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Adaptive Radar Detection in the Presence of Missing-Data

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***Abstract*—This paper deals with the problem of adaptive radar detection in a context with missing-data where the complete observations (i.e., downstream information loss mechanisms) are characterized by homogeneous Gaussian disturbance with an unknown but possibly structured covariance matrix. The detection problem, formulated as a composite hypothesis test, is tackled by resorting to sub-optimal design strategies, leveraging the generalized likelihood ratio (GLR) criterion demanding appropriate maximum likelihood estimates (MLEs) of the unknowns under both hypotheses. Capitalizing on some possible a-priori knowledge about the interference covariance matrix structure, the optimization problems involved in the MLEs computation are handled by employing the expectation-maximization (EM) algorithm or its expectation-conditional maximization (ECM) and multi-cycle EM (M-EM) variants. At the analysis stage, the performance of the devised architectures is assessed both via Monte Carlo simulations and on measured data for some covariance matrix structures of practical interest.**

***Index Terms*—Adaptive Radar Detection, EM-algorithm, Missing Data**

I. Introduction

Adaptive radar detection has long been a popular and widely discussed topic in signal processing [1]–[6]. Significant efforts have been made in the last decades to conceive practical detectors for point-like targets embedded in additive Gaussian interference (due to for instance thermal noise plus clutter and/or jammers) with unknown spectral properties [7]–[9].

In multi-channel sensor array setting, most of the devised procedures have been designed under the ideal conditions that all the data at the output of the array would be available. Nevertheless, in practical radar systems, measurement errors due to acquisition equipment, random sensor failure [10] caused by impulsive noise [11], range ambiguous echo returns affecting useful signal samples [12], as well as reception failures (e.g., in distributed radar architecture [13], [14]), can determine the lack of some observations. It becomes mandatory to address the design of adaptive detection architectures capable of operating in such non-ideal conditions, accounting for the presence of missing-data [15]–[18]. To this end, several studies have addressed missing-data situations, as for instance [19] and [20] in the context of synthetic aperture radar (SAR), [21] for subspace tracking (subspace estimation from a data matrix corrupted by outliers and missing observations), and [22] for nonparametric complex spectral estimation. Besides, a structured covariance matrix estimator, accounting for missing-data and leveraging possible a-priori structural knowledge, has been recently proposed in [23].

In this paper, the problem of detecting a prospective target embedded in Gaussian interference with unknown (but possibly structured) covariance matrix is addressed for a missing-data context. It is assumed the existence of a secondary data set, i.e., returns free of useful target echoes gathered from range cells spatially adjacent to that under test. Complete primary and secondary data share the same interference second order statistics, which is tantamount to considering the so-called homogeneous environment [24]–[30].

The detection problem is formulated as a composite hypothesis test characterized by different unknowns under the two hypotheses. The presence of these unknowns precludes the implementation of the optimum solution (in the Neyman-Pearson sense), given by the likelihood ratio test (LRT). In detection theory jargon, this means that a uniformly most powerful (UMP) test does not exist and hence, to come up with practical detector, a sub-optimal generalized likelihood ratio test (GLRT) architecture is designed. Besides, a variation of the conventional GLRT, i.e., the adaptive matched filter (AMF) [3] test (also known as the two-step GLRT), is derived, too. In the absence of missing-data and for unstructured interference covariance matrix,the AMF receiver is commonly used in radar detection, due to its performance level comparable to Kelly's GLRT as well as its low computational complexity and higher robustness to useful signal mismatches [31]. The two-step GLRT only requires the maximum likelihood estimate (MLE) of the covariance matrix under the null hypothesis, which can be computed from secondary data, and the MLE of the complex target echo amplitude, assuming known the interference covariance matrix.

The devised detectors demand the optimization of appropriate observed-data likelihood functions over the unknowns (under one or both the hypotheses), for which closed-form solutions could not exist. Precisely, under the null hypothesis, the MLE of the interference covariance matrix is required. To this end, resorting to the expectation-maximization (EM) [32]–[34] framework, an effective iterative covariance estimator capable of managing diverse covariance structures is devised in [23]. On the other hand, the alternative hypothesis (with reference

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to the GLRT receiver) involves the joint ML estimation of the complex target echo parameter and the covariance matrix. To handle this challenging task, an EM-based framework is proposed to determine optimized solutions, with some quality guarantees, to the maximization problem at hand. The resulting procedure involves only closed-form expressions for the cases of unstructured estimation and covariance matrix with a centro-Hermitian structure. For the case of the covariance matrix with a more general structure, the plain EM leads to an optimization problem where closed-form solutions could not be available. Therefore, this case is addressed using two EM variations, i.e., the expectation-conditional maximization (ECM) and multi-cycle EM (M-EM) techniques, requiring the optimization of individual subsets of the unknowns at a given loop [35]–[37], each of them updated in closed-form for some covariance structures of practical interest.

At the analysis stage, the performance of the devised detectors is assessed in terms of probability of detection (P_D) versus the signal-to-interference-plus-noise ratio (SINR) on both simulated and measured data. This latter evaluation is critical for validating the robustness of the proposed detection strategies on real data, including potential mismatches (due to hardware imperfections) that are not taken into account at the design stage. For comparison purposes, two additional detectors are considered. The former is a benchmark that assumes direct access to the complete-data set, whereas the latter replaces the missing values via linear interpolation.

Summarizing, the main contributions of this paper are as follows.

- The development of adaptive architectures for target detection in the presence of missing-(complex) data, accounting for uncertainty sets of practical interest for radar signal processing applications[1]. Specifically, two adaptive detectors are derived, resorting to the GLR and AMF criteria, respectively. To the best of our knowledge, there are no similar works in the open literature addressing the target detection problem in the presence of missing-data and capitalizing on specific covariance matrix structures.
- The devised detectors demand the ML estimate of the complex target echo parameter α, which rules the expectation term under the $\mathcal{H}_1$ hypothesis, and the structured covariance matrix $\boldsymbol{M}$. Computing the estimate of the aforementioned unknowns, capitalizing on the general EM framework and encompassing possible covariance matrix structure, is one of the main technical contributions of the paper. In this regard, some attempts to estimate mean and covariance matrix in a context with missing-data can be found in the statistical literature [34], with reference to real valued observations/unknowns and without a structured mean. In contrast, this paper focuses on the estimation problem for the case of complex parameters and deals with a structured mean (due to the possible target echo return in the received signal) and a constrained covariance matrix. The proposed estimates are then exploited to address challenging radar detection problems.
- Finally, the performance of the proposed decision architectures are investigated both numerically and on measured data, assuming some covariance matrix uncertainty sets of practical relevance.

[1] In radar signal processing, the geometry (for instance uniform linear array or other regular array structures) and the characteristics of the acquisition system (for instance knowledge of the thermal noise level, or the available number of bits that induces an upper bound to the covariance condition number), impose specific structures of the covariance matrix.

The paper is organized as follows. The data model and target detection problem with missing-data are presented in Section II. In Section III, the detection problem is addressed resorting to sub-optimal design criteria, i.e., one-step and two-step GLR, which demand the optimization of appropriate observed-data likelihood functions under the two hypotheses. Hence, in Section III, an EM-based framework is also devised to tackle the resulting optimization problems and derive practical detectors. The performance of the mentioned detectors is analyzed in Section IV, whereas conclusions are drawn in Section V.

A. Notation

Boldface is used for vectors $\boldsymbol{a}$ (lower case), and matrices $\boldsymbol{A}$ (upper case). The (k,l)-entry (or l-entry) of a generic matrix $\boldsymbol{A}$ (or vector $\boldsymbol{a}$) is indicated as $\boldsymbol{A}(k,l)$ (or $\boldsymbol{a}(l)$). $\boldsymbol{I}$ and $\boldsymbol{0}$ denote respectively the identity matrix and the square matrix with zero entries (their size is determined from the context). Besides, $\mathbf{diag}(\boldsymbol{x})$ indicates the diagonal matrix whose i-th diagonal element is $\boldsymbol{x}(i)$. $\mathbb{R}^N$, $\mathbb{C}^N$, and $\mathbb{H}^N$ are respectively the sets of N-dimensional column vectors of real numbers, of N-dimensional column vectors of complex numbers, and of $N \times N$ Hermitian matrices. The transpose, the conjugate, and the conjugate transpose operators are denoted by the symbols $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$, and $(\cdot)^\dagger$, respectively. The determinant and the trace of the matrix $\boldsymbol{A} \in \mathbb{C}^{N\times N}$ are indicated with $\det(\boldsymbol{A})$ and $\mathrm{tr}\{\boldsymbol{A}\}$, respectively. The curled inequality symbol $\succeq$ (and its strict form $\succ$) is used to denote generalized matrix inequality: for any $\boldsymbol{A} \in \mathbb{H}^N$, $\boldsymbol{A} \succeq \boldsymbol{0}$ means that $\boldsymbol{A}$ is a positive semi-definite matrix ($\boldsymbol{A} \succ \boldsymbol{0}$ for positive definiteness). The letter j represents the imaginary unit (i.e., $j = \sqrt{-1}$). For any complex number x, $\Re(x)$, $\Im(x)$, and $|x|$ are used to denote the real part, the imaginary part, and the modulus of x, respectively. Furthermore, for any $x, y \in \mathbb{R}$, $\max(x,y)$ returns the maximum between the two argument values. Finally, $\mathbb{E}[\cdot]$ stands for statistical expectation.

