FLUIDODINAMICA
A BASSI NUMERI DI REYNOLDS.
Introduzione

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PROGRAMMA

Parte “facile”: meccanica del continuo Newtoniano
il mondo a bassi Reynolds (<<1)

Parte “difficile”: effetti di tempo (“memoria”) e/o spaziali
la tecnica delle espansioni asintotiche

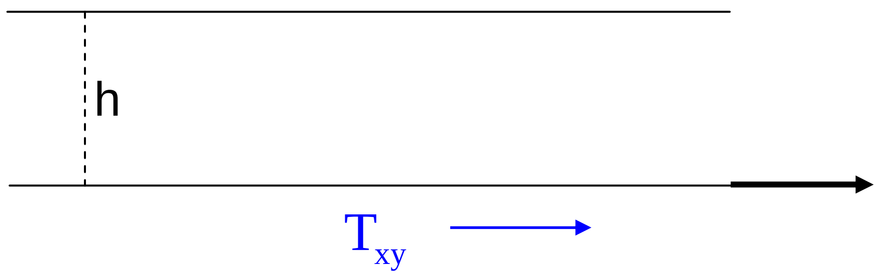
$$\boxed{Re = \frac{\rho}{\eta} L_c V_c}$$

If $Re \ll 1$:

Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

(E. Purcell, 1976)

Start-up di shear



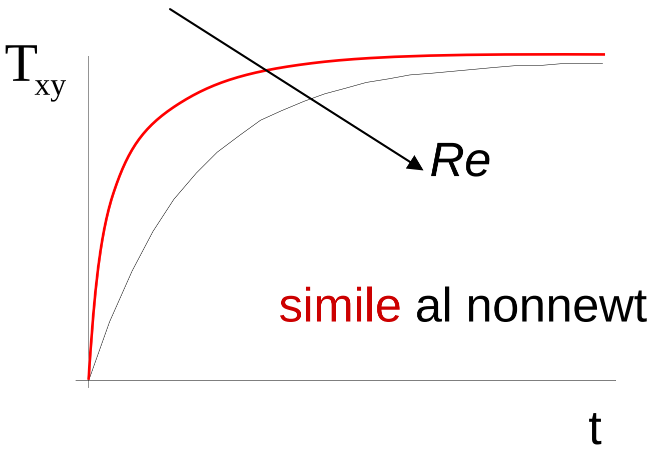
$$\begin{cases} v(t)=0 & t\le 0 \\ v(t)=1 & t>0 \end{cases} \qquad y=0$$

$$v(t)=0 \qquad \forall t \qquad y=1$$

$$v(t)=0 \qquad t\le 0 \qquad \forall y\in|0,1|$$

$$v_x = v_x(y,t) \quad \Rightarrow \quad \boxed{Re\,\frac{\partial v_x}{\partial t}=\frac{\partial^2 v_x}{\partial y^2}} \quad \Rightarrow \quad T_{xy} = \eta\,\frac{dv_x}{dy}$$

Navier – Stokes lineare !



$$\boxed{T_{xy} = \eta\dot{\gamma}\left(1+\sum_n s_n e^{-\frac{n^2\pi^2\dot{\gamma}t}{Re}}\right)}$$

<u>*osservazione:*</u>

i) “memoria”

ii) Effetto di *Re* $\approx \tau$

Vediamo la velocita'

$$\boxed{Re\frac{\partial v_x}{\partial t} = \frac{\partial^2 v_x}{\partial y^2}}$$

separazione di variabili

$$Re\frac{\partial v}{\partial t}w = v\frac{\partial^2 w}{\partial y^2}$$

$$Re\frac{\partial v}{\partial t}\frac{1}{v} = \frac{1}{w}\frac{\partial^2 w}{\partial y^2} = -K$$

cioe'

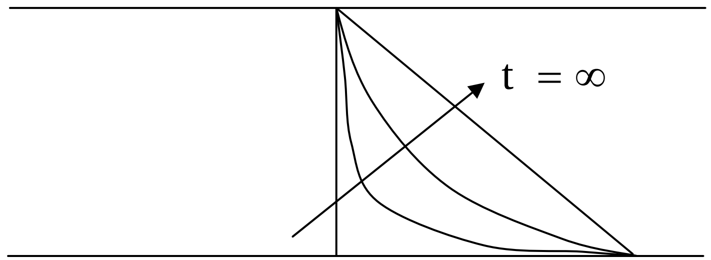
$$\frac{\partial v}{\partial t} = -\frac{K}{Re}v$$

$$\frac{\partial^2 w}{\partial y^2} = -Kw$$

$$v_x = e^{-\frac{K}{Re}t}\left(a_1\cos\sqrt{K}y + a_2\sin\sqrt{K}y\right)$$

$$\begin{cases} v_x(t) = 1 & y = 0 \quad \Rightarrow \quad a_1 = 0 \\ v_x(t) = 0 & y = 1 \quad \Rightarrow \quad \sqrt{K} = n\pi \end{cases}$$

