On the conductor of a surface at a point whose projectivized tangent cone is a generic union of lines

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ABSTRACT. Let $(A, \mathfrak{m})$ be the local ring at a singular point x of a surface V with regular normalization $\overline{A}$. Let $e(A) = e$ and $emdim(A) = r + 1$ denote the multiplicity and embedding dimension of A and $\mathfrak{b}$ denote the conductor of A in $\overline{A}$. Assume that $\sqrt{\mathfrak{b}} = \mathfrak{m}$, $Spec(G(A))$ is reduced and the projective tangent cone $Proj(G(A))$ to V at x is a union of e lines in generic position. In this paper we show that with a few exceptions $\mathfrak{b} = \mathfrak{m}^s$, where $s = Min\{n \in \mathbb{N} \mid (e-1)(n+1) < \binom{n+r}{r}\}$.

1 INTRODUCTION

In [1] the conductor of the local ring at a multiple point x, of an algebraic reduced curve C was computed under the assumption that the projectivized tangent cone $Proj(G(A))$ is reduced and consists of points in generic position. In this case it was shown that the conductor is a power of the maximal ideal of the local ring. In [4] the conductor of the local ring of an algebraic reduced variety X at a general point x of a multiple irreducible subvariety Y of codimension one was computed under the assumption that the tangent cone to X at x of Y consists of linear varieties in generic position. In this case it was shown that the conductor is a power of the prime defining Y. No result is known about the computation of the conductor of a non-normal variety whose singular locus has codimension > 1. In this paper we want to start filling this gap in the case of surfaces. In fact let $(A, \mathfrak{m})$ be a reduced non-normal local ring of multiplicity $e = e(A)$ and Hilbert function $H(A, n)$. Let

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$r+1 = emdim(A) = H(A,1)$ be the embedding dimension of A. Let $\mathfrak{b}$ be the conductor of A in its normalization $(\overline{A}, \mathfrak{J})$ (where $\mathfrak{J}$ is the Jacobson radical of $\overline{A}$) Suppose that $\sqrt{\mathfrak{b}} = \mathfrak{m}$ and that the tangent cone $Spec(G(A))$ is reduced. First we show that the natural homomorphism $G(A) \to G(\overline{A})$ is injective so that $G(A)$ can be identified to a subring of $G(\overline{A})$. Then we prove that if $G(A)$ is reduced and $G(\mathfrak{J}^n)$ is the conductor of $G(A)$ in $G(\overline{A})$, for some integer n, then $\mathfrak{m}^n = \mathfrak{J}^n$ is the conductor of A in $\overline{A}$. Let now A be the local ring of a surface X at a singular point x. Suppose that $\overline{A}$ is regular. Assume that $Spec(G(A))$ is reduced and $Proj(G(A)) \subset \mathbb{P}^r$ consists of e skew lines $\{l_1, \ldots, l_e\}$. We say that $Proj(G(A)) = \{l_1, \ldots, l_e\}$ is in generic e-position if the Hilbert function of A (that is of the coordinate ring of the lines $G(A)$) is maximal i.e.

$$H(A,n) = Min\{\binom{n+r}{r}, en+e\}$$

$Proj(G(A))$ is in generic $e-1$-position if any $e-1$ lines of $Proj(G(A))$ are in generic position. By using the previous result on the conductor of $G(A)$ and the computation of the conductor of reduced rings given in [1] we prove the main result of the paper:

let $Proj(G(A))$ be in generic $e-1, e$-position and

$$s = Min\{n \in \mathbb{N} \mid (e-1)(n+1) < \binom{n+r}{r}\}$$

Then $\mathfrak{b} = \mathfrak{m}^s$ if and only if $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$.

Throughout the paper all ring are supposed to be commutative, with identity and noetherian.

