1. Introduction

In the development of modern aircraft, aeroelastic problems have far-reaching effects upon structural and aerodynamic design. Aeroelastic effects are a result of the mutual interaction of elastic and inertial structural forces, i. e. aerodynamic forces induced by static or oscillatory deformations of the structure and external disturbance forces.

If structures, when exposed to an airstream were to remain perfectly rigid, aeroelastic problems would not exist.

Aircraft structures, however, are more or less flexible, and their flexibility is fundamentally responsible for the various types of aeroelastic phenomena. Questions of aeroelastic stability arise when structural deformations induce additional aerodynamic forces and these aerodynamic forces produce, in turn, additional structural deformations, which again will induce still greater aerodynamic forces.

Generally speaking, aeroelasticity is considered a branch of applied mechanics, which is concerned with the motion of a deformable body through an airstream. Although aeroelasticity had its origin in aircraft engineering, where aeroelastic problems turned out to be important since the beginning of powered flight, aeroelastic aspects have been gaining more importance in many other engineering branches, as well. There is a general trend to build greater, and thus more elastic constructions and buildings in civil engineering, turbomachinery and hydraulic plants, where aero- and hydro-elastic aspects appear increasingly more often as limiting factors. From this, it is apparent that the field of aeroelasticity is still in expansion.

Most destructive aeroelastic phenomena arise with aeronautical engineering constructions, and aeroelastic problems are generally accepted as problems of primary concern in the design of modern high-speed aircraft structures. Since World War II, aeroelastic considerations have become significantly more influential in the development of aircraft.
With the increase of flight speed and the necessary change of airframe geometry, as well as with the design and development of aerodynamically controlled flight vehicles, consistently new aeroelastic problems appeared and new solutions had to be found. In this introduction a survey and a classification of aeroelastic problems is presented. The interaction of the aeroelastic system forces is demonstrated by means of functional diagrams, and the application of aeroelastic operators for the formulation of aeroelastic problems is shown.

2. Aeroelastic Operators and Functional Diagrams

There are certain unifying features cornrnon to all aeroelastic problems. These features provide a convenient framework for introducing and classifying the entire subject. The features offer the possibility of diagramming aeroelastic phenomena through functional diagrams, and the casting of the governing aeroelastic equations in operator form.

2.1 Aeroelastic Functional Diagrarns

The rneaning of functional diagrams in aeroelasticity can be best illustrated by the action of an elastic wing, see Fig. 1.

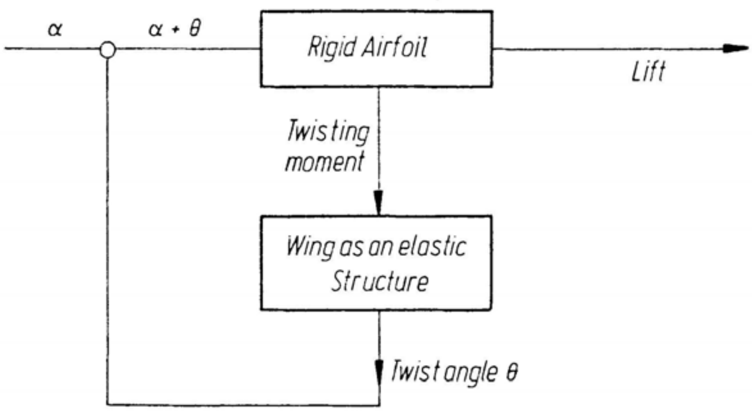

Fig. 1: Aeroelastic Action of an Elastic Wing

Statically, the basic "aero-elastic" action of a wing con be considered as being cornposed of two elements: the wing as a lift-producing mechanism, and the wing as an elastic structure. With respect to aerodynamics, each configuration of the wing can be considered rigid. With respect to the elastic deformation, the action of aerodynamic forces is the same as any other system of exterior forces. Thus, in a static aeroelastic analysis, the two solutions of the partial problems in steady aerodynamics and elasticity, which may be solved independently in the classical manner, must be properly combined to account for the "aeroelastic" behavior of the elastic wing. The airfoil, at an angle of attack α and having an elastic deflection ϑ, produces an aerodynamic pressure distribution which, in turn, produces an elastic deflection ϑ. Then the geometrical configuration may simply be written as $\alpha + \vartheta$, with respect to which the aerodynamic force is computed.
Thus, the rigid-airfoil-elastic-structure system forms a loop in the sense of a feedback system. If the deflection ϑ tends to become infinite, i.e. if the lift problem has no unique solution, the wing is said to be critically divergent.