$$\boxed{v_x = (1-y) + \sum_n a_n e^{-\frac{n^2\pi^2\dot{\gamma}t}{Re}} \sin n\pi y}$$



diverso dal nonnewt

ATTENZIONE

$$\boxed{\begin{aligned}\frac{v_x}{V} &= (1-y)+\sum_n a_n e^{-\frac{n^2\pi^2\dot{\gamma}t}{Re}}\sin n\pi y\\ \frac{T_{xy}}{\eta\dot{\gamma}} &= 1+\sum_n s_n e^{-\frac{n^2\pi^2\dot{\gamma}t}{Re}}\end{aligned}}$$

tempi

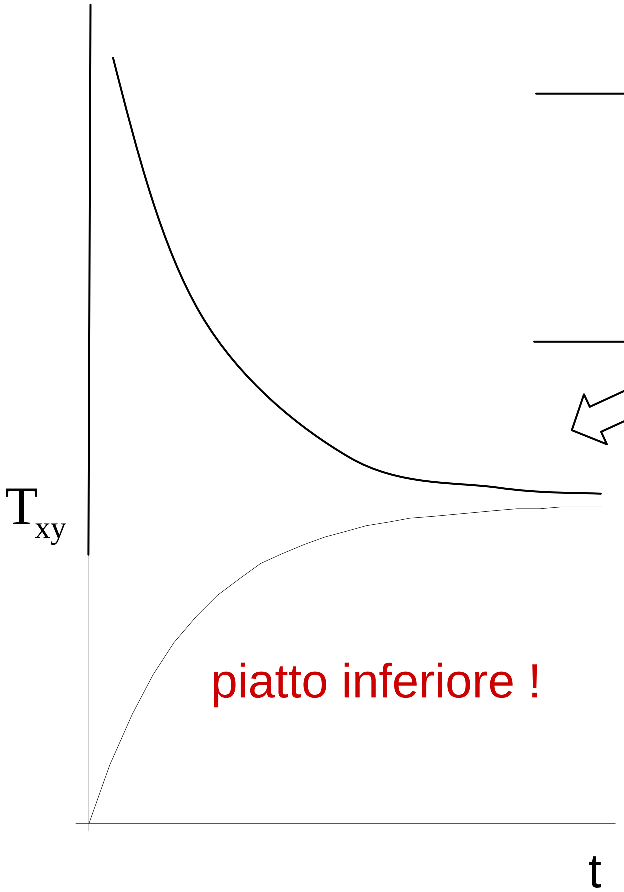
"diffusivita'"

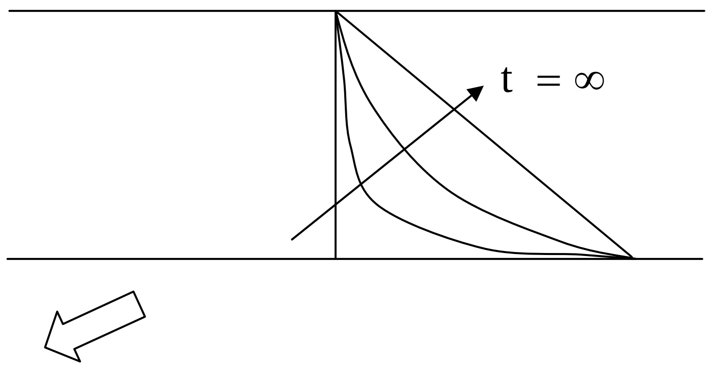
$$\frac{\dot{\gamma}t}{Re} = \frac{\frac{V}{h}t}{\frac{Vh}{\nu}} = \frac{\nu}{h^2}t = \frac{\nu}{\frac{h^2}{t}}$$

ambiguita'

$$\boxed{\frac{\partial v_x}{\partial t} = \frac{\eta}{\rho}\frac{\partial^2 v_x}{\partial y^2}}$$

$$Re = \frac{hV}{\nu} = \frac{h^2\dot{\gamma}}{\nu}$$

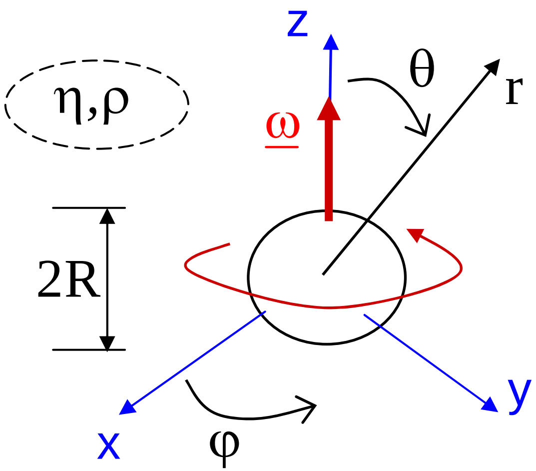




NEL TRANSITORIO
SI ROMPE LA SIMMETRIA SPAZIALE

(diverso dal nonnewt)