Let S be a semilocal ring. By $G(S)$ we denote the associated graded ring $\bigoplus_{n\geq 0}(\mathfrak{J}^n/\mathfrak{J}^{n+1})$ with respect to the Jacobson radical $\mathfrak{J} = \mathfrak{m}_1 \cap \cdots \cap \mathfrak{m}_{\mathfrak{e}}$. If $x \in S$, $x \neq 0$, $x \in \mathfrak{J}^n - \mathfrak{J}^{n+1}$, $n \in \mathbb{N}$, we say that x has degree n and the image $x^* \in \mathfrak{J}^n/\mathfrak{J}^{n+1}$ of x in $G(S)$ is said to be the initial form of x. If $\mathfrak{a}$ is an ideal of S, by $G(\mathfrak{a})$ we denote the ideal of $G(S)$ generated by all the inital forms of the elements of $\mathfrak{a}$.

If $(A, \mathfrak{m})$ is local with maximal ideal $\mathfrak{m}$ and residue field $k = A/\mathfrak{m}$, $H(A,n) = dim_k(\mathfrak{m}^n/\mathfrak{m}^{n+1})$, $n \in \mathbb{N}$, denotes the Hilbert function of A and $e(A)$ the multiplicity of A at $\mathfrak{m}$. The embedding dimension $emdim(A)$ of A is given by $H(A,1)$.

If $R = \bigoplus_{n\geq 0} R_n$ is a standard graded finitely generated algebra over a field k, of maximal homogeneous ideal $\mathfrak{n}$, $H(R,n) = dim_k R_n = H(R_{\mathfrak{n}}, n)$ denotes the Hilbert function of R and $emdim(R) = H(R,1) = emdim(R_{\mathfrak{n}})$ the embedding dimension of R. The multiplicity of R is $e(R) = e(R_{\mathfrak{n}})$. One has $e(A) = e(G(A))$ and $emdim(A) = emdim(G(A))$.

If B is any ring $\overline{B}$ denotes the normalization of B. If A is a subring of B $Ann_A(B/A)$ is the conductor of A in B (that is the largest ideal of A and B).

2 THE CONDUCTOR OF REDUCED ASSOCIATED GRADED RINGS

In this section we assume that $(A, \mathfrak{m})$ is a reduced equidimensional local ring with finite normalization $(\overline{A}, \mathfrak{J})$ (where $\mathfrak{J}$ is the Jacobson radical of $\overline{A}$). We assume also that the conductor $\mathfrak{b}$ of A in $\overline{A}$ has radical $\sqrt{\mathfrak{b}} = \mathfrak{m}$ (this is equivalent to saying that $A_{\mathfrak{p}}$ is regular for any prime $\mathfrak{p}$ of codimension one). In this case there is an integer h such that $\mathfrak{J}^h \subset A$. We assume also that $G(A)$ is reduced.

PROPOSITION 2.1 *The natural homomorphism $G(A) \to G(\overline{A})$ is injective.*

Proof. $G(A) \to G(\overline{A})$ is injective if and only if $\mathfrak{m}^n/\mathfrak{m}^{n+1} \to \mathfrak{J}^n/\mathfrak{J}^{n+1}$ is injective for any n, but this means $\mathfrak{J}^{n+1} \cap \mathfrak{m}^n = \mathfrak{m}^{n+1}$. Then it is enough to prove that $\mathfrak{J}^n \cap A = \mathfrak{m}^n$ for any $n \geq 0$. We prove first, by induction, that $\mathfrak{J}\mathfrak{m}^n \cap \mathfrak{m} = \mathfrak{m}^{n+1}$ for every $n \geq 0$. It is obvious that $\mathfrak{J} \cap \mathfrak{m} = \mathfrak{m}$. Suppose $\mathfrak{J}\mathfrak{m}^{n-1} \cap \mathfrak{m} = \mathfrak{m}^n$. If $x \in \mathfrak{J}\mathfrak{m}^n \cap \mathfrak{m}$, $x \in \mathfrak{J}\mathfrak{m}^{n-1} \cap \mathfrak{m} = \mathfrak{m}^n$. Now, since by assumption $\sqrt{\mathfrak{b}} = \mathfrak{m}$ we have $\mathfrak{J}^h \subset \mathfrak{m}$ for some integer h. Then $x^h \in \mathfrak{J}^h\mathfrak{m}^{hn} \subset \mathfrak{m}^{hn+1}$ i.e. the leading form of x in $G(A)$ is nilpotent. But $G(A)$ is reduced, so $x \in \mathfrak{m}^{n+1}$. As a consequence we have that $\mathfrak{J}^n \cap \mathfrak{m}^{n-1} = \mathfrak{m}^n$. In fact if $x \in \mathfrak{J}^n \cap \mathfrak{m}^{n-1}$ we have $x^h = xx^{h-1} \in \mathfrak{J}^n\mathfrak{m}^{(n-1)(h-1)} = \mathfrak{J}^{n-1}(\mathfrak{J}\mathfrak{m}^{(n-1)(h-1)}) = \mathfrak{J}^{n-1}\mathfrak{m}^{(n-1)(h-1)+1} = \cdots = \mathfrak{m}^{(n-1)h+1}$ i.e. the leading form of x is nilpotent. So $x \in \mathfrak{m}^n$. Thus we can conclude that

