Robotic Non-prehensile Object Transportation

Viviana Morlando
DIETI
University of Naples Federico II
Naples, Italy
viviana.morlando@unina.it

Mario Selvaggio
DIETI
University of Naples Federico II
Naples, Italy
mario.selvaggio@unina.it

Fabio Ruggiero
DIETI
University of Naples Federico II
Naples, Italy
fabio.ruggiero@unina.it

***Abstract*—This document revises the latest results related to robotic non-prehensile object transportation. The problem consists of a robotic arm transporting an object along the desired trajectory on a tray, guaranteeing a sticking behaviour and other constraints. The solution in [1], where an optimal control problem has been devised, is revised together with the case in [2], where a quadruped/mobile robot is considered. A model predictive control approach is also described to solve the same problem.**

***Index Terms*—Dexterous manipulation, Object transportation, Optimal control problem, Model predictive control**

I. Introduction

Transporting an object in a non-prehensile configuration (i.e., without any form- or force-closure grasp [3], [4]) is representative of many situations in which the robot cannot firmly hold the object and constrain its motion induced by inertial/external forces. In these cases, the object is free to slide or break contact with the robot end-effector. A solution to such a non-prehensile manipulation problem is known as *dynamic grasp* [5], defined as the condition in which friction forces prevent the object from moving relative to the manipulator. Dynamic grasp (i.e., non-sliding manipulation) is achieved by regulating the robot motion such that the object remains stationary to the end-effector. Applications of dynamic grasp can be the remote manipulation of contaminated objects of very different size and shape to achieve a faster decommissioning of nuclear sites. Another example is a robot able to carry a meal on a tray to a patient: a dish, a glass, a bottle, and pieces of cutlery are placed on the tray, and they must be safely transported to the patient.

An optimal control problem preventing the sliding of the object from the tray is devised in [1]. The control design is integrated within a shared-control telemanipulation framework omitted here. In the case of non-prehensile transportation with a mobile robot (i.e., a quadruped robot), a whole-body control must be conceived. Finally, a model predictive control (MPC) can be employed to satisfy further constraints.

II. Preliminaries

Several assumptions must be considered to model an object transported by a tray-like robotic end-effector [1], [2]. The main reference frames and quantities used in this work are illustrated in Fig. 2. Let $q_b = (p_b, R_b) \in SE(3)$ be the pose of the object frame $\{\mathcal{B}\}$ attached to the object's center of mass (CoM), in the inertial reference frame $\{\mathcal{W}\}$, with $p_b \in \mathbb{R}^3$ the position vector of the object and $R_b \in SO(3)$ the orientation of $\{\mathcal{B}\}$ in $\{\mathcal{W}\}$. The object dynamics can be written as

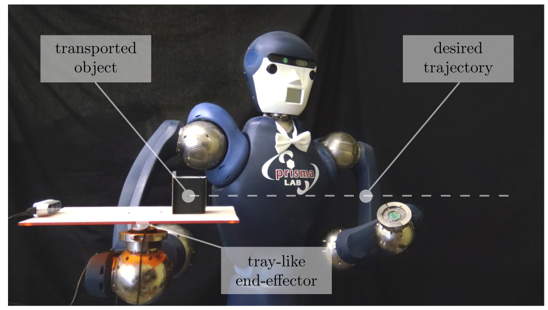

Fig. 1. Example of a robotic non-prehensile object transportation task.