In dynamic aeroelastic problems, the wing perfoms three distinct functions: it produces

1) unsteady airload,
2) inertia force and
3) elastic deforrnation.

The unsteady aerodynamic forces acting on the elastic airframe may stem from different origins, which can be classified as follows:

a) The airframe or its structural components moves unsteadily throught the undisturbed free atmosphere due to structural vibrations;
b) The flow around the aircraft is separated in some regions, for example on parts on the wing, thus creating a greater or lesser turbulent (and sometimes periodic) flow behind

the separation points. This unsteady flow may be transferred to the attached flow regions by induction effects;

c) The free atrnosphere is not undisturbed, but rather contains unsteady motions resulting in continuous atmospheric turbulence or impulsive-type gusts.

The mutual dynamic aeroelastic interaction of these different aerodynamic forces, and the structural motions of the elastic airframe can be illustrated through a functional diagram, as shown in Fig. 2:

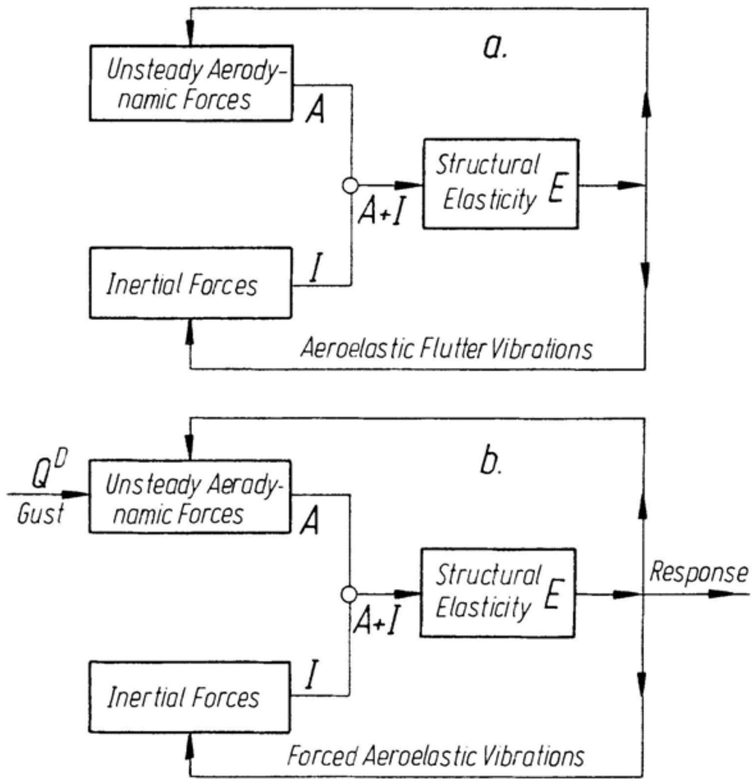

Fig. 2: Dynamic Aeroelastic Interaction

a) Closed Loop Flutter Instability
b) Open Loop Dynamic Response

Different dynamic aeroelastic phenomena can be attributed to each or several of the particular unsteady airloads resulting in undesirable closed or open loop structural vibrations. Fig. 2a illustrates the important case of the interaction of elastic, inertial and unsteady aerodynamic forces in a closed loop flutter instability. Closed loop aeroelastic flutter vibrations are of a self-excited nature, in which, in a linear system the motion-induced unsteady airloads (refer back to 2.1a) give rise to oscillatory amplitudes of increasing magnitude, which may become disastrous.