Sfera in rotazione. Seconda puntata



Navier-Stokes

$$\boxed{\begin{array}{l} \underline{\nabla}\cdot\underline{v}=0 \\ Re\,\underline{v}\cdot\underline{\nabla}\,\underline{v}=-\underline{\nabla}p+\nabla^2\underline{v} \end{array}}$$

BD CD

$$\begin{cases} v_\varphi=\omega R\sin\theta \\ v_r=v_\theta=0 \end{cases} \quad r=R$$

$$\begin{cases} v_\varphi=v_r=v_\theta=0 \\ p=0 \end{cases} \quad r\to\infty$$

soluzione creeping

$$\boxed{\begin{cases} p=0 \\ \underline{v}=\dfrac{1}{r^3}\underline{\omega}\times\underline{r} \end{cases}}$$

⇨ velocita' ovunque su xy

$Re \neq 0$

metodo di soluzione per espansione regolare

$$\begin{cases} p=p_0+Re\,p_1+Re^2p_2+\dots \\ \underline{v}=\underline{v}_0+Re\,\underline{v}_1+Re^2\underline{v}_2+\dots \end{cases}$$

⇨ sequenza di equazioni $Re=0$, Re, Re^2, ...

ordine Re

$$\underline{\nabla}\cdot\underline{v}_1=0$$

$$-\underline{\nabla}p_1+\nabla^2\underline{v}_1=\underline{v}_0\cdot\underline{\nabla}\,\underline{v}_0\propto\frac{\omega}{r^5}\left(\cos\varphi\,\underline{i}_x+\sin\varphi\,\underline{i}_y\right)$$

"forzante" ovunque su xy

Riformulazione del problema matematico

TEOREMA:
il problema

$$\boxed{-\underline{\nabla} p + \nabla^2 \underline{v} = \underline{h}}$$

nei casi assialsimmetrici diventa

$$E^4(\psi) = r \sin\theta \left(\underline{\nabla} \times \underline{h}\right)_\varphi \qquad \text{con}: \ \psi(r,\theta)$$

$$E^4(\ldots) = E^2\left[E^2(\ldots)\right] \qquad E^2(\ldots) = \frac{\partial^2(\ldots)}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial(\ldots)}{\partial\theta}\right)$$

trovato $\psi(r,\theta)$ $\Rightarrow$ $v_r = -\frac{1}{r^2 \sin\theta}\frac{\partial\psi}{\partial\theta}$, $v_\theta = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r}$ $\Rightarrow$ si risolve per p

attenzione alle BD CD !!

Nel nostro caso:

$$\boxed{E^4(\psi) \propto \frac{\omega}{r^5}\sin^2\theta\cos\theta}$$

$$\psi = f(r)\sin^2\theta\cos\theta$$

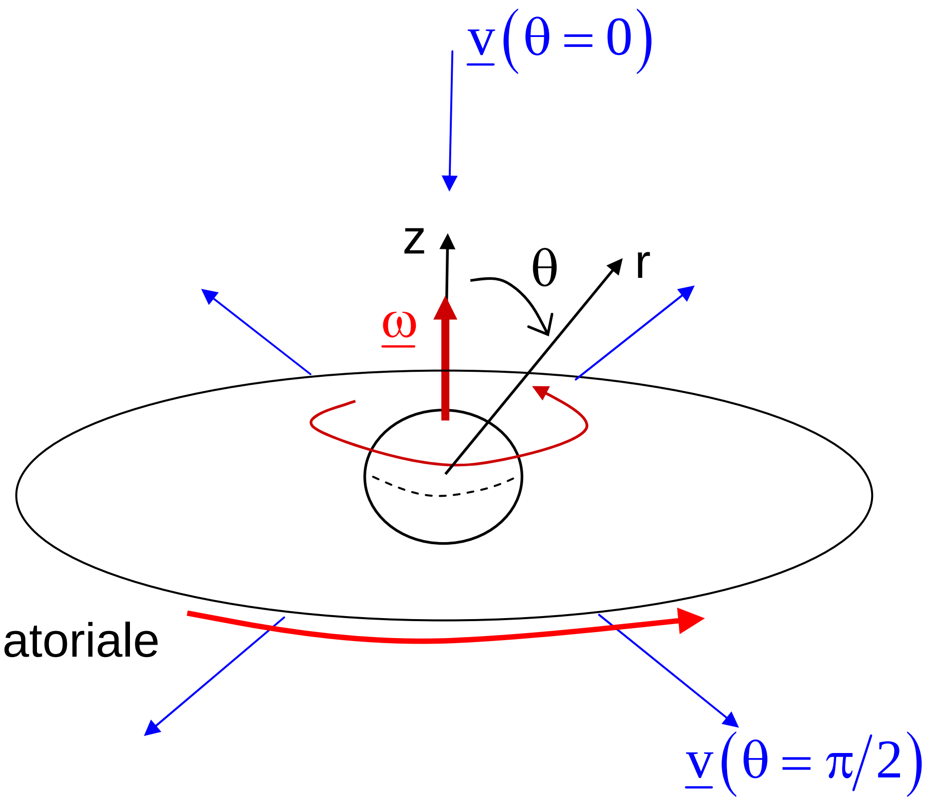
trial function

La soluzione

$$\psi = \left(\omega R r^{-1} + c_1 + c_2 r^{-2} + c_3 r^3 + c_4 r^5\right)\sin^2\theta\cos\theta$$

$$< 0$$

$$\begin{cases} v_{1r} = -\dfrac{1}{8}\omega R\dfrac{R^2}{r^2}\left(1-\dfrac{R}{r}\right)^2\left(3\cos^2\theta - 1\right) \\ v_{1\theta} = \dfrac{1}{4}\omega R\dfrac{R^3}{r^3}\left(1-\dfrac{R}{r}\right)\sin\theta\cos\theta \end{cases}$$



Gli effetti inerziali (*Re* ≠ 0) generano *<u>FLUSSI SECONDARI</u>*

? e i fluidi NON-NEWTONIANI ?

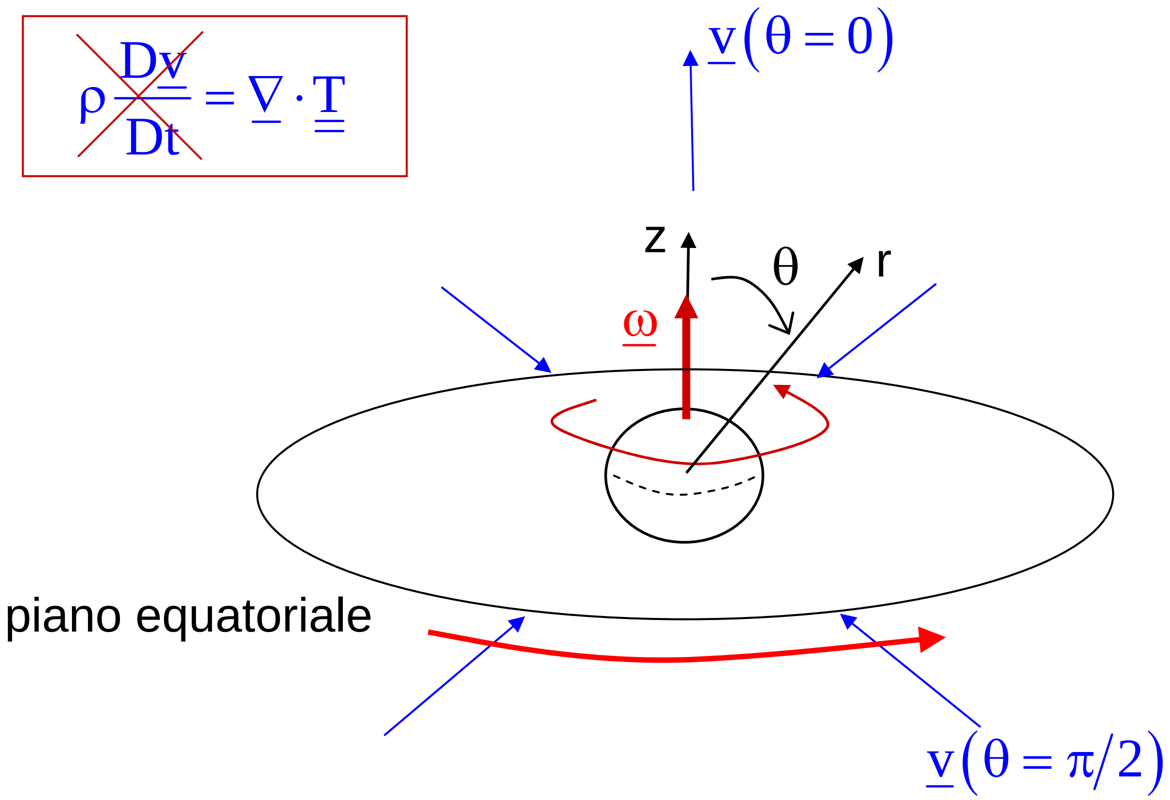
Problema e soluzione per i fluidi non-Newtoniani

$$\tau \frac{D\underline{\underline{T}}}{Dt} + \underline{\underline{T}} = \eta\left(\underline{\nabla}\,\underline{v} + \underline{\nabla}\,\underline{v}^{T}\right)$$

$$\underline{\nabla}\cdot\underline{v} = 0$$

$$\cancel{\rho\frac{D\underline{v}}{Dt}} = \underline{\nabla}\cdot\underline{\underline{T}}$$

$$\begin{cases} v_{1r} = \overset{>0}{F(r)}\left(3\cos^{2}\theta - 1\right) \\ v_{1\theta} = G(r)\sin\theta\cos\theta \end{cases}$$