II. Problem Formulation

Let us consider a radar system collecting spatial data via a linear array composed of N antennas and operating in the presence of noise and interference, with unknown spectral characteristics.

Under the ideal conditions of complete access to the set of space-time observations, the problem of detecting a prospective target located at range R and elevation θ_0 with respect to the array boresight (under the narrowband radar probing signal assumption), can be formulated as the following composite binary hypothesis testing problem

$$\begin{cases} \mathcal{H}_0 : \begin{cases} \boldsymbol{r} = \boldsymbol{n} \\ \boldsymbol{r}_i = \boldsymbol{n}_i, \quad i = 1, \dots, K \end{cases} \\ \mathcal{H}_1 : \begin{cases} \boldsymbol{r} = \alpha \boldsymbol{p} + \boldsymbol{n} \\ \boldsymbol{r}_i = \boldsymbol{n}_i, \quad i = 1, \dots, K \end{cases} \end{cases} \tag{1}$$

where $\boldsymbol{r}$ is the primary data, α is an unknown complex parameter which accounts for the target reflectivity and the channel propagation effects, whereas $\boldsymbol{p}$ denotes the spatial steering vector evaluated at θ_0, which is assumed known at the design stage. Besides, a set of secondary data $\boldsymbol{r}_i,\ i = 1, \dots, K$, free of the useful signal and with the same covariance matrix as the primary data (homogeneous environment) [2], [3], [38], [39], is supposed available. The interference plus noise components $\boldsymbol{n}$ and $\boldsymbol{n}_i,\ i = 1, \dots, K$, are modeled as independent and identically distributed (iid) zero-mean circularly symmetric Gaussian random vectors, with unknown (but possibly structured) covariance matrix given by

$$\boldsymbol{M}(\boldsymbol{\theta}) = \mathbb{E}[\boldsymbol{n}\boldsymbol{n}^\dagger] = \mathbb{E}[\boldsymbol{n}_i\boldsymbol{n}_i^\dagger], \quad i = 1, \dots, K \tag{2}$$

where $\boldsymbol{\theta}$ denotes the vector of the unknowns parameterizing the structure of $\boldsymbol{M}$.

Let us now frame the detection problem in a context with missing-data caused by random failures of some array elements [10], [11], [40], [41] or possible transmission-reception faults experienced by distributed radar systems (DRS) [13] or wirelessly networked aperstructure digital phased array radars (WNADPARs) [14], etc. For the case at hand, the observed primary data is modeled as

$$\boldsymbol{z} = \boldsymbol{A}\boldsymbol{r} \tag{3}$$

where $\boldsymbol{A}$ is an appropriate $p \times N$ selection matrix; specifically, denoting by $\kappa_1, \kappa_2, \dots, \kappa_{N-p} \in \{1, \dots, N\}$ the indices of the channels where a missing-data occurs in the snapshot from the cell under test (CUT), $\boldsymbol{A}$ is obtained from the $N \times N$ identity matrix, by deleting the rows indexed by $(\kappa_1, \kappa_2, \dots, \kappa_{N-p})$. Similarly, each secondary observed snapshot can be modeled as

$$\boldsymbol{z}_i = \boldsymbol{A}_i\boldsymbol{r}_i,\ i = 1, \dots, K \tag{4}$$

with $\boldsymbol{A}_i$ the $p_i \times N$ selection matrix of the i-th snapshot which is defined similarly to $\boldsymbol{A}$. In the following, the vectors $\boldsymbol{r}, \boldsymbol{r}_i,\ i = 1, \dots, K$, and $\boldsymbol{z}, \boldsymbol{z}_i,\ i = 1, \dots, K$, will be referred to as the *complete* and the *observed* data, respectively. Accordingly, the variables p and p_i indicate the number of the actual available channels, i.e., the number of observed elements, in the primary $\boldsymbol{r}$ and the i-th secondary snapshot $\boldsymbol{r}_i,\ i = 1, \dots, K$, respectively[2].

Hence, leveraging the observed-data model in (3) and (4), the target detection problem in the presence of missing-data can be cast as

$$\begin{cases} \mathcal{H}_0 : \begin{cases} \boldsymbol{z} = \boldsymbol{A}\boldsymbol{n} \\ \boldsymbol{z}_i = \boldsymbol{A}_i\boldsymbol{n}_i, \quad i = 1, \dots, K \end{cases} \\ \mathcal{H}_1 : \begin{cases} \boldsymbol{z} = \alpha \boldsymbol{A}\boldsymbol{p} + \boldsymbol{A}\boldsymbol{n} \\ \boldsymbol{z}_i = \boldsymbol{A}_i\boldsymbol{n}_i, \quad i = 1, \dots, K \end{cases} \end{cases} \tag{5}$$

where the unknowns are $\boldsymbol{\theta}$ under $\mathcal{H}_0$ and α, $\boldsymbol{\theta}$ under $\mathcal{H}_1$.

III. Design of Decision Rules

Pursuing the classical approach based on the Neyman-Pearson criterion, the optimal decision statistic to the hypotheses testing problem (5), i.e., maximizing P_D for a given P_{FA}, could be devised. Unluckily, the resulting LRT, relies on the complete knowledge of the probability density functions (PDFs) under both hypotheses which requires a perfect knowledge of the parameters α and $\boldsymbol{\theta}$, reasonably not available in practical situations. As a result, a UMP test for the aforementioned problem does not exist. Hence, practically implementable receivers have to be designed resorting to sub-optimal criteria, such as GLR, which demands the ML estimation of the parameters under both hypotheses.

In this respect, it is worth pointing out that the existence of affordable low-complexity optimal solutions to the optimization problems involved in the estimation process under the two hypotheses is essential for the design of practically implementable detectors. Unfortunately, quite often, closed-form solutions are not available [42].

In light of the above considerations, the EM framework represents a viable means to determine approximated MLE of the parameters from the observed-data. Specifically, it alternates between an expectation (E)-step (in which the conditional expectation of the more analytically tractable complete-data likelihood is evaluated using the current estimate of the parameters) and a maximization (M)-step, in which the E-step score function is optimized in order to update the estimates. The above steps are then repeated until a convergence condition is achieved.

An iterative EM-based solution $\hat{\boldsymbol{\theta}}_{EM,0}$ to the optimization problem under $\mathcal{H}_0$ has been devised in [23]. More specifically, accounting for some possible a-priori knowledge on the covariance matrix structure, estimator devised in [23] involves only closed-form updates, at each iteration of the procedure, for a wide class of covariance structures. Detailed analysis on the convergence properties, as well as on the convergence rate, are also provided in [23].

The work in [23], however, does not address the optimization problem under $\mathcal{H}_1$, which is necessary to solve the detection problem (5). This challenging step is here addressed, yielding an EM-based framework for the joint estimation of α and $\boldsymbol{M}$. From an optimization theory point of view this represents the main innovation of this paper.

A. Parameters estimation under $\mathcal{H}_1$

The EM procedure starts with an initial guess of the parameters, i.e., $\bar{\boldsymbol{\theta}}^{(0)} = [\alpha^{(0)}, \boldsymbol{\theta}^{(0)\mathrm{T}}]^{\mathrm{T}}$, and iterates between the E-step and the M-step, until convergence [32]. Specifically,

[2] From a physical point of view, the selection matrices $\boldsymbol{A}$ and $\boldsymbol{A}_i$, associated with $\boldsymbol{r}$ and $\boldsymbol{r}_i,\ i = 1, \dots, K$, respectively, provide the components of the complete-data vectors into the observed-data space. As a result, the number of rows p (p_i) of the selection matrix $\boldsymbol{A}$ ($\boldsymbol{A}_i$) denotes the dimension of the complex space where the observed-data in the considered (i-th secondary) snapshot lies.

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at the h-th iteration, the E-step involves the evaluation of the score function

$$Q\left(\alpha,\boldsymbol{\theta}|\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)}\right) = \mathbb{E}[\mathcal{L}_{\boldsymbol{r}}(\alpha,\boldsymbol{\theta},\mathcal{H}_1)|\boldsymbol{z},\boldsymbol{Z},\boldsymbol{A},\boldsymbol{A}_1,\ldots,\boldsymbol{A}_K,\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1] \tag{6}$$

where

- $\boldsymbol{Z}=\{\boldsymbol{z}_1,\ldots,\boldsymbol{z}_K\}$ is the set of observed secondary data;
- $\alpha^{(h-1)}$ and $\boldsymbol{\theta}^{(h-1)}$ are the estimates at the $(h-1)$-th iteration;
- $\mathcal{L}_{\boldsymbol{r}}(\alpha,\boldsymbol{\theta},\mathcal{H}_1)$ is the complete-data log-likelihood given by

$$\begin{aligned}\mathcal{L}_{\boldsymbol{r}}&(\alpha,\boldsymbol{\theta},\mathcal{H}_1)\\ &=-(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]\\ &\quad-\operatorname{tr}\left\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}[(\boldsymbol{r}-\alpha\boldsymbol{p})(\boldsymbol{r}-\alpha\boldsymbol{p})^\dagger+\boldsymbol{S}]\right\}\end{aligned} \tag{7}$$

- $\boldsymbol{S}=\sum_{i=1}^K \boldsymbol{r}_i\boldsymbol{r}_i^\dagger$ is proportional, via K, to the conventional secondary data sample covariance matrix.