In contrast to this, open loop aeroelastic vibrations are of the forced type with finite response amplitudes. The gust response of an elastic wing represents an important example of a dynamic aeroelastic response problem.

2.2 Aeroelastic Operators

It is convenient to represent the airframe's triple functions - elastic, inertial and aerodynamic - as three independent operators, which can form the basis of an easier analytical treatment of aeroelasticity, and which may be employed as building blocks in the synthesis of aeroelastic problems. The majority of aeroelastic problems in aircraft design are concerned with continuous elastic bodies and with forces, which are distributed in one way or another over the body surface or throughout its volume. The operator forms the functional relation between the generalized displacement q, at a specified point and in a specified direction, and a distributed generalized force Q, which can be expressed symbolically by

$$Q = \mathscr{S}(q) \tag{1}$$

where $\mathscr{S}$ is a structural operator. In the simplest case this structural operator is the spring constant, which gives the functional relationship between the force on the spring and the resulting displacement. One can also conceive of an inverse structural operator, which may be denoted by $\mathscr{S}^{-1}$.

Then the symbolic relationship

$$q = \mathscr{S}^{-1}(Q) \tag{2}$$

represents one of the solutions, q, of Eq. (1).

Similarly, the functional relation between the deformation of the structure and the motion-induced aerodynamic forces can be represented with the symbolic operator forms

$$Q_A = \mathscr{A}(q)\ ,\ q = \mathscr{A}^{-1}(Q_A) \tag{3}$$

where $\mathscr{A}$ now is an aerodynamic operator and $\mathscr{A}^{-1}$ its inverse.

Since in aeroelasticity, the aerodynamic drag force and the skin friction (boundary layer effects) are generally neglected, the most important aerodynamic operator is concerned with the (steady and/or unsteady) aerodynamic lift and moment. The complexity of the aerodynamic operator is one of the major difficulty in aeroelastic analyses.

Finally, in dynamic aeroelastic problems, the relation between the displacement of the elastic system and the inertial forces, which arise as a result of the time variation of this (oscillatory) displacement, can be expressed symbolically by

$$Q_I = \mathscr{I}(q)\ ,\ q = \mathscr{I}^{-1}(Q_I) \tag{4}$$

where $\mathscr{I}$ is the inertial operator and $\mathscr{I}^{-1}$ its inverse.

Then, replacing the generalized force Q in Eq. (1) by the three individual forces Q_A, Q_I and Q_D where Q_D is an explicitely known external disturbance force independent of the system's displacement and motion, the following fundamental operator (equations of aeroelasticity) are obtained:

$$Q_A + Q_I + Q_D = \mathscr{A}(q) + \mathscr{I}(q) + Q_D = \mathscr{S}(q) \tag{5}$$

or

$$q = \mathscr{S}^{-1}(Q) = \mathscr{S}^{-1}\left[\mathscr{A}(q) + \mathscr{I}(q) + Q_D\right] \tag{6}$$

Equations (5) and (6) may be employed alternatively as a basis for forming aeroelastic problems. The choice depends on the exact nature of the problem to be solved.

The aeroelastic operators may be algebraic, differential, or integral operators, as well as combinations thereof. They may, in general, be nonlinear; but in the great majority of aeroelastic problems linearized operators are employed, because the assumption of (structural and aerodynamic) linearity is a necessity for obtaining analytical or numerical solutions.

In the case of linearity, Eauations (5) and (6) can be written in the respective forms: It can be said that Eq. (7) represents the most generul form of aeroelasticity of dynamic systems. Upon introduction of the analytical relations of the various operutors in Eq. (7) one obtains the goveming aeroelastic equations for the different types of aeroelastic problems.

3. Classification of Aeroelastic Problems

3.1 Collar's Aeroelastic Triangle of Forces

The various aeroelastic phenomena can best be classified by means of Collar's [1] well known triangle of forces, illustrated in Fig. 3.