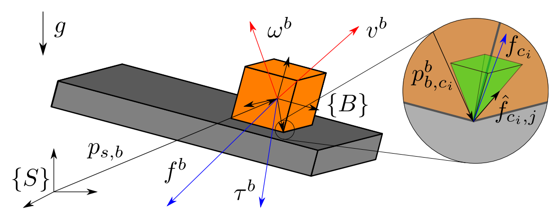

Fig. 2. Relevant quantities of the nonprehensile manipulation robotic system.

$$M_b \dot{\mathcal{V}}_b + C_b(\mathcal{V}_b)\mathcal{V}_b + N_b(R_b) = \mathcal{F}_b, \tag{1}$$

with $M_b \in \mathbb{R}^{6\times 6}$ the constant and positive-definite object's mass matrix, constructed from the object's mass $m \in \mathbb{R}_{\geq 0}$ and the constant symmetric and positive-definite inertia matrix $I_b \in \mathbb{R}^{3\times 3}$; $C_b \in \mathbb{R}^{6\times 6}$ the matrix accounting centrifugal/Coriolis effect; $N_b(R_b) \in \mathbb{R}^6$ the vector containing gravity terms; $\mathcal{V}_b = (v_b, \omega_b) \in \mathbb{R}^6$ the body object's twist with $v_b \in \mathbb{R}^3$ the linear velocity and $\omega_b \in \mathbb{R}^3$ the angular velocity; $\mathcal{F}_b \in \mathbb{R}^6$ is the wrench exerted at the object's center of mass, expressed in $\{\mathcal{B}\}$. The body wrench $\mathcal{F}_b$ is dictated by the tray/object contact forces. A suitable contact model is adopted to control the tray/object interaction behaviour [1]. In order to obtain safe object transportation, the contact model must be characterised by a non-sliding behaviour, meaning that each contact force vector $f_{c_i} \in \mathbb{R}^3$ must be contained inside the $i-$th friction cone FC_i. The $i-$th friction cone can be defined as the set of generalised contact forces realisable

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given the friction coefficient μ, between the object and the tray. Whenever $F_c \in FC = FC_1 \times \cdots \times FC_{n_c}$, the object can be manipulated without sliding with respect to the tray. This constraint can be expressed in linear form by approximating the i-th friction cone with a polyhedral cone generated by a finite set of unit vectors $\hat{f}_{c_i,j} \in \mathbb{R}^3$. The number of unit vectors $k \in \mathbb{N}_{>0}$ that constitute the approximated friction cone's edges is free to be picked. To approximate the friction cone with an inscribed pyramid, $k = 4$ is considered work [1]. The constraint is formulated expressing f_{c_i} as a non-negative linear combination of unit vectors $\hat{f}_{c_i,1} \dots \hat{f}_{c_i,k} \in \delta FC_i$, with δFC_i denoting the boundary of the i-th cone manifold.

By denoting $\Lambda_b = \begin{bmatrix} \lambda_{c_{1,1}}, ..., \lambda_{c_{n_c,k}} \end{bmatrix} \in \mathbb{R}^{kn_c}$ and $\hat{F}_c = \text{blockdiag}(\hat{F}_{c,1}, ..., \hat{F}_{c,n_c})$, with $\hat{F}_{c,i} = \begin{bmatrix} \hat{f}_{c_i,1}, ..., \hat{f}_{c_i,k} \end{bmatrix}$, the stacked vector of contact forces can be compactly rewritten as $F_c = \hat{F}_c \Lambda_b$. with $n_c > 0$ the number of considered contacts (e.g., $n_c = 4$ in the considered cases).

III. Optimal control problem

The optimisation problem has the following form

$$\underset{\zeta}{\text{minimize}} \quad f(\zeta) \tag{2}$$

$$\text{subject to} \quad A\zeta = b, \tag{3}$$

$$D\zeta \leq c. \tag{4}$$

1) Cost Function $f(\zeta)$: The desired body wrench $\mathcal{F}_b^\star$ can be obtained using an inverse dynamics control law, which can be derived from (1). A possible choice of the cost function is given by minimising the difference between the desired and the actual body wrench and the contact forces, that is $f(\zeta) = \|\mathcal{F}_b^\star - \mathcal{F}_b\|_H^2 + \|\Lambda_b\|^2$, where $H \in \mathbb{R}^{6\times 6}$ can be used to specify the relative weight. In this case, the chosen vector of control variables $\zeta = \begin{bmatrix} \mathcal{F}_b^T & \Lambda_b^T \end{bmatrix}^T \in \mathbb{R}^{6+3n_c}$. Given the calculated optimal $\mathcal{F}_b$ and the corresponding F_c, from Λ_b, the goal is, in turn, to find the manipulator generalised control forces that realise the desired end-effector/object motion [1].