Computing the conditional expectation involved in (6) yields (see Appendix A for details on the statistical expectation evaluation)

$$\begin{aligned}Q&\left(\alpha,\boldsymbol{\theta}|\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)}\right)\\ &=-(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]\\ &\quad-\operatorname{tr}\left\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}[(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})^\dagger+\boldsymbol{\Sigma}^{(h-1)}]\right\}\end{aligned} \tag{8}$$

where (the detailed expression is provided in (46) and (47))

$$\boldsymbol{\mu}^{(h-1)}=\mathbb{E}[\boldsymbol{r}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1] \tag{9}$$

and (see (48)-(56) for the detailed derivation)

$$\begin{aligned}\boldsymbol{\Sigma}^{(h-1)}=&\left(\sum_{i=1}^K\mathbb{E}[\boldsymbol{r}_i\boldsymbol{r}_i^\dagger|\boldsymbol{z}_i,\boldsymbol{A}_i,\boldsymbol{\theta}^{(h-1)}]\right)\\ &-\boldsymbol{\mu}^{(h-1)}\boldsymbol{\mu}^{(h-1)\dagger}+\mathbb{E}[\boldsymbol{r}\boldsymbol{r}^\dagger|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1].\end{aligned} \tag{10}$$

After the computation of the E-step, the M-step is performed, i.e., the score function (8) is maximized providing the following updated estimate of the unknowns

$$\left(\alpha^{(h)},\boldsymbol{\theta}^{(h)}\right)=\underset{\alpha,\,\boldsymbol{\theta}:\boldsymbol{M}(\boldsymbol{\theta})\in\mathcal{C}}{\operatorname{argmax}}\,Q\left(\alpha,\boldsymbol{\theta}|\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)}\right). \tag{11}$$

Still, as in the $\mathcal{H}_0$ case [23], different solution strategies to the optimization problem (11) are connected to diverse feasible sets $\mathcal{C}$. In this regard, some relevant cases of interest are analyzed in the following.

1) Unconstrained estimation: For this special and relevant case, the optimal solution to the M-step is available in closed-form, i.e., [2]

$$\alpha^{(h)}=\frac{\boldsymbol{p}^\dagger\,[\boldsymbol{\Sigma}^{(h-1)}]^{-1}\,\boldsymbol{\mu}^{(h-1)}}{\boldsymbol{p}^\dagger\,[\boldsymbol{\Sigma}^{(h-1)}]^{-1}\,\boldsymbol{p}} \tag{12}$$

and

$$\boldsymbol{M}(\boldsymbol{\theta}^{(h)})=\frac{(\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})^\dagger+\boldsymbol{\Sigma}^{(h-1)}}{K+1}. \tag{13}$$

2) Centro-Hermitianity constraint: Centro-Hermitian is a particular matrix structure, commonly satisfied by covariance matrices encountered in many radar signal processing applications, e.g., radar systems utilizing standard rectangular, hexagonal, uniform circular, or cylindrical array [33].

Enforcing this structure is tantamount to considering $\boldsymbol{M}$ belonging to the following constraint set [43]

$$\mathcal{C}=\begin{cases}\boldsymbol{M}=\boldsymbol{J}\boldsymbol{M}^*\boldsymbol{J}\\ \boldsymbol{M}\succeq\boldsymbol{0}\end{cases} \tag{14}$$

with $\boldsymbol{J}$ the $N\times N$ permutation matrix given by

$$\boldsymbol{J}=\begin{bmatrix}0&0&\cdots&0&1\\0&0&\cdots&1&0\\ \vdots&\vdots&\ddots&\vdots&\vdots\\1&0&\cdots&0&0\end{bmatrix}. \tag{15}$$

Following the same line of reasoning as in [44] and modeling $\boldsymbol{p}$ as a persymmetric vector [45], i.e., $\boldsymbol{p}=\boldsymbol{J}\boldsymbol{p}^*$, the E-step (8) can be recast as

$$\begin{aligned}Q&\left(\alpha,\boldsymbol{\theta}|\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)}\right)\\ &=-(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]\\ &\quad-\operatorname{tr}\left\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}[\boldsymbol{P}+\boldsymbol{\Sigma}_{FB}^{(h-1)}]\right\}\end{aligned} \tag{16}$$

where

$$\begin{aligned}\boldsymbol{P}=\frac{1}{2}&\Big[(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})^\dagger\\ &+\boldsymbol{J}\left((\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})^\dagger\right)^*\boldsymbol{J}\Big]\end{aligned} \tag{17}$$

and

$$\boldsymbol{\Sigma}_{FB}^{(h-1)}=\frac{1}{2}\left[\boldsymbol{\Sigma}^{(h-1)}+\boldsymbol{J}\left(\boldsymbol{\Sigma}^{(h-1)}\right)^*\boldsymbol{J}\right] \tag{18}$$

which plays the role of a forward-backward (FB) averaged estimator [33].

Expression (16) allows the computation of the optimizers for the M-step in closed form as

$$\alpha^{(h)}=\frac{\boldsymbol{p}^\dagger\,[\boldsymbol{\Sigma}_{FB}^{(h-1)}]^{-1}\,\boldsymbol{\mu}^{(h-1)}}{\boldsymbol{p}^\dagger\,[\boldsymbol{\Sigma}_{FB}^{(h-1)}]^{-1}\,\boldsymbol{p}} \tag{19}$$

and

$$\hat{\boldsymbol{M}}(\boldsymbol{\theta}^{(h)})=\frac{\boldsymbol{P}^{(h)}+\boldsymbol{\Sigma}_{FB}^{(h-1)}}{K+1} \tag{20}$$

with

$$\begin{aligned}\boldsymbol{P}^{(h)}=\frac{1}{2}&\Big[(\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})^\dagger\\ &+\boldsymbol{J}\left((\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha^{(h)}\boldsymbol{p})^\dagger\right)^*\boldsymbol{J}\Big].\end{aligned} \tag{21}$$

3) General structured covariance matrix: For the case of an arbitrary constraint set $\mathcal{C}$, closed-form expressions for the joint estimation of α and $\boldsymbol{\theta}$, as involved in each M-step, could not be available. To this end, variations of the plain EM strategy are demanded, for which the resulting update step is easier to handle. Two relevant methods are analyzed in the following, i.e., the ECM and M-EM [35], [36] which turn out to be very useful when the marginal optimization of the E-step score function over a single or sub-groups of unknowns can be

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conducted in closed form. In the former, which is a particular GEM algorithm [35], [37], the E-step is given by (8), while the M-step demands a sequence of conditional maximizations (CM), where in each of them a parameter is optimized while the others are held fixed. Formally, the CM over α is cast as

$$\alpha^{(h)} = \underset{\alpha}{\operatorname{argmax}}\, Q\left(\alpha, \boldsymbol{\theta}^{(h-1)} \mid \alpha^{(h-1)}, \boldsymbol{\theta}^{(h-1)}\right) \quad (22)$$

whose solution is given by (12), whereas

$$\boldsymbol{\theta}^{(h)} = \underset{\boldsymbol{\theta}:\boldsymbol{M}(\boldsymbol{\theta})\in\mathcal{C}}{\operatorname{argmax}}\, Q\left(\alpha^{(h)}, \boldsymbol{\theta} \mid \alpha^{(h-1)}, \boldsymbol{\theta}^{(h-1)}\right) \quad (23)$$

is the CM related to the parameter vector $\boldsymbol{\theta}$.

As to the M-EM procedure, each iteration consists of several "cycles", each focused on the optimization over a single parameter. Specifically, in the m-th cycle of the i-th iteration, the m-th variable is updated by maximizing the E-step score function computed with respect to the currently available parameters estimate. For the problem at hand, two cycles are considered. In the first, the E-step is given by evaluating the score function (6) at the point $(\alpha, \boldsymbol{\theta}^{(h-1)})$, i.e.,

$$\begin{aligned} &Q\left(\alpha, \boldsymbol{\theta}^{(h-1)} | \alpha^{(h-1)}, \boldsymbol{\theta}^{(h-1)}\right) = \\ &\mathbb{E}[\mathcal{L}_{\boldsymbol{r}}(\alpha, \boldsymbol{\theta}^{(h-1)}, \mathcal{H}_1) | \boldsymbol{z}, \boldsymbol{Z}, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \alpha^{(h-1)}, \boldsymbol{\theta}^{(h-1)}, \mathcal{H}_1] \end{aligned} \quad (24)$$

whereas the M-step yields

$$\alpha^{(h)} = \underset{\alpha}{\operatorname{argmax}}\, Q\left(\alpha, \boldsymbol{\theta}^{(h-1)} | \alpha^{(h-1)}, \boldsymbol{\theta}^{(h-1)}\right) \quad (25)$$

with the optimal solution provided by (12).