The three types of forces: aerodynamic, elastic and inertial represented by the symbols A, E and I, respectively, are placed at the vertices of an equilateral triangle. Each aeroelastic phenomenon can be located in Fig. 3 and classified according to its relation to the three vertices. Thus static aeroelastic phenomena from the interaction of A and E lie outside the triangle on the upper left side, whereas dynamic aeroelastic phenomena lie within the triangle, since they involve all three kinds of forces. Static aeroelastic problems can generally be represented in operator form by equating the inertia operator in Eq. (5) with zero, which gives

$$\mathscr{S}(q) - \mathscr{A}(q) = Q_D \tag{7}$$

Equation (7) encompasses the phenomena of lifting surface static load distribution, control effectiveness, and control reversal . These static aeroelastic problems are concerned with the effect of elastic deformations of the airframe on the distribution of aerodynamic pressure over the structure and on the controllability and static flight stability of an airplane. The homogeneous form of Eq. (7)

$$\mathscr{S}(q) - \mathscr{A}(q) = 0 \tag{8}$$

represents the static stability phenomenon of lifting surface divergence, a very important static aeroelastic stability problem. The dynamic aeroelastic problems, located within the aeroelastic triangle of forces are generally represented in operator form by the complete Eq. (5).

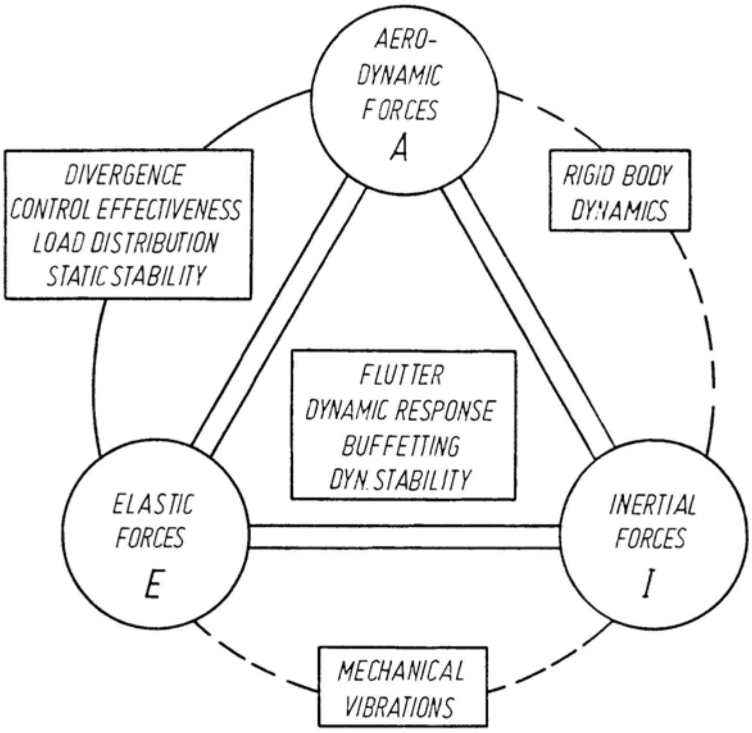

Fig. 3: The Aeroelastic Triangle of Forces

It includes within its scope the problems of the dynamic response of aircraft to gusts and other external disturbance forces, as well as to buffeting and forced vibrations in flight. The homogeneous form of Eq. (5), obtained by placing $Q_D = 0$,

$$\mathscr{S}(q) - \mathscr{A}(q) - \mathscr{I}(q) = 0 \tag{9}$$

represents a more restricted class of dynamic aeroelastic problems in which the dynamic stability of an aeroelastic system is the foremost example. In this class the important problems of flutter and dynamic stability of flight vehicles are included. But attention may be also directed in this context to those borderline fields which are connected to the vertices in Fig. 3 by dotted lines, namely the domain of mechanical vibrations and the domain of classical rigid-body dynamics. The first one is of basic importance for the analytical treatment of dynamic aeroelastic problems. On the other hand, it is very likely that in certain cases the dynamic flight stability problem is influenced by the structural flexibility. It would therefore be moved within the triangle where it would be regarded as a dynamic aeroelastic problem, taking into account the structural and aerodynamic coupling between the rigid body motions and elastic free body vibrations.