$$\mathfrak{m}^n = \mathfrak{J}^n \cap \mathfrak{m}^{n-1} = \mathfrak{J}^n \cap \mathfrak{J}^{n-1} \cap \mathfrak{m}^{n-2} = \mathfrak{J}^n \cap \mathfrak{m}^{n-2} = \cdots = \mathfrak{J}^n \cap A$$

□

THEOREM 2.2 *Let $\mathfrak{b}$ be the conductor of A in $\overline{A}$. and G the conductor of $G(A)$ in $G(\overline{A})$ then:*

(a) *$G(\mathfrak{b}) \subset G$;*

(b) *if $G = G(\mathfrak{J}^n)$ for some integer n then $\mathfrak{b} = \mathfrak{m}^n = \mathfrak{J}^n$.*

Proof. (a) Let $b^* \in G(\overline{A})$ of degree m and $x^* \in G(\mathfrak{b})$ of degree n. We have $bx \in \mathfrak{J}^{m+n} \cap A = \mathfrak{m}^{m+n}$ (see Proposition 2.1) i.e. $b^*x^* \in G(A)$ which proves the inclusion.

(b) If $G(\mathfrak{J}^n) = \bigoplus_{i \geq n}(\mathfrak{J}^i/\mathfrak{J}^{i+1})$ is the conductor of $G(A)$ in $G(\overline{A})$ then $\mathfrak{J}^i \subset \mathfrak{m}^i + \mathfrak{J}^{i+1}$ for any $i \geq n$ and so

$$\mathfrak{J}^n = \mathfrak{m}^n + \mathfrak{J}^{n+1} = \mathfrak{m}^n + \mathfrak{J}^{n+2} = \mathfrak{m}^n + \mathfrak{J}^i$$

for any $i \geq n$. But there exists an integer h such that $\mathfrak{J}^h \subset A$. Hence $\mathfrak{J}^n \subset A$ and $\mathfrak{J}^n$ is an ideal of A. So $\mathfrak{J}^n \subset \mathfrak{b}$ and $G(\mathfrak{J}^n) \subset G(\mathfrak{b})$. Thus $G(\mathfrak{J}^n) = G(\mathfrak{b})$. This implies that $\mathfrak{J}^n = \mathfrak{b}$. Finally $\mathfrak{J}^n = \mathfrak{J}^n \cap A = \mathfrak{m}^n$ by Proposition 2.1. □

In the following we will need also a general result on the computation of the conductor. Let R be a reduced ring and $\mathfrak{p}_i$, $i = 1, \ldots, n$, be the minimal primes of R. Set $R_i = R/\mathfrak{p}_i$. The canonical homomorphisms $\pi_i : R \to R_i$ induce an embedding

$$\pi_i : R \to \prod_{i=1}^{n} R_i = R' \tag{2.3}$$

Thus we can identify R with a subring of R'. Furthermore R' is integral over R.

PROPOSITION 2.4 *The conductor of R in R' is the ideal*

$$\bigcap_{i=1}^{n}\Big(\mathfrak{p}_i+\bigcap_{j\neq i}\mathfrak{p}_j\Big)=\bigoplus_{i=1}^{n}\Big(\bigcap_{j\neq i}\mathfrak{q}_j\Big)$$

where $\mathfrak{q}_j$ is the image of the ideal $\mathfrak{p}_j$ under the homomorphism $R\to R_i$ (for any j). Furthermore if the rings R_i are normal then R' is the normalization of R.