As to the second, the E-step is cast as the score function (6) evaluated at the point $(\alpha^{(h)}, \boldsymbol{\theta})$ given the knowledge of $\alpha^{(h)}$ and $\boldsymbol{\theta}^{(h-1)}$, i.e.,

$$\begin{aligned} &Q\left(\alpha^{(h)}, \boldsymbol{\theta} \mid \alpha^{(h)}, \boldsymbol{\theta}^{(h-1)}\right) = \\ &\mathbb{E}[\mathcal{L}_{\boldsymbol{r}}(\alpha^{(h)}, \boldsymbol{\theta}, \mathcal{H}_1) | \boldsymbol{z}, \boldsymbol{Z}, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \alpha^{(h)}, \boldsymbol{\theta}^{(h-1)}, \mathcal{H}_1] \end{aligned} \quad (26)$$

while the M-step is given by

$$\boldsymbol{\theta}^{(h)} = \underset{\boldsymbol{\theta}:\boldsymbol{M}(\boldsymbol{\theta})\in\mathcal{C}}{\operatorname{argmax}}\, Q\left(\alpha^{(h)}, \boldsymbol{\theta} \mid \alpha^{(h)}, \boldsymbol{\theta}^{(h-1)}\right). \quad (27)$$

A case study is analyzed in the following.

a) Constraint on the lower bound of the white noise power level: Let us consider $\boldsymbol{M}$ belonging to the uncertainty set

$$\mathcal{C} = \begin{cases} \boldsymbol{M} = \sigma_n^2 \boldsymbol{I} + \boldsymbol{R}_e \\ \boldsymbol{R}_e \succeq \boldsymbol{0} \\ \sigma_n^2 \geq \sigma^2 \end{cases} \quad (28)$$

where $\boldsymbol{R}_e$ accounts for colored interference and clutter, $\sigma_n^2 > 0$ is the power of the white disturbance term, and $\sigma^2 > 0$ is a known lower bound on σ_n^2.

Denoting by $\boldsymbol{U}\boldsymbol{\Lambda}_{\boldsymbol{\Sigma}}\boldsymbol{U}^\dagger$ the eigenvalue decomposition (EVD) of

$$\boldsymbol{\Sigma}_1^{(h-1)} = \frac{(\boldsymbol{\mu}^{(h-1)} - \alpha^{(h)}\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)} - \alpha^{(h)}\boldsymbol{p})^\dagger + \boldsymbol{\Sigma}^{(h-1)}}{K+1}$$

and by $\tilde{\lambda}_v,\ v = 1, \ldots, N$, its eigenvalues, the Fast ML (FML) procedure [39], [46] provides the solution to the optimization problems (23) and (27), i.e.,

$$\hat{\boldsymbol{M}}(\boldsymbol{\theta}^{(h)}) = \boldsymbol{U}\boldsymbol{\Lambda}_{FML}\boldsymbol{U}^\dagger \quad (29)$$

with

$$\boldsymbol{\Lambda}_{FML} = \textbf{diag}(\lambda_{1,FML}, \ldots, \lambda_{N,FML}) \quad (30)$$

and $\lambda_{v,FML} = \max(\tilde{\lambda}_v,\ \sigma^2),\ v = 1, \ldots, N$.

Remark: Unitary invariant constraints. In many practical cases, the covariance matrix belongs to the feasible set of covariance matrices defined via unitary invariant continuous functions of the matrix entries [47]. Interestingly, many of these uncertainty sets can be described in terms of convex functions of the covariance matrix eigenvalues, paving the way for the development of tailored solutions to the ML estimation problems (23) and (27). However, even for some non-convex uncertainty sets, efficient algorithms can still be derived [47]. Uncertainty sets defined via unitary invariant functions encompass those resulting from an upper bound on the covariance condition number or a constraint on the maximum number of uncorrelated interfering sources, just to mention a few [47].

B. Decision rules

This subsection provides practical detectors stemming from[3] the GLR [2] and AMF [3] design criteria. Specifically, the following detectors are considered:

1) GLRT detector

$$\tau_{\text{GLRT-EM}} = \frac{f_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_1, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_1)}{f_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_0, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_0)} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} T \quad (31)$$

where $f_{\boldsymbol{z}}(\cdot)$ represents the likelihood function of the observations (under the appropriate hypothesis), $\bar{\boldsymbol{\theta}}_0 = [0, \hat{\boldsymbol{\theta}}_{EM,0}]$, $\bar{\boldsymbol{\theta}}_1 = [\hat{\alpha}_{EM}, \hat{\boldsymbol{\theta}}_{EM,1}]$ with $\hat{\boldsymbol{\theta}}_{EM,0}$ and $\hat{\boldsymbol{\theta}}_{EM,1}$ the estimates of $\boldsymbol{\theta}$ under $\mathcal{H}_0$ and $\mathcal{H}_1$, respectively, provided by the bespoke EM-based procedures, and T is an appropriate detection threshold[4] set to ensure a desired P_{fa}. Equation (31) is statistically equivalent to

$$\begin{aligned} \tau_{\text{GLRT-EM}} =& \mathcal{L}_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_1, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_1) \\ &- \mathcal{L}_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_0, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_0) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} T \end{aligned} \quad (32)$$

where, for $h = 0, 1$,

$$\begin{aligned} &\mathcal{L}_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_h, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_h) \\ &= \log f_{\boldsymbol{z}}(\boldsymbol{z}, \boldsymbol{Z}; \bar{\boldsymbol{\theta}}_h, \boldsymbol{A}, \boldsymbol{A}_1, \ldots, \boldsymbol{A}_K, \mathcal{H}_h) \\ &= -\left(p + \sum_{i=1}^{K} p_i\right)\ln(\pi) - \ln(\det(\boldsymbol{A}\boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{A}^{\mathrm{T}})) \\ &\quad - \operatorname{tr}\{(\boldsymbol{A}\boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{A}^{\mathrm{T}})^{-1}\boldsymbol{C}_h\} \\ &\quad - \sum_{i=1}^{K}\left[\ln(\det(\boldsymbol{A}_i\boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{A}_i^{\mathrm{T}}))\right. \\ &\qquad \left. + \operatorname{tr}\{(\boldsymbol{A}_i\boldsymbol{M}(\boldsymbol{\theta})\boldsymbol{A}_i^{\mathrm{T}})^{-1}\boldsymbol{z}_i\boldsymbol{z}_i^\dagger\}\right] \end{aligned} \quad (33)$$

[3]Notice that other sub-optimal criteria, such as Rao [25] and Wald [48] tests, can be pursued as well.

[4]With a slight abuse of notation, the same symbol is used to denote the detection threshold and its possible modifications introduced later, see, e.g., (32) and (34).

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with $\boldsymbol{C}_0 = \boldsymbol{z}\boldsymbol{z}^\dagger$ and $\boldsymbol{C}_1 = (\boldsymbol{z} - \hat{\alpha}_{EM}\boldsymbol{A}\boldsymbol{p})(\boldsymbol{z} - \hat{\alpha}_{EM}\boldsymbol{A}\boldsymbol{p})^\dagger$.

2) AMF counterpart to (31), also referred to as two-step GLRT (it computes the GLRT of the observed primary data over the parameter α and then substitutes in the resulting GLRT the estimate of the covariance parameters obtained from the secondary data)

$$\tau_{\text{AMF-EM}} = \frac{\left| \boldsymbol{z}^\dagger \left(\boldsymbol{A}\boldsymbol{M}(\hat{\boldsymbol{\theta}}_{EM,2})\boldsymbol{A}^{\mathrm{T}} \right)^{-1} \boldsymbol{A}\boldsymbol{p} \right|^2}{\boldsymbol{p}^\dagger \boldsymbol{A}^{\mathrm{T}} \left(\boldsymbol{A}\boldsymbol{M}(\hat{\boldsymbol{\theta}}_{EM,2})\boldsymbol{A}^{\mathrm{T}} \right)^{-1} \boldsymbol{A}\boldsymbol{p}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} T \quad (34)$$

where $\hat{\boldsymbol{\theta}}_{EM,2}$ indicates the estimate of $\boldsymbol{\theta}$ obtained via the EM algorithm fed by secondary data only.

IV. Performance Analysis

In this section, the effectiveness of the detectors devised to counteract the presence of missing-data is assessed. Specifically, the observables (subjected to downstream information loss mechanisms) are considered gathered by a radar system employing a uniform linear array (ULA) pointing at the boresight direction ($\theta_0 = 0$). The array comprises $N = 16$ antennas, unless otherwise stated, separated by $d_x = \lambda_0/2$, with λ_0 the radar operating wavelength. For each complete-data snapshot, the information provided by the output of $L = 3$ randomly selected channels is assumed missed. Therefore, each selection matrix (including $\boldsymbol{A}$ of the primary data and $\boldsymbol{A}_i$ of the i-th secondary data snapshot, $i = 1, \dots, K$) is constructed removing, independently from the other snapshots, L rows from the identity matrix, with the subset of L indexes randomly picked up from $\{1, \dots, N\}$ without replacement.