Gli effetti NON-NEWTONIANI (τ ≠ 0) generano *FLUSSI SECONDARI*

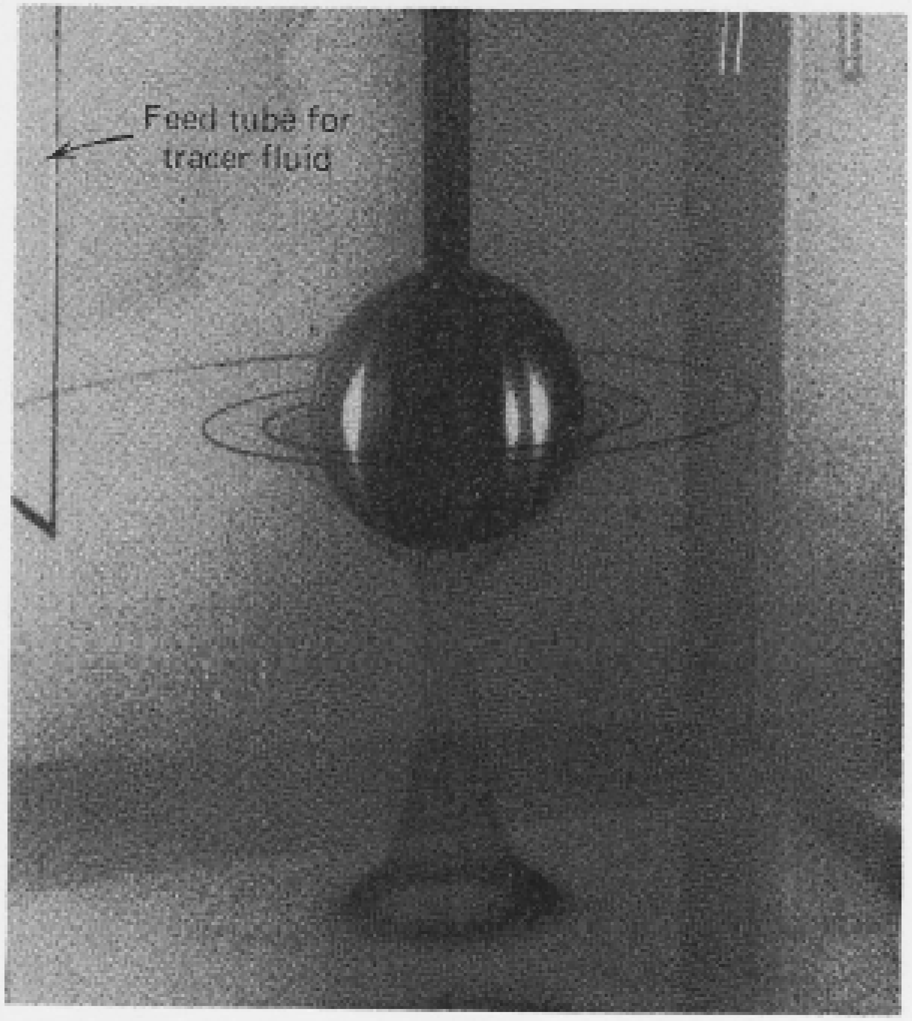
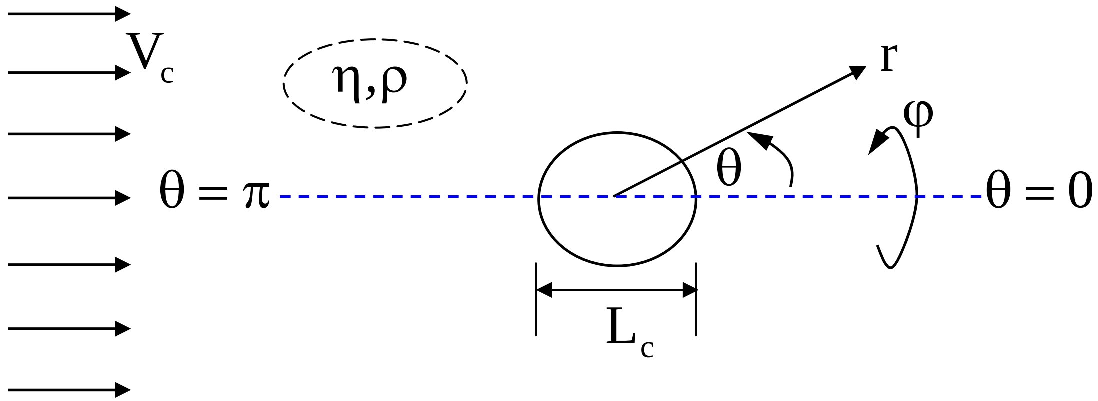