Proof. See [1], Proposition 2.5. $\square$

3 LINES IN GENERIC POSITION AND CONDUCTOR OF SURFACES

In this section we assume that k is an algebraically closed field of characteristic zero.

THEOREM 3.1 *Let R be the homogeneous coordinate ring of e lines $\{l_1,\dots,l_e\}\subset\mathbb{P}^r_k$, $r\geq 3$. Then $H(R,n)\leq Min\{\binom{n+r}{r},en+e\}$. Furthermore let $x_j=a_{ij}s+b_{ij}t$, $j\in\{0,\dots,r\}$, be a parametric representation of the line l_i, $i\in\{1,\dots,s\}$, and let $(a_{10},b_{10},\dots,a_{sr},b_{sr})\in\mathbb{A}^q$, $q=2e(r+1)$ be the q-tuple of all the coefficients of the representations of the lines l_i. Then the set $U\subset\mathbb{A}^q_k$ of the q-tuples $(a_{10},\dots,a_{sr},b_{10},\dots,b_{sr})$ for which $H(R,n)=Min\{\binom{n+r}{r},en+e\}$ is open and non-empty (and consists of skew lines).*

Proof. All claims are proved in Proposition 1.3 and 1.5 of [5] except for the crucial fact that the set U is non-empty. This is proved in Theorem 0.1 of [2]. $\square$

DEFINITION 3.2 A set of lines $X=\{l_1\cup\cdots\cup l_e\}\subset\mathbb{P}^r_k$, $r\geq 3$ is *in generic position* if its homogeneous coordinate ring R has Hilbert function $H(R,n)=Min\{\binom{n+r}{r},en+e\}$. X is *in generic t-position*, $t\leq e$, if every t-subset of X is in generic position (then generic e-position is generic position).

THEOREM 3.3 *Let R be the coordinate ring of a set of lines $X=\{l_1\cup\cdots\cup l_e\}\subset\mathbb{P}^r_k$, $r\geq 3$ and set $s=Min\{n\in\mathbb{N}\mid(e-1)(n+1)<\binom{n+r}{r}\}$. Then X is in generic position if and only if $H(R,s)=(e+1)s$ and $H(R,s-1)=\binom{s-1+r}{r}$ that is X is not contained in any hypersurface of degree $<s$ and X is contained in $\binom{s+r}{r}-(e+1)s$ linearly independent hypersurfaces.*

Proof. See Theorem 1.6 of [5]. $\square$

THEOREM 3.4 *Let R be the homogeneous coordinate ring of e lines $\{l_1,\dots,l_e\}\subset\mathbb{P}^r_k$, $r\geq 3$ in generic $e-1,e$ position. Let $emdim R=r+1$, $\mathfrak{n}$ be the maximal homogeneous ideal of R and $\mathfrak{n}_i$, $i=1,\dots,e$, be the maximal homogeneous ideals of $\overline{R}$. Set $s=Min\{n\in\mathbb{N}\mid(e-1)(n+1)<\binom{n+r}{r}\}$ and let $\mathfrak{c}$ be the conductor of R in $\overline{R}$. Then:*

(a) $\mathfrak{c}\subset\mathfrak{n}^s\subset\bigcap_{i=1}^{e}\mathfrak{n}_i^s$

(b) $\mathfrak{c} = \mathfrak{n}^s = \bigcap_{i=1}^e \mathfrak{n}_i^s$ *if and only if* $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$.

Proof. Let $\mathfrak{p}_i$, $i = 1, \dots, n$, be the minimal primes of $R = k[x_0, \dots, x_r]$. Then by assumption $R_i = R/\mathfrak{p}_i \cong k_i[X, Y]$, $k_i = k$ and, by Proposition 2.4, we can identify $\overline{R}$ with the ring $\prod_{i=1}^e k_i[X, Y]$. Hence $\bigcap_{i=1}^e \mathfrak{n}_i^s = \bigoplus_{i=1}^e (X, Y)^s k_i[X, Y]$. (a) Since the lines $\{l_1 \cup \dots \cup l_e\} - \{l_i\} \subset \mathbb{P}_k^r$ are in generic position we have that the ideal $\bigcap_{j\neq i} \mathfrak{p}_j$ is generated by forms of degree $\geq s$ (see Theorem 3.3) and the same happens to its image in R_i. Hence $\bigcap_{j\neq i} \mathfrak{q}_j \subset (X, Y)^s k_i[X, Y]$