The performance of the detectors is analyzed in terms of P_D estimated via standard Monte Carlo counting techniques over 10^4 independent trials. Besides, the detection thresholds of the receivers are set to guarantee $P_{fa} = 10^{-4}$ and are evaluated using $100/P_{fa}$ independent Monte Carlo trials.

In the reported case studies, the disturbance covariance matrix is modeled as $\boldsymbol{M} = \boldsymbol{M}_J + \sigma_a^2\boldsymbol{I}$, where σ_a^2 is the noise power level (assumed without loss of generality equal to 0 dB) and

$$\boldsymbol{M}_J = \sum_{l=1}^{J_{NB}} \sigma_l^2\, \boldsymbol{p}(\theta_l)\boldsymbol{p}(\theta_l)^\dagger \quad (35)$$

is the covariance contribution of J_{NB} uncorrelated narrow-band jammers, with

$$\boldsymbol{p}(\theta_l) = [1, e^{j\frac{2\pi}{\lambda_0}d_x\sin(\theta_l)}, \dots, e^{j(N-1)\frac{2\pi}{\lambda_0}d_x\sin(\theta_l)}]^{\mathrm{T}} \in \mathbb{C}^N \quad (36)$$

the steering vector in the direction θ_l of the l-th jammer and σ_l^2 the power, or jammer to noise ratio (JNR) σ_l^2/σ_a^2, of the l-th jammer. Moreover, the complete-data SINR is defined as [5]

$$\text{SINR} = |\alpha|^2\boldsymbol{p}^\dagger\boldsymbol{M}^{-1}\boldsymbol{p}. \quad (37)$$

Finally, denoting by $\tilde{\boldsymbol{z}}$ and $\tilde{\boldsymbol{z}}_i$, $i = 1, \dots, K$, the observed primary and secondary data snapshots with missing values replaced by zero-elements, the devised EM-based estimation procedures are initialized with

$$\alpha^{(0)} = \frac{\boldsymbol{p}^\dagger\, \boldsymbol{S}_1^{-1}\, \tilde{\boldsymbol{z}}}{\boldsymbol{p}^\dagger\, \boldsymbol{S}_1^{-1}\, \boldsymbol{p}} \quad (38)$$

and

$$\boldsymbol{\theta}^{(0)} = \boldsymbol{S}_1 \quad (39)$$

where[5] $\boldsymbol{S}_1 = 1/K\sum_{i=1}^K \tilde{\boldsymbol{z}}_i\tilde{\boldsymbol{z}}_i^\dagger$.

The values of the system parameters involved in the analyzed case studies are summarized in Table I.

TABLE I
Simulation Parameters.

Parameter	L	d_x	θ_0	σ_a^2	JNR_1	JNR_2	θ_1	θ_2
Value	3	$\lambda_0/2$	$0°$	1	30 dB	40 dB	$-10°$	$15°$

A. Unconstrained estimation

Fig. 1 reports the P_D curves of the devised EM-based detection strategies[6] versus SINR for the case of unconstrained interference covariance, with $K = 48$ and $K = 64$ in Figs. 1(a) and 1(b), respectively. As benchmarks, the GLRT and AMF detectors, with direct access to the complete-data set, are included for comparison. Besides, two additional heuristic counterparts, comprising the Kelly's GLRT and AMF detectors computed on a data set where the missing values are replaced by linear interpolation (LI) (in the following referred to as GLRT-LI and AMF-LI, respectively) are reported too (see Appendix B for details).

The curves highlight that the devised procedures attain almost the same P_D levels, with performance comparable to the benchmarks, i.e., a loss of 4 dB for $K = 48$ and smaller than 2 dB for $K = 64$ at $P_D = 0.9$. As expected, if the sample support size increases, higher P_D levels can be achieved, with performance closer and closer to the benchmarks. The results reveal that for the considered unstructured case, the devised two-step as strategy is an effective and less computational demanding detector compared with the GLRT architecture. Indeed, it only requires the estimation of the covariance matrix under $\mathcal{H}_0$. As to the LI-based methods, they achieve P_D levels far below the benchmarks, with a significant loss in all the analyzed scenarios. This confirms the requirement to develop appropriate detection procedures capable of dealing with missing-data.

B. Centro-Hermitianity constraint

Fig. 2 presents the detection performance of the proposed receivers assuming the uncertainty set (14) for a symmetric ULA composed of $N = 15$ antennas, i.e.,

$$\boldsymbol{p}(\theta) = [e^{j\frac{2\pi}{\lambda_0}x_0\sin(\theta)}, e^{j\frac{2\pi}{\lambda_0}x_1\sin(\theta)}, \dots, e^{j\frac{2\pi}{\lambda_0}x_{N-1}\sin(\theta)}]^{\mathrm{T}} \in \mathbb{C}^N \quad (40)$$

where $x_i = d_x\ \left(i - \left(\frac{N-1}{2}\right)\right),\ i = 0, 1, \dots, N-1$.

[5]It is worth noting that some selection matrix configuration can lead, with non-zero probability, to a rank-deficient $\boldsymbol{S}_1$. The considered Monte Carlo trials do not include such realizations.

[6]For covariance regularization purposes, a diagonal loading of 10^{-2} is applied to $\boldsymbol{M}(\hat{\boldsymbol{\theta}}_{EM,0})$, $\boldsymbol{M}(\hat{\boldsymbol{\theta}}_{EM,1})$, and $\boldsymbol{M}(\hat{\boldsymbol{\theta}}_{EM,2})$.

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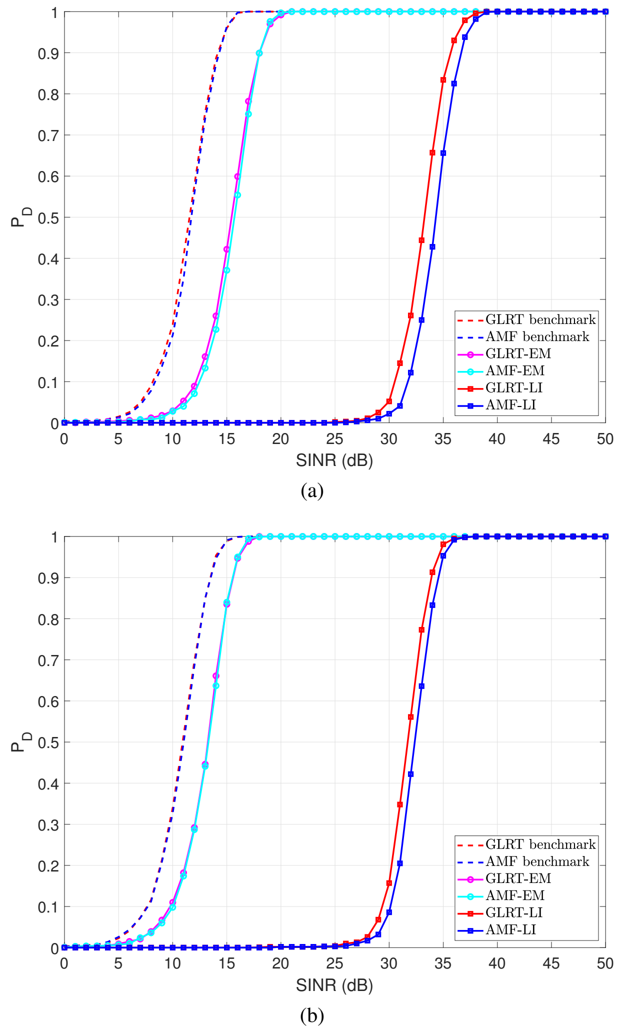

Fig. 1. Detection performance for an ULA with $N = 16$ antennas and unconstrained estimation. Different sample support sizes are considered, i.e., (a) $K = 48$ and (b) $K = 64$.

Specifically, two different sample support sizes are considered, i.e., $K = 30$ in Fig. 2(a) and $K = 45$ in Fig. 2(b). For comparison purposes, tailored GLRT and AMF detectors, leveraging the centro-Hermitianity (CH) structure for the estimation of the covariance matrix, are reported, too. Specifically, those computed on the complete-data set serve as benchmarks, whereas those evaluated on the observed-data set, with missing-data replaced by appropriately interpolated values, are considered as counterparts.

Inspection of the results shows that the proposed detectors ensure performance levels close to the benchmark with a gap between the curves less than 2 dB at $P_D = 0.9$ and $K = 30$. This is an indirect proof that capitalizing on the centro-Hermitian structure, accurate estimation of the unknowns could be obtained under both hypotheses, resulting in improved detection performance even with a reduced number of secondary data. Besides, both the devised one-step and two-step strategies achieve similar performance levels, with P_D values closer and closer to the benchmark as K increases, further corroborating the effectiveness of the bespoke detectors. Summarizing, the proposed detectors outperform all the considered (practically implantable) counterparts in the analyzed scenarios, confirming the capabilities of the devised adaptive architectures to operate in contexts with missing-data and structured covariance matrix.

