

**FLUIDODINAMICA
A BASSI NUMERI DI REYNOLDS.
Introduzione**

**Francesco Greco
IRC - CNR**

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PROGRAMMA

Parte “facile”: meccanica del continuo Newtoniano
il mondo a bassi Reynolds ($\ll 1$)

Parte “difficile”: effetti di tempo (“memoria”) e/o spaziali
la tecnica delle espansioni asintotiche

Richiami sui fondamenti della meccanica (dei fluidi)

Bilancio di massa (incomprimibili)

$$\underline{\nabla} \cdot \underline{v} = 0$$

Bilancio di quantità di moto
(Cauchy, 1821)

$$\rho \frac{D\underline{v}}{Dt} = \underline{\nabla} \cdot \underline{T}$$

(**NO** effetti termici)

I campi ρ, \underline{v} , etc. sono medie di quantità microscopiche

L'ipotesi del continuo:

$$\begin{array}{ccccc} \delta & \ll & V^{1/3} & \ll & L \\ \text{atomico} & & \text{medie} & & \text{"macro"} \end{array}$$

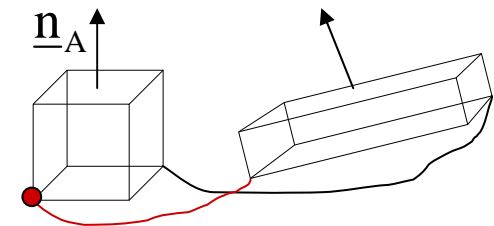
NOTA: 1 mm³ di aria
contiene
 $\approx 10^{16}$ molecole

osservazioni:

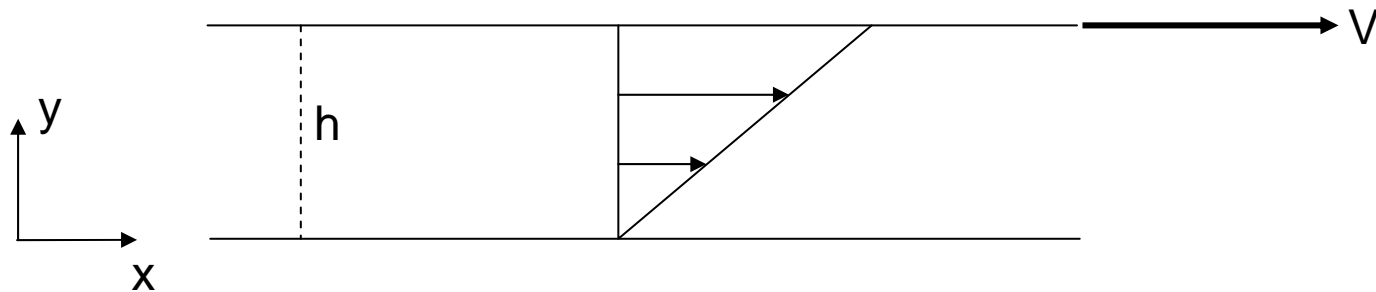
i) $D(\dots)/Dt = \partial(\dots)/\partial t + \underline{v} \cdot \underline{\nabla}(\dots)$

ii) $\underline{T} \cdot \underline{n}_A A = \underline{F}_A$ esempio: $\underline{n}_A = \hat{k} \Rightarrow \underline{F}_A = (T_{xz}, T_{yz}, T_{zz})$

iii) c'è necessità (ma non troppo!) di una equazione costitutiva