(b) The condition $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$ is equivalent to saying that $(e-1)(s+1) > \binom{s+r}{r}$ or $e(s+1) \leq \binom{s+r}{r}$. Since by assuption we have $(e-1)(s+1) < \binom{s+r}{r}$ $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$ is equivalent to $e(s+1) \leq \binom{s+r}{r}$.

Fix an integer i, $1 \leq i \leq e$. Let $(f_1, \dots, f_u)$ be the elements of degree s of a minimal set of generators of the ideal $\bigcap_{j\neq i} \mathfrak{p}_j$. By the minimality $(f_1, \dots, f_u)$ are linearly independent modulo $\mathfrak{p}_i$ and then their images

$$(\overline{f}_1, \dots, \overline{f}_u) \subset \bigcap_{j\neq i} \mathfrak{q}_j \subset (X, Y)^s k_i[X, Y]$$

are linearly independent forms. But $u = H(R, s) - H(R/\bigcap_{j\neq i} \mathfrak{p}_j, s)$. Then if $e(s+1) \leq \binom{s+r}{r}$ we have $u = e(s+1) - (e-1)(s+1) = s+1$ and $(\overline{f}_1, \dots, \overline{f}_u) = \bigcap_{j\neq i} \mathfrak{q}_j = (X, Y)^s k_i[X, Y]$. Then

$$\mathfrak{c} = \bigoplus_{i=1}^n \Big(\bigcap_{j\neq i} \mathfrak{q}_j \Big) = \bigoplus_{i=1}^e (X, Y)^s k_i[X, Y] = \bigcap_{i=1}^e \mathfrak{n}_i^s$$

If $e(s+1) > \binom{s+r}{r}$ we have

$$u = \binom{s+r}{r} - (s-1)(e+1) < s(e+1) - (s-1)(e+1) = e+1$$

and

$$(\overline{f}_1, \dots, \overline{f}_u) = \bigcap_{j\neq i} \mathfrak{q}_j \neq (X, Y)^s k_i[X, Y]$$

□

REMARK 3.5 The numbers e of lines in generic $e, e-1$ position for which $e = \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$ ($s = Min\{n \in \mathbb{N} \mid (e-1)(n+1) < \binom{n+r}{r}\}$) are sparse. For example if $e \leq 20$, these numbers are $4, 10, 19$ in $\mathbb{P}^3$, 9 in $\mathbb{P}^4$, 10 in $\mathbb{P}^6$, 19 in $\mathbb{P}^9$.

The following two examples were checked with CoCoA [3].

EXAMPLE 3.6 Let R be the homogeneous coordinate ring of the following 4 skew lines

$$l_1 : w - x = y - z = 0, \quad l_2 : w + x = y + z = 0$$

$$l_3 : w - 2x = y - 2z = 0, \quad l_4 : 2w - x = 2y + z = 0$$

in generic $e-1, e$ position.The conductor of R in $\overline{R}$ is the ideal

$$(y^2z, w^2x, w^3, y^3, x^3, z^3, wx^2, yz^2, xy, wy, wz, xz)$$

which is not a power of the maximal homogeneous ideal (x, y, z, w) of R.

EXAMPLE 3.7 Let R be the homogeneous coordinate ring of the following $e = 5$ skew lines

$$l_1 : w-x+y = y-z+x = 0,\ l_2 : 6w+x+2y = y+7z-x = 0,\ l_3 : w-2x = y-2z = 0$$

$$l_4 : 2w-3x = 2y+5z = 0,\ l_5 : 3w-2x = 3y+2z = 0$$

which are in generic $e-1, e$ position. Then by Theorem 3.4, (b), the conductor of R in $\overline{R}$ is $\mathfrak{n}^3$ where $\mathfrak{n}$ is the maximal homogeneous ideal of R.