FIGURE 2.4-1. Secondary flow around a rotating sphere in a polyacrylamide solution. [Reproduced from H. Giesekus in E. H. Lee, ed., *Proceedings of the Fourth International Congress on Rheology*, Wiley-Interscience, New York (1965), Part 1, pp. 249–266.]

Sfera in trascinamento. Seconda puntata



BD CD

$$v_r = v_\theta = 0 \qquad r = 1$$

$$\begin{cases} v_r = \cos(\theta) \\ v_\theta = -\sin(\theta) \\ p = 0 \end{cases} \qquad r \to \infty$$

soluzione
creeping

$$\boxed{\begin{cases} p = -\dfrac{3}{2r^3}\underline{V}_c \cdot \underline{r} \\ \underline{v} = \left(1 - \dfrac{3}{4r} - \dfrac{1}{4r^3}\right)\underline{V}_c - \left(\dfrac{3}{4r^3} - \dfrac{3}{4r^5}\right)\underline{V}_c \cdot \underline{r}\,\underline{r} \end{cases}}$$

metodo di soluzione
per espansione regolare

$$\begin{cases} p = p_0 + Re\, p_1 + Re^2 p_2 + \ldots \\ \underline{v} = \underline{v}_0 + Re\, \underline{v}_1 + Re^2 \underline{v}_2 + \ldots \end{cases}$$

sequenza di equazioni
$Re = 0$, Re , Re^2 , ...

ordine *Re*

$$\underline{\nabla} \cdot \underline{v}_1 = 0$$
$$-\underline{\nabla} p_1 + \nabla^2 \underline{v}_1 = \underline{v}_0 \cdot \underline{\nabla}\, \underline{v}_0$$

TEOREMA

$\Rightarrow$

$$E^4\psi = r\sin\theta\left(\underline{\nabla} \times \left(\underline{v}_0 \cdot \underline{\nabla}\,\underline{v}_0\right)\right)_\varphi$$

$$\text{con: } \psi(r,\theta)$$

La soluzione

L'equazione

$$E^4(\psi) = \frac{9}{4}\left(\frac{2}{r^2} - \frac{3}{r^3} + \frac{1}{r^5}\right)\sin^2\theta\cos\theta$$

$$\psi = f(r)\sin^2\theta\cos\theta$$

trial function

soluzione (per serie)

$$\boxed{\psi = \sum_1^{\infty}\left(a_n r^{n+3} + b_n r^{n+1} + c_n r^{2-n} + d_n r^{-n}\right)Q_n(\theta) + \frac{3}{32}\left(2r^2 - 3r - \frac{1}{r}\right)\sin^2\theta\cos\theta}$$

polinomi di Gegenbauer

$$Q_0 \propto (\cos\theta + 1)$$

$$Q_1 \propto (\cos^2\theta - 1)$$

$$Q_2 \propto \sin^2\theta\cos\theta$$

......

Ricorda: $v_{1r} = -\frac{1}{r^2\sin\theta}\frac{\partial\psi}{\partial\theta} \to 0\,,\quad v_{1\theta} = \frac{1}{r\sin\theta}\frac{\partial\psi}{\partial r} \to 0$

$$r \to \infty$$

GRAVE PROBLEMA!!

!! NON SI PUO' SODDISFARE BDCD ALL'INFINITO !!

Whitehead, 1889

Perche' non funziona?

L'equazione originaria (generale)

$$\boxed{\boxed{Re\,\underline{v}\cdot\underline{\nabla}\,\underline{v} = -\underline{\nabla}p + \nabla^2\underline{v}}}$$

$$\frac{Re}{r^2} \quad << \quad \frac{1}{r^3}$$

NON sempre vera !!!!!

