests include cognitive radio networks and nanonetworks.

Dr. Cacciapuoti is an Area Editor of the *Computer Networks Journal* (Elsevier).

Marcello Caleffi (M'12) received the Dr.Eng. degree (*summa cum laude*) in computer science engineering from the University of Lecce, Lecce, Italy, in 2005 and the Ph.D. degree in electronic and telecommunications engineering from the University of Naples Federico II, Naples, Italy, in 2009.

He is currently an Assistant Professor with the Department of Electrical Engineering and Information Technologies, University of Naples Federico II. From 2010 to 2011, he was a Visiting Researcher with the Broadband Wireless Networking Laboratory, Georgia Institute of Technology, Atlanta, GA, USA. In 2011, he was also a Visiting Researcher with the NaNoNetworking Center in Catalunya (N3Cat), Universitat Politècnica de Catalunya, Barcelona, Spain. His research interests include cognitive radio networks and biological networks.

Dr. Caleffi serves as an Area Editor for *Elsevier Ad Hoc Networks*.

Luigi Paura (M'11) received the Dr.Eng. degree (*summa cum laude*) in electronic engineering from the University of Naples Federico II, Naples, Italy, in 1974.

From 1979 to 1984, he was with the Department of Biomedical, Electronic, and Telecommunications Engineering, University of Naples Federico II, first as an Assistant Professor and then as an Associate Professor. Since 1994, he has been a Full Professor of telecommunications, first with the Department of Mathematics, University of Lecce, Lecce, Italy; then with the Department of Information Engineering, Second University of Naples, Naples; and finally, since 1998, with the Department of Biomedical, Electronic and Telecommunications Engineering, University of Naples Federico II. From 1985 to 1986 and in 1991, he was a Visiting Researcher with the Signal and Image Processing Laboratory, University of California, Davis, CA, USA. His research interests include digital communication systems and cognitive radio networks.