Instead, when a quadruped robot with an arm is considered, the vector of control variables is $\zeta = \begin{bmatrix} \dot{v}^T & \Lambda_{gr}^T & \Lambda_b^T \end{bmatrix}^T \in \mathbb{R}^{6+n+kn_{st}+kn_c}$, with $v \in \mathbb{R}^{6+n}$ including the linear and angular velocity of the quadruped base and the velocity of the leg joints. The vector $\Lambda_{gr}^T \in \mathbb{R}^{kn_{st}}$, with $n_{st} > 0$ the number of stance legs, as Λ_b, is related to the friction cones at the stance legs. The optimisation can be defined as two objective functions aiming to track the desired wrench at the robot's CoM and the desired wrench at the object's CoM [2].

2) Equality constraints $A\zeta = b$: Equality constraints are present only in the quadruped robot case. Three equality constraints need to be imposed. The first constraints the control variable to be consistent with the legged system dynamic equations; the second guarantees that the contact of the stance feet is maintained; and the third imposes the velocity at the arm's end-effector to track the desired object motion [2].

3) Inequality constraints $D\zeta \leq c$: Object/tray contact forces must be constrained inside the friction cones and guarantee non-sliding behaviour. To this aim, it is sufficient to impose $\Lambda_b \geq 0$. In the case of a quadruped robot, this must hold for the ground reaction forces, too (i.e., $\Lambda_{gr} \geq 0$). In the mobile robot case, two other inequality constraints are imposed. The first considers that the joint torques need always to be limited. The second imposes to "almost" track the desired trajectory for the swing feet by introducing slack variables.

IV. Model predictive control

In this section, a nonlinear MPC approach is instead briefly introduced. The underlying optimisation problem is as in (2)-(4). The difference is that the optimal sequence of control inputs and the corresponding state trajectory are computed over a finite-length prediction horizon, subject to several constraints. Besides, only the first sample is applied before the controller runs again, and new control inputs are computed.

Let x be the system state and u be the control input. The cost function is optimised with respect to $\zeta = \begin{bmatrix} x^T & u^T \end{bmatrix}^T$. The cost function has the following expression $f(\zeta) = \|x_e^* - x_e\|_{Q_e}^2 + \sum_{i=0}^{e-1} \|x_{i+1}^* - x_{i+1}\|_{Q_i}^2 + \|u_i\|_{R_i}^2$, with $e > 0$ indicating the steps of the prediction horizon. It comprises two weighted two-norms: the state difference from the desired values x^* and the input u. The extended state vector reads as $x = \begin{bmatrix} \tau^T & q^T & \dot{q}^T & \Lambda_b^T \end{bmatrix}^T$, with q the manipulator joints and τ the related torques. The control input is instead $u = \dot{\tau}$, giving rise to a continuous torque profile, which constitutes the real input to the robotic system. The reference values, x^*, can be computed using a standard inverse kinematics routine assuming that the object is rigidly attached to the manipulator.

The equality constraints address the initial state definition and follow the dynamic evolution considering the mathematical modelling of the combined manipulator-object system and contact parameterised forces. The inequality constraints consider non-sliding manipulation condition and lower and upper bounds on joint positions, velocities, torques, contact force coefficients, system states, and control inputs.

V. Results

The developed code, the results, and the video for the non-prehensile object transportation with a manipulator (both optimal control and MPC) can be found at https://github.com/prisma-lab/nonprehensile-object-transp. Results, code, and video about the non-prehensile object transportation with a legged system are available at https://github.com/prisma-lab/legged-nonprehensile-manip.

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