THEOREM 3.8 *Let A be the local ring at a singular non-normal point x of an equidimensional surface X with regular normalization $\overline{A}$. Assume that the conductor $\mathfrak{b}$ of A in $\overline{A}$ has radical $\sqrt{\mathfrak{b}} = \mathfrak{m}$ (that is x is not contained in any multiple curve of X) and that the tangent cone $Spec(A)$ is reduced and consists of $e = e(A)$ planes.Let $Proj(G(A))$ be in generic $e-1, e$- position and set $s = Min\{n \in \mathbb{N} \mid (e-1)(n+1) < \binom{n+r}{r}\}$.*

(a) $\mathfrak{b} \subset \mathfrak{m}^s$

(b) $\mathfrak{b} = \mathfrak{m}^s$ *if and only if* $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$.

Proof. First we prove that $G(\overline{A})$ is the normalization of $G(A)$. The natural splitting $\mathfrak{J}^e/\mathfrak{J}^{e+1} = \bigoplus_n^e (\mathfrak{m}^e/\mathfrak{m}^{e+1})$ induces the isomorphism $G(\overline{A}) \cong \prod_{i=1}^e G(\overline{A})_{\mathfrak{m}_i}$. But since $k = \overline{A}/\mathfrak{m}_i$ and $\overline{A}_{\mathfrak{m}_i}$ is regular we have $G(\overline{A})_{\mathfrak{m}_i} \cong k_i[X,Y]$, where $k_i = k$. Then we can identify $G(\overline{A})$ with the ring $\prod_{i=1}^e k_i[X,Y]$. Now by assumption $G(A)$ is the coordinate ring of e planes. Thus if $\mathfrak{p}_i$ are the minimal primes of $G(A)$ we have $G(A)/\mathfrak{p}_i \cong k[X,Y]$. Then by 2.3 and Proposition 2.4 we have $\overline{G(A)} = \prod_{i=1}^e k_i[X,Y] = G(\overline{A})$Let G be the conductor of $G(A)$ in its normalization $G(\overline{A})$.

(a) By Theorem 2.2 and Theorem 3.4, (a), we have $G(\mathfrak{b}) \subset G = G(\mathfrak{m})^s = G(\mathfrak{m}^s)$. Hence $\mathfrak{b} \subset \mathfrak{m}^s$

(b) If $\mathfrak{b} = \mathfrak{m}^s$ by Theorem 2.2, (a), $G(\mathfrak{b}) = G(\mathfrak{m}^s) = G(\mathfrak{m})^s \subset G$. But $G \subset G(\mathfrak{m})^s$ by Theorem 3.4. Hence $G = G(\mathfrak{m})^s = G(\mathfrak{m})^s G(\overline{A}) = (G(\mathfrak{m})G(\overline{A}))^s = \bigcap_{i=1}^e G(\mathfrak{m}_i)^s$, where $G(\mathfrak{m}_i)$ are the maximal homogeneous ideals of $G(\overline{A})$, and $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$ by Theorem 3.4, (b).

Viceversa if $e \neq \lfloor \binom{s+r}{r}/(s+1) \rfloor + 1$ $G = \bigcap_{i=1}^e G(\mathfrak{m}_i)^s = G(\bigcap_{i=1}^e \mathfrak{m}_i^s) = G(\mathfrak{J}^s)$, by Theorem 3.4, (b) and the claim follows from Theorem 2.2, (b). □

EXAMPLE 3.9 Let A be the local ring at the origin $x = 0, y = 0, z = 0, w = 0$ of the surface of $\mathbb{A}^4$

$$xz - yw = z^3 + z^2 - w^2 = yz^2 + yz - xw = y^2 z - x^2 + y^2 = 0.$$

Then it is easily shown that

$$G(A) = k[x,y,z,w]/(xz - yw, z^2 - w^2, yz - xw, x^2 - y^2)$$

and then the projectivized tangent cone $Proj(G(A))$ is the union of the two skew lines $l_1 : x - y = z - w = 0$, $l_2 : x + y = z + w = 0$. Since two skew lines are always in generic position by Theorem 3.8, (b), we get that the conductor of A in $\overline{A}$ is the maximal ideal of A ($s = 1$).

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