C. Lower bound of the white noise power level constraint

The performance of the devised detectors assuming a ULA with $N = 16$ antennas and the uncertainty set (28) is depicted in Fig. 3. Specifically, Figs. 3(a) and 3(b) consider $K = 24$ and $K = 48$, respectively. The performance of two GLRT detectors, using respectively the ECM and M-EM for the estimation of the parameters under $\mathcal{H}_1$ hypothesis, are analyzed. In addition, a tailored (covariance structure aware) two-step receiver, using the EM-based structured procedure devised in [23] for the covariance matrix estimation under the $\mathcal{H}_0$ hypothesis and referred to in the following as AMF-EM-FML, is also reported. Besides, the clairvoyant receiver, based on a perfect knowledge of the covariance matrix, is considered as benchmark.

The curves show that the devised architectures provide detection probabilities quite close to the optimum, highlighting the capabilities of the proposed detectors to leverage a-priori knowledge about the covariance matrix structure to keep the loss due to missing-data. The results are in line with the centro-Hermitian case and confirm the intuition that a-priori knowledge exploitation represents a viable means to perform an improved adaptation process, especially in the presence of missing observations. More specifically, for $K = 24$, the loss with respect to the clairvoyant is less than 1 dB for the AMF-EM-FML and less than 2 dB for the two GLRT-based receivers. Besides, as K increases, the loss reduces progressively more and more, as depicted in Fig. 3(b).

D. Analysis on measured data

In this subsection, the performance of the devised detectors is analyzed on the measured data set collected in [49]. Specifically, the test-bed used for the acquisition process consists of

- a low-cost software defined radio (SDR) coherent receiver made up of four RTL-SDR dongles (based on the RTL2832U chipset manufactured by Realtek [50]) that share the same clock source;
- a standard personal computer, used to calibrate the devices and run algorithms;
- a ULA comprising four dipole antennas with an inter-element space of $\lambda_0/2$.

The data recording process has been conducted in an anechoic chamber using two SDR transmitters (each feeding a horn antenna with an azimuth beamwidth of 90°, at the considered operating frequency 1 GHz) to mimic the presence of two jammers occupying different spectral intervals and located at $\theta_1 = 0.5°$ and $\theta_2 = 14.5°$, respectively [51].

In [49], the data set have been used to validate the effectiveness of the covariance matrix estimator proposed in [23] on measured data. Here, the measured data set is used to validate the robustness to missing-data endowed by the proposed detection strategies. To this end, 9 prospective point-like targets

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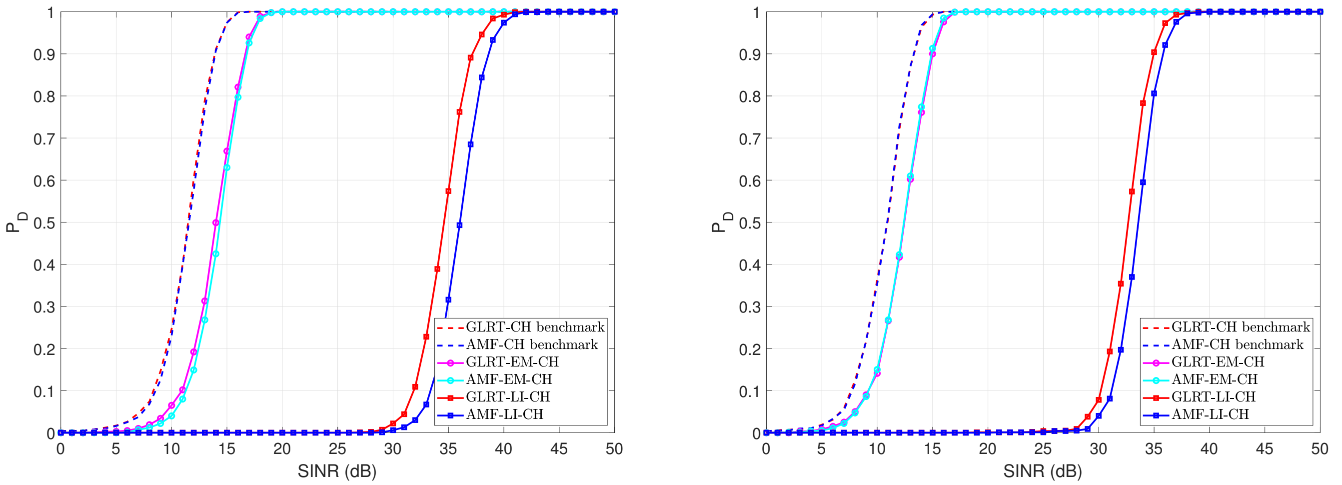
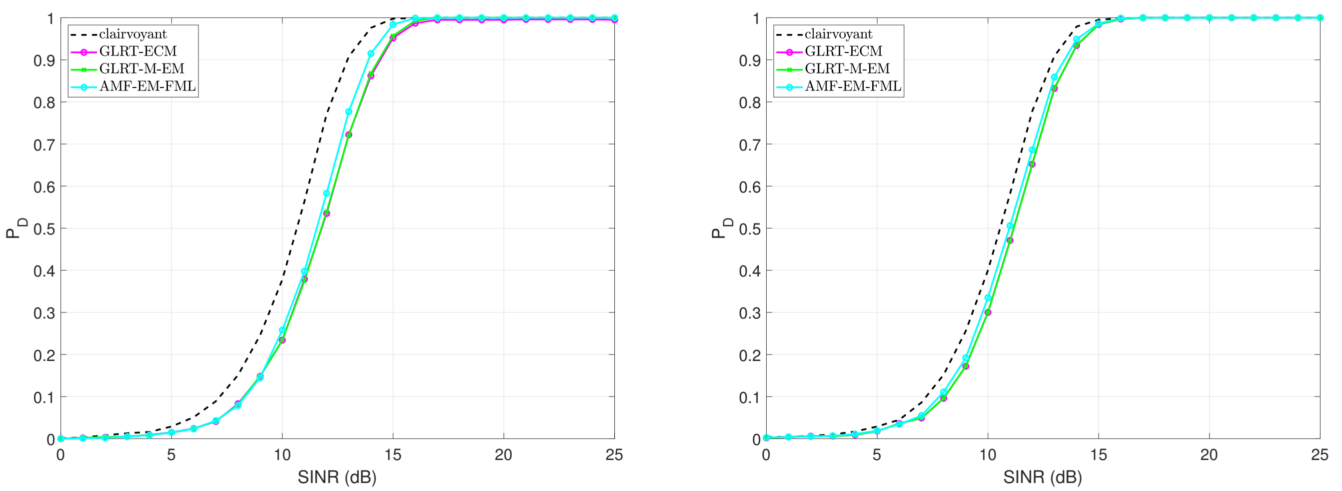


(a) (b)

Fig. 2. Detection performance for a symmetric ULA with $N = 15$ antennas assuming the covariance matrix with a centro-Hermitian structure, see (14). Different sample support sizes are considered, i.e., (a) $K = 30$ and (b) $K = 45$.



(a) (b)

Fig. 3. Detection performance for a ULA with $N = 16$ antennas assuming the uncertainty set in (28). Different sample support sizes are considered, i.e., (a) $K = 24$ and (b) $K = 48$.

causing echo returns with a SINR of 20 dB are synthetically simulated and injected in the dataset. The angle positions and ranges of the considered targets are reported in Table II. Furthermore, for the analyzed case studies, the missing-data context is emulated considering a missing element at the output of a channel (chosen at random) for each observed snapshot.

TABLE II
TARGETS RANGE AND ANGLE LOCATIONS.

Range cell	30	40	78	85	120	145	180	245	280
Angle	7°	9°	9°	8°	9°	9°	8°	7°	9°

Figs. 4 and 5 display the decision statistics of one-step and two-step strategies versus time/range resolution cell index, for a window of 300 bins. Besides, K snapshots, selected from a distinct but homogeneous temporal window, are used as a secondary data set. Fig. 4 report the behavior of the considered detectors in the unstructured case, assuming $K = 40$. The output of the GLRT-EM detector is depicted in Fig. 4(a), whereas the output of the two-step counterpart strategy is shown in Fig. 4(b). Inspection of the results clearly reveals the presence of the targets, with peak levels higher than the interference-only floor level. Precisely, the GLRT-EM exhibits peaks greater than the floor of 2 dB in the worst case, whereas 5 dB peak gains are reached by the AMF-EM. The results corroborate the effectiveness of the proposed detectors highlighting their robustness on measured data.

In Fig. 5, the analysis is conducted assuming a sample support size of $K = 24$ and the covariance matrix belonging to the uncertainty set in (28). Fig. 5(a) shows the one-step decision strategies based on the ECM and M-EM methods (whose performance curves are substantially overlapped), while the output of the AMF-EM-FML detector is illustrated in Fig. 5(b).

As in the unstructured case, it is evident the presence of

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visible peaks located in correspondence of the targets range. In particular, peak levels greater than interference-only floor of at least 1 dB are reached by the two GLRT detectors (apart from the target at 180 where the peak strength is about 0.5 dB), whereas the AMF-EM-FML detector provides peak gains larger than 5 dB. The results highlight that also on measured data reliable detection performance can be obtained with a reduced number of secondary data if bespoke a-priori structural knowledge is exploited, corroborating the capabilities of the devised detection strategies also on a measured data scenario.