3.2 Garrick's Aerothermoelastic Tetrahedron

Aerodynamic thermal effects associated with high-speed flight vehicles may introduce deformations, stresses, and change in material properties, which can greatly extend the field of aeroelasticity. A new term has been introduced by Garrick [2] for this new field of aeroelasticity: Aerothermoelasticity.

Garrick ingeneously extended Collar's classical triangle of forces into a tetrahedron by including the effects of heat inputs, see Fig. 4.

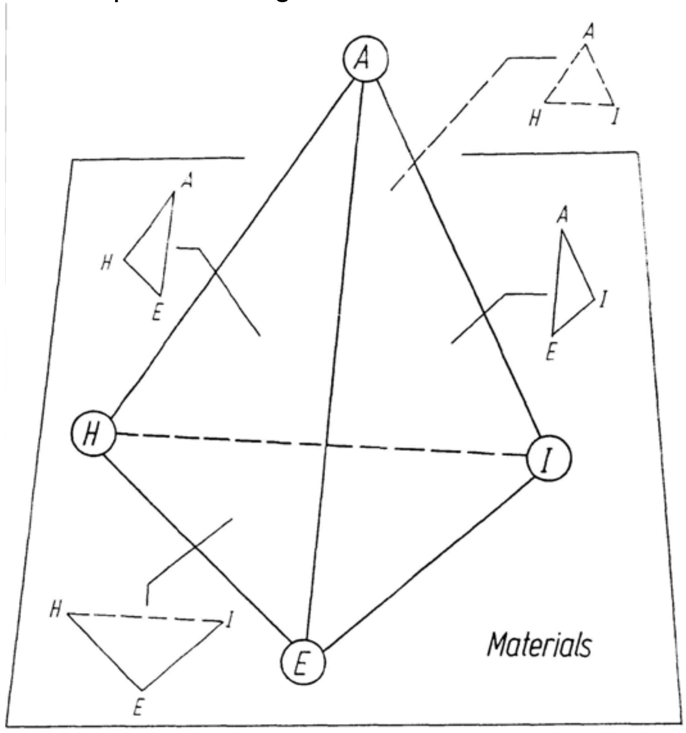

Fig. 4: Aeroelastic Tetrahedron illustrating the Interrelationships of Aeroelasticity

This aerothermoelastic tetrahedron provides an excellent background for discussion and classification of aerothermoelastic phenomena and makes apparent the broad interdisciplinary aspects of aerothermoelasticity.

For the four triangular surfaces, representing domains of interaction of the respective fields designated by their boundary links, we have:

A E I	Aeroelasticity	This is Collar's classical aeroelastic triangle
A H I	Stability and Heat	This covers the field of stability and control modified by thermal effects
A H E	Static Aerothermoelasticity	This represents the effects of heat on static aeroelasticity
H E I	Vibration and heat	Effects of heat on the vibration behavior of structures, on moduli of materials, on fatigue, and so on

A E I H = Aerothermoelasticity

The base of the aerothermoelastic tetrahedron vests on materials, not only in the sense of determination of their properties for given engineering materials, but also in the more fundamental sense of development of materials of desired properties. One can ask whether the problems of aerothermoelasticity lie near the surfaces and along the linear links, or whether they lie deep within the volume of the tetrahedron. This question is a significant one with respect to the separation or combining of problem areas, or more generally, with respect to whether problems can be treated sequentially by aerothermal, thermoelastic, and aeroelastic methods, by static or quasi-steady time-varying procedures, or by truely unsteady flow methods. The answers to this question depend on various time factors in the domain, on vehicle missions and corridors of flight, on structural concepts and detail design. In fact, aerothermoelastic problems can frequently be treated sequentially, since the time constants of the thermoelastic processes are usually considerably greater than those for the aerothermal and aeroelastic process. The structural designer is strongly motivated to plan to separate, as far as is possible, the severest regions of these three processes, which leads to a considerable reduction of computation work in practical analyses.