Creeping flow attorno alla sfera

$$\boxed{\begin{cases} v_r = \left(1 - \dfrac{3}{2r} + \dfrac{1}{2r^3}\right)\cos\theta \\ v_\theta = -\left(1 - \dfrac{3}{4r} - \dfrac{1}{4r^3}\right)\sin\theta \\ p = -\dfrac{3}{2r^2}\cos\theta \end{cases}}$$

CIOE': il termine inerziale NON E' dovunque piccolo

CIOE': lontano dalla sfera deve esistere una ADIMENSIONALIZZAZIONE diversa

CIOE': lontano dalla sfera $\left(r' \geq \dfrac{a}{Re}\right)$ la soluzione creeping E' SBAGLIATA

Il nuovo quadro

POSIZIONE r' (DIM)

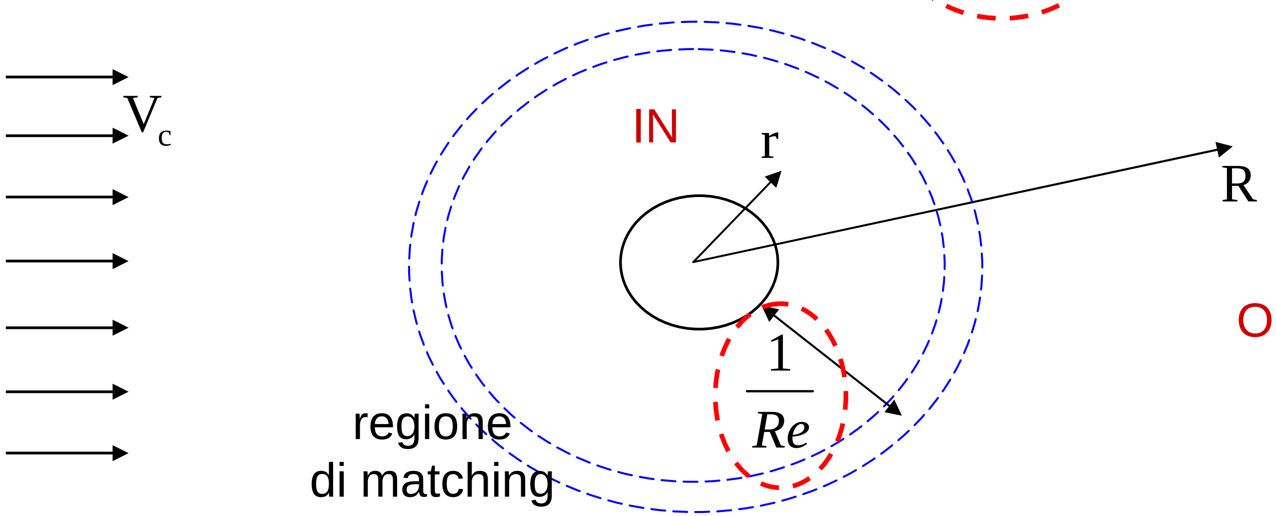
vicino alla sfera

$$L_c = a \quad \Rightarrow \quad r = \frac{r'}{a}$$

POSIZIONE r' (DIM)

lontano dalla sfera

$$L_c = \frac{a}{Re} >> a \quad \Rightarrow \quad R = \frac{r'}{a/Re}$$



$$r >> 1 \quad \Leftrightarrow \quad R << 1$$

NOTA BENE:

$$R = Re\, r \quad \Rightarrow \quad Re\, \underline{v} \cdot \underline{\nabla}_R\, \underline{v} = -\underline{\nabla}_R p + Re \nabla_R^2 \underline{v}$$

Definendo anche: $p_R = \dfrac{p}{Re}$

OUT

$$\Rightarrow \quad \boxed{\underline{v} \cdot \underline{\nabla}_R\, \underline{v} = -\underline{\nabla}_R p_R + \nabla_R^2 \underline{v}}$$

Espansioni asintotiche: come si procede 1

OUT 0

$$\boxed{\underline{v}_0 \cdot \underline{\nabla}_R\, \underline{v}_0 = -\underline{\nabla}_R\, p_{R0} + \nabla_R^2 \underline{v}_0}$$

IN 0

$$\boxed{0 = -\underline{\nabla} p_0 + \nabla^2 \underline{v}_0}$$

STOKES

ovvero

$$E_R^4\left(\psi_{R0}\right) = \Im\left[\partial_{R,\theta}\psi_{R0}\right]$$

ovvero

$$E^4\left(\psi_0\right) = 0$$

matching

$$R << 1 \;\Leftrightarrow\; r >> 1$$

SOL

$$\psi_{R0} = R^2 Q_1$$

SOL

$$\psi_0 = \left(r^2 - \frac{3}{2} r + \frac{1}{2r}\right) Q_1$$

e si ritorna a pressione e velocita'.