V. Conclusion

In this paper, adaptive detection architectures accounting for the presence of missing-data, have been proposed. Specifically, the problem of detecting a potential target echo return buried in Gaussian interference with a possibly structured covariance is formulated as a composite hypothesis test. The problem is handled via the GLR criterion, leading to the design of one-step and two-step GLRT detectors, which require the maximization of proper likelihood functions. Leveraging specific covariance structures, tailored estimation procedures relying upon the EM framework are developed. Specifically, for some covariance structures of practical interest, the optimization procedures involve only closed-form solutions at each iteration. Conversely, the case of a quite arbitrary constraint set is addressed resorting to ECM and M-EM frameworks, yielding more tractable optimization problems than classic EM strategy. The performance of the devised detection strategies has been assessed via Monte Carlo simulations for some a-priori structural covariance models. The results have highlighted the potentialities of the proposed detectors showing a performance level comparable to the benchmarks, which assume access to the entire set of observables. Besides, the effectiveness of the detectors has been validated on measured data, collected in a controlled environment using an inexpensive four-channel receiver.

Future research studies might concern the extension of the framework to the case of distributed (range-spread) targets, the inclusion of other relevant covariance structures as well as the generalization of the devised architectures to the partially homogeneous and heterogeneous environment scenarios. Finally, it would be of great interest to consider the case of a multistatic radar where the different sensors observe the same scene but the missing data are diverse from sensor to sensor.

Appendix

A. *Closed-form expression of the score function* (8)

Let us first rewrite equation (7) as

$$\begin{aligned}\mathcal{L}_{\boldsymbol{r}}(\alpha,\boldsymbol{\theta},\mathcal{H}_1) &= -(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]\\ &- \mathrm{tr}\left\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}\left[\boldsymbol{r}\boldsymbol{r}^\dagger+|\alpha|^2\boldsymbol{p}\boldsymbol{p}^\dagger-2\Re\{\alpha\boldsymbol{p}\boldsymbol{r}^\dagger\}+\boldsymbol{S}\right]\right\}.\end{aligned} \tag{41}$$

It follows that

$$\begin{aligned}&Q\left(\alpha,\boldsymbol{\theta}|\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)}\right)=\\ &-(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]\\ &-\mathrm{tr}\Big\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}\Big[\mathbb{E}[\boldsymbol{r}\boldsymbol{r}^\dagger|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\\ &\qquad+|\alpha|^2\boldsymbol{p}\boldsymbol{p}^\dagger-2\Re\{\alpha\boldsymbol{p}\boldsymbol{\mu}^{(h-1)\dagger}\}+\boldsymbol{X}^{(h-1)}\Big]\Big\}=\\ &-(K+1)\left[N\ln(\pi)+\ln(\det(\boldsymbol{M}(\boldsymbol{\theta})))\right]-\mathrm{tr}\Big\{\boldsymbol{M}(\boldsymbol{\theta})^{-1}\\ &\Big[(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})(\boldsymbol{\mu}^{(h-1)}-\alpha\boldsymbol{p})^\dagger+\boldsymbol{\Sigma}^{(h-1)}\Big]\Big\}\end{aligned} \tag{42}$$

where

$$\boldsymbol{X}^{(h-1)}=\sum_{i=1}^{K}\mathbb{E}[\boldsymbol{r}_i\boldsymbol{r}_i^\dagger|\boldsymbol{z}_i,\boldsymbol{A}_i,\boldsymbol{\theta}^{(h-1)}] \tag{43}$$

and

$$\begin{aligned}\boldsymbol{\Sigma}^{(h-1)}&=\boldsymbol{X}^{(h-1)}-\boldsymbol{\mu}^{(h-1)}\boldsymbol{\mu}^{(h-1)\dagger}\\ &\quad+\mathbb{E}[\boldsymbol{r}\boldsymbol{r}^\dagger|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1].\end{aligned} \tag{44}$$

To proceed further, let us denote

$$\boldsymbol{B}=[\boldsymbol{A}^{\mathrm{T}}\ \bar{\boldsymbol{A}}^{\mathrm{T}}]^{\mathrm{T}} \tag{45}$$

where $\bar{\boldsymbol{A}}$ is the $N-p\times N$ selection matrix complementary to $\boldsymbol{A}$ (obtained removing from $\boldsymbol{I}$ the p rows not removed in the definition of $\boldsymbol{A}$) and $\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B}=\boldsymbol{I}$. Hence [34],

$$\begin{aligned}\boldsymbol{\mu}^{(h-1)}&=\boldsymbol{B}^{\mathrm{T}}\mathbb{E}[\boldsymbol{B}\boldsymbol{r}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\\ &=\boldsymbol{B}^{\mathrm{T}}[\boldsymbol{z}^{\mathrm{T}},\ \boldsymbol{\zeta}^{(h-1)\mathrm{T}}]^{\mathrm{T}}\end{aligned} \tag{46}$$

where

$$\begin{aligned}&\boldsymbol{\zeta}^{(h-1)}=\mathbb{E}[\bar{\boldsymbol{A}}\boldsymbol{r}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\\ &=\bar{\boldsymbol{A}}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}^{\mathrm{T}}(\boldsymbol{A}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}^{\mathrm{T}})^{-1}(\boldsymbol{z}-\alpha^{(h-1)}\boldsymbol{A}\boldsymbol{p})\\ &\quad+\alpha^{(h-1)}\bar{\boldsymbol{A}}\boldsymbol{p}.\end{aligned} \tag{47}$$

Besides,

$$\begin{aligned}&\mathbb{E}[\boldsymbol{r}\boldsymbol{r}^\dagger|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\\ &=\boldsymbol{B}^{\mathrm{T}}\,\mathbb{E}[\boldsymbol{B}\boldsymbol{r}\boldsymbol{r}^\dagger\boldsymbol{B}^{\mathrm{T}}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\,\boldsymbol{B}.\end{aligned} \tag{48}$$

As to the expectation term,

$$\begin{aligned}&\mathbb{E}[\boldsymbol{B}\boldsymbol{r}\boldsymbol{r}^\dagger\boldsymbol{B}^{\mathrm{T}}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\\ &=\begin{bmatrix}\boldsymbol{z}\boldsymbol{z}^\dagger & \boldsymbol{z}\boldsymbol{\zeta}^{(h-1)\dagger}\\ \boldsymbol{\zeta}^{(h-1)}\boldsymbol{z}^\dagger & \mathbb{E}\left[\bar{\boldsymbol{A}}\boldsymbol{r}\boldsymbol{r}^\dagger\bar{\boldsymbol{A}}^{\mathrm{T}}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1\right]\end{bmatrix}\end{aligned} \tag{49}$$

where

$$\begin{aligned}&\mathbb{E}\left[\bar{\boldsymbol{A}}\boldsymbol{r}\boldsymbol{r}^\dagger\bar{\boldsymbol{A}}^{\mathrm{T}}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1\right]\\ &=\boldsymbol{G}+\boldsymbol{\zeta}^{(h-1)}\boldsymbol{\zeta}^{(h-1)\dagger}\end{aligned} \tag{50}$$

with

$$\begin{aligned}\boldsymbol{G}&=\bar{\boldsymbol{A}}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\bar{\boldsymbol{A}}^{\mathrm{T}}-\bar{\boldsymbol{A}}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}^{\mathrm{T}}\\ &(\boldsymbol{A}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}^{\mathrm{T}})^{-1}\boldsymbol{A}\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\bar{\boldsymbol{A}}^{\mathrm{T}}.\end{aligned} \tag{51}$$

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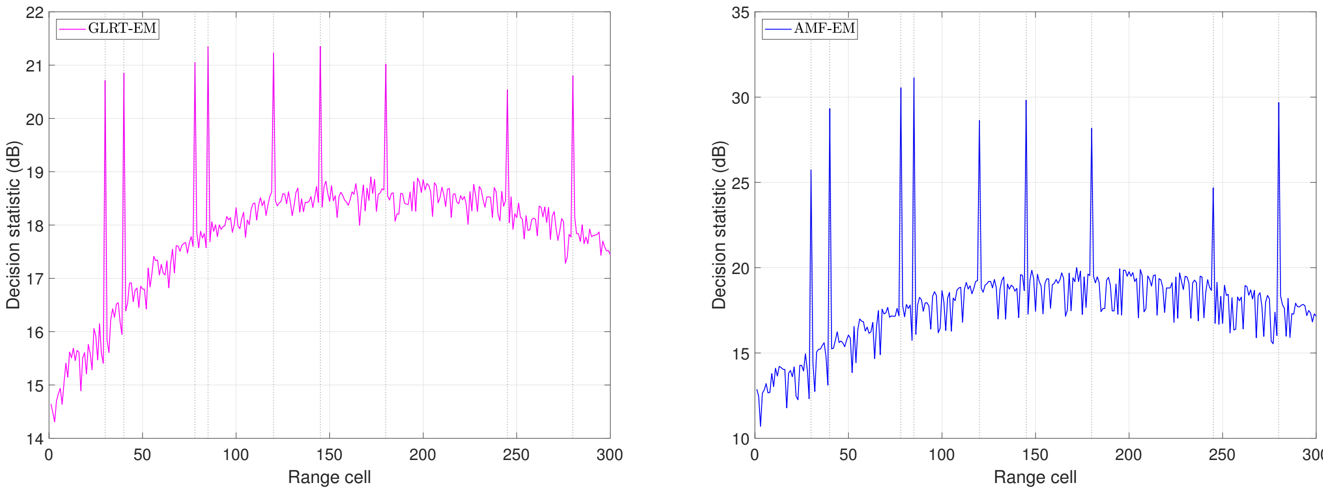
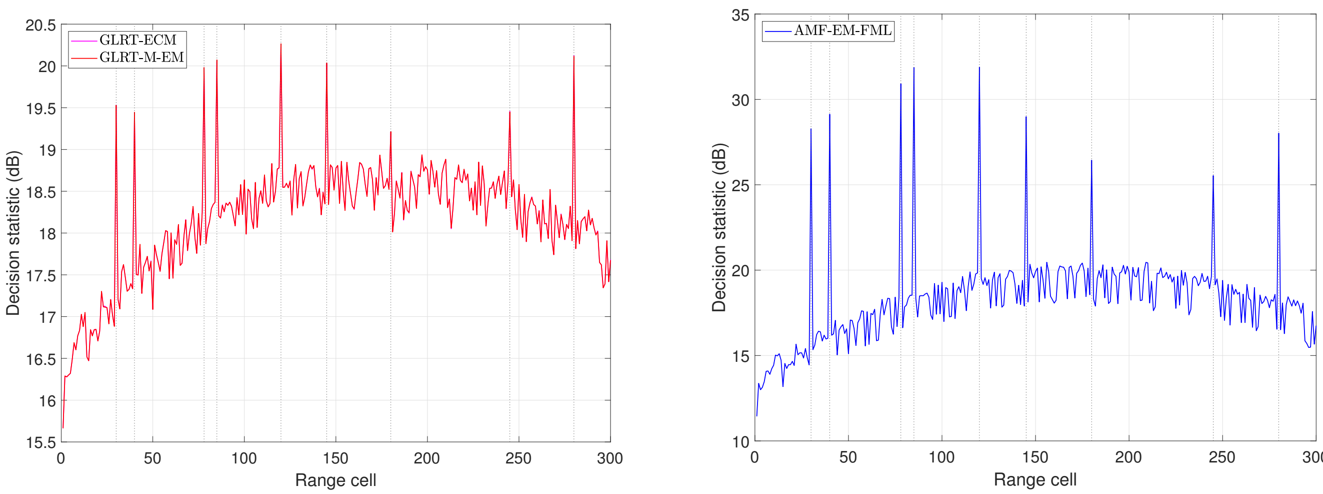


(a) (b)

Fig. 4. Detection performance for a ULA with $N = 4$ antennas and unconstrained estimation assuming $K = 40$ secondary data. Target locations are indicated by black dotted lines.



(a) (b)

Fig. 5. Detection performance for a ULA with $N = 4$ antennas assuming the uncertainty set in (28) and $K = 24$ secondary data. Target locations are indicated by black dotted lines.

Therefore,

$$\begin{aligned} &\boldsymbol{B}^{\mathrm{T}}\mathbb{E}[\boldsymbol{B}\boldsymbol{r}\boldsymbol{r}^{\dagger}\boldsymbol{B}^{\mathrm{T}}|\boldsymbol{z},\boldsymbol{A},\alpha^{(h-1)},\boldsymbol{\theta}^{(h-1)},\mathcal{H}_1]\boldsymbol{B} \\ &= \boldsymbol{A}^{\mathrm{T}}\boldsymbol{z}\boldsymbol{z}^{\dagger}\boldsymbol{A} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{\zeta}^{(h-1)}\boldsymbol{z}^{\dagger}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{z}\boldsymbol{\zeta}^{(h-1)^{\dagger}}\bar{\boldsymbol{A}} \\ &\quad + \bar{\boldsymbol{A}}^{\mathrm{T}}(\boldsymbol{G} + \boldsymbol{\zeta}^{(h-1)}\boldsymbol{\zeta}^{(h-1)^{\dagger}})\bar{\boldsymbol{A}} \\ &= (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{z} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{\zeta}^{(h-1)})(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{z} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{\zeta}^{(h-1)})^{\dagger} + \bar{\boldsymbol{A}}^{\mathrm{T}}\boldsymbol{G}\bar{\boldsymbol{A}}. \end{aligned} \tag{52}$$

Finally, denoting by $\bar{\boldsymbol{A}}_i$ the $N - p_i \times N$ selection matrix defined similarly to $\bar{\boldsymbol{A}}$, equation (43) can be recast as

$$\boldsymbol{X}^{(h-1)} = \sum_{i=1}^{K} \boldsymbol{C}_i^{(h-1)} \tag{53}$$

where

$$\begin{aligned} \boldsymbol{C}_i^{(h-1)} =& (\boldsymbol{A}_i^{\mathrm{T}} + \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i)\boldsymbol{z}_i\boldsymbol{z}_i^{\dagger}(\boldsymbol{A}_i^{\mathrm{T}} + \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{\Gamma}_i)^{\dagger} \\ &+ \bar{\boldsymbol{A}}_i^{\mathrm{T}}\boldsymbol{G}_i\bar{\boldsymbol{A}}_i \end{aligned} \tag{54}$$

with

$$\boldsymbol{\Gamma}_i = \bar{\boldsymbol{A}}_i\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}_i^{\mathrm{T}}(\boldsymbol{A}_i\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\boldsymbol{A}_i^{\mathrm{T}})^{-1} \tag{55}$$

and

$$\boldsymbol{G}_i = \bar{\boldsymbol{A}}_i\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\bar{\boldsymbol{A}}_i^{\mathrm{T}} - \boldsymbol{\Gamma}_i\boldsymbol{A}_i\boldsymbol{M}(\boldsymbol{\theta}^{(h-1)})\bar{\boldsymbol{A}}_i^{\mathrm{T}}. \tag{56}$$

B. Detailed expressions for LI-based detectors

In order to formally define the adopted LI procedure, let us observe that given two vectors (x_1, y_1) and (x_2, y_2) belonging to $\mathbb{R}^2$, the equation of the line connecting these two points is

$$y = y_1 + \frac{(y_2 - y_1)}{x_2 - x_1}(x - x_1). \tag{57}$$

Hence, the value at a point x^* can be predicted via interpolation as

$$y^* = y_1 + \frac{(y_2 - y_1)}{x_2 - x_1}(x^* - x_1). \tag{58}$$

For the case at hand, based on the location $\hat{x}$ of the missing element within the snapshot, the two points (x_1, y_1) and (x_2, y_2) with closest spatial locations to $\hat{x}$ are used to interpolate the value at point $\hat{x}$. For the two special cases when $\hat{x}$ is located at the beginning (the end) of the array, the sample points are chosen as the first two (last two) elements of the observed-data snapshot, respectively. Moreover, due to complex-valued nature of the data, each missing element is estimated by linearly interpolating real and imaginary components separately. Then, denoting by $\breve{z}_i \in \mathbb{C}^N, i = 1, \ldots, K$, the interpolated secondary data snapshot, the following detectors, based on the GLR and AMF criteria, respectively, can be implemented, i.e.,

$$\tau_{\mathrm{GLRT-LI}} = \frac{1}{1 + \boldsymbol{z}^\dagger \left(\boldsymbol{A}\breve{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} \boldsymbol{z}} \frac{\left|\boldsymbol{z}^\dagger \left(\boldsymbol{A}\breve{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} \boldsymbol{A}\boldsymbol{p}\right|^2}{\boldsymbol{p}^\dagger \boldsymbol{A}^{\mathrm{T}} \left(\boldsymbol{A}\breve{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} \boldsymbol{A}\boldsymbol{p}} \quad (59)$$

and

$$\tau_{\text{AMF-LI}} = \frac{\left|\boldsymbol{z}^\dagger \left(\boldsymbol{A}\breve{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} \boldsymbol{A}\boldsymbol{p}\right|^2}{\boldsymbol{p}^\dagger \boldsymbol{A}^{\mathrm{T}} \left(\boldsymbol{A}\breve{\boldsymbol{S}}\boldsymbol{A}^{\mathrm{T}}\right)^{-1} \boldsymbol{A}\boldsymbol{p}} \quad (60)$$

where $\breve{\boldsymbol{S}} = \sum_{i=1}^K \breve{\boldsymbol{z}}_i\, \breve{\boldsymbol{z}}_i^\dagger$.

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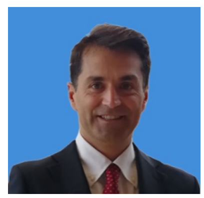

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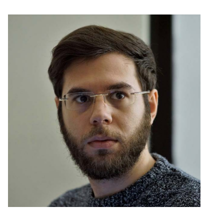

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