3.3 Aeroservoelasticity

With the application of power boosted servo-controls in modern aircraft, the interaction between an aircraft's structural dynamics, aerodynamics and automatic flight control system emerged as an important design consideration. This interaction has been termed "aeroservoelasticity" and i s illustrated in Fig. 5.

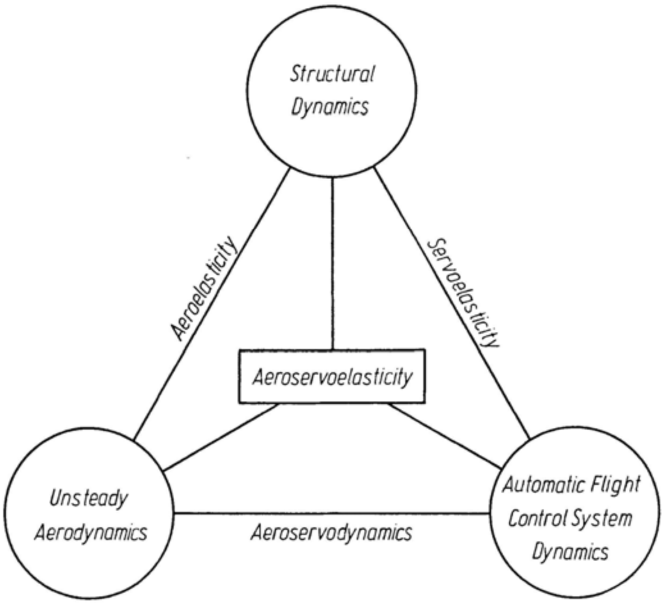

Fig. 5: Aeroservoelastic interaction triangle

In this aeroservoelastic interaction triangle classical dynamic aeroelastic problems are represented by the left leg of the interaction triangle (aeroelastic). The lower leg of the triangle (aeroservodynamic) represents classical rigid body flight control system synthesis. Finally, the right leg of the triangle (servoelastic) represents the interaction of structural dynamics and automatic flight control system dynamics, and all together the field of aeroservoelasticity. Historically, the importance of aeroservoelastic design considerations was caused by increases in the aircraft's structural efficiency, e.g., minimum weight to meet strength requirements, and by the increased use of high authority, high response automatic flight control systems. As aircraft designers strove to reduce the structural weight of each new aircraft, the flexibility of the structure necessarily increased. At the same time flight control developed new automatic flight control system functions, which improved aircraft performance, stability, service life, ride quality, etc. Unfortunately, these efforts went on more or less independently until the effects of these two trends overlapped, and significantly and frequentiy degrading interactions occured. Perhaps the most interesting aspect of aeroservoelasticity is the possibility of applying the aerodynamic active controls of flight vehicles for an active flutter suppression, and for an improvement of the ride quality, and for gust load alleviation. Active flutter suppression is a feedback control concept, in which the signal of a vibration transduces in electronically processed and fed as an input signal into the automatic control system such that this control surface is commanded to oscillate at the proper frequency creating aerodynamic loads, which interact with the corresponding lifting surface airloads in such a way that flutter is prevented.

4. Final Remarks

The emergence of aeroelastic problems may be said to have coincided almost exactly with the first achievement of powered flight. A large body of evidence can be brought forward to show that since World War II the interaction between aerodynamic loads and structural deformations gained considerable importance relative to many other factors affecting the design of aircraft and missiles. This was caused by the rapid in- crease in flight speed and the drastic changes in lifting surface geometry and in the application of new light weight construction concepts. The most constructive of the aeroelastician is therefore to find ways of avoiding, or of at least minimizing undesired aeroelastic interactions or, in a few cases, of putting these interaction to valuable use.