Espansioni asintotiche: come si procede 2

OUT 1

$$\boxed{\underline{v}_1 \cdot \underline{\nabla}_R\, \underline{v}_0 + \underline{v}_0 \cdot \underline{\nabla}_R\, \underline{v}_1 = -\underline{\nabla}_R\, p_{R1} + \nabla^2_R \underline{v}_1}$$

ovvero

$$E^4_R\left(\psi_{R1}\right) = \aleph\left[\partial_{R,\theta}\psi_{R1}\right]$$

SOL

$$\psi_{R1} \propto \left(1+\cos\theta\right)\left(1-e^{-R(1-\cos\theta)/2}\right)$$

IN 1

$$\boxed{\underline{v}_0 \cdot \underline{\nabla}\underline{v}_0 = -\underline{\nabla} p_1 + \nabla^2 \underline{v}_1}$$

Whitehead

ovvero

$$E^4\left(\psi_1\right) = \frac{9}{4}\left(\frac{2}{r^2} - \frac{3}{r^3} + \frac{1}{r^5}\right)\sin^2\theta\cos\theta$$

SOL

$$\psi_1 = \ldots\ldots + \underline{r^2 \sin^2\theta\cos\theta}$$

matching

$$R \ll 1 \iff r \gg 1$$

$$\psi_{R1} = \ldots. + R^2 \sin^2\theta\cos\theta$$ **!!!!!**

e cosi' via… (Proudman, Pearson 1958)

Statistical mechanics of a gas-fluidized particle

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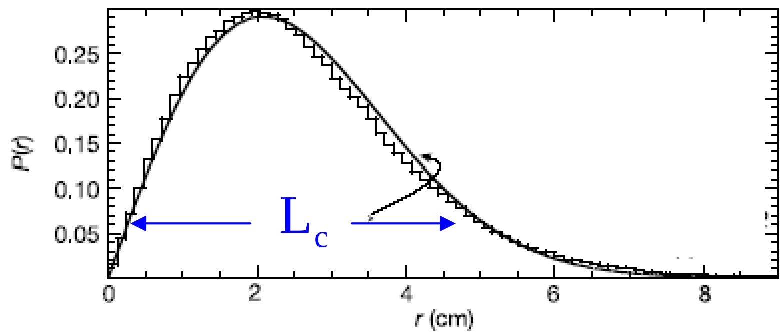
Characterization of the microscopic fluctuations in systems that are far from equilibrium is crucial for understanding the macroscopic response. One approach is to use an 'effective temperature'—such a quantity has been invoked for chaotic fluids[1,2], spin glasses[3,4], glasses[5,6] and colloids[7,8], as well as non-thermal systems such as flowing granular materials[9–14] and foams[15]. We therefore ask to what extent the concept of effective temperature is valid. Here we investigate this question experimentally in a simple system consisting of a sphere placed on a fine screen in an upward flow of gas; the sphere rolls because of the turbulence it generates in the gas stream. In contrast to many-particle systems, in which it is difficult to measure and predict fluctuations, our system has no particle–particle interactions and its dynamics can be captured fully by video imaging. Surprisingly, we find that the sphere behaves exactly like a harmonically bound brownian particle. The random driving force and frequency-dependent drag satisfy the fluctuation–dissipation relation, a cornerstone of statistical mechanics. The statistical mechanics of near-equilibrium systems is therefore unexpectedly useful for studying at least some classes of systems that are driven far from equilibrium.

$Re >> 1$

L_c

To fluidize a granular system, energy must be injected, for example by mechanical agitation or an upflow of gas strong enough to counter gravity[16,17]. The latter method is used industrially in gas-fluidized bed reactors[18], where the medium is a catalyst powder and the gas is a reactant. The nature of the fluidization transition versus flow rate, and the mechanical properties of the fluidized medium, are topics that still pose a formidable research challenge; this is largely because the relative importance of collisional, cohesive, and gas-mediated interactions is not well understood. In part to isolate gas-mediated interactions, we constructed a gas-fluidized bed for large, easily visualized spheres (see Methods) to serve as a paradigm for more typical fluidized systems, where the collisions take place on length and timescales too small to be captured by video imaging.

The nature of fluctuations in other experimental driven systems has been explored previously[19–24]. Gaussian speed distributions were found for rolling[19] and sliding[20] particles, but not for all systems of vertically vibrated beads[21–23]. In addition, Boltzmann energy distributions were found for a particle driven randomly by design[24]. Here we focus on a single sphere rolling stochastically in an upflow of gas (see Supplementary video 1 and the Methods section). In contrast with previous work, we measure not only the speed and position statistics, but also the time-dependent dynamics. This unprece-



Nature, **427** (2004)
p.521-523

CONCLUSIONI

$$Re = \frac{\rho}{\eta} L_c V_c$$

L’esistenza di *Re* finiti implica sempre la nascita di una scala di tempo / lunghezza

Lo studio di *Re*<1 non e’ il passato remoto
(simulazioni, microfluidica, …)

“Nella logica formale una contraddizione e’ il segno della sconfitta,

ma nell’evoluzione della vera conoscenza e’ il primo passo verso una vittoria”

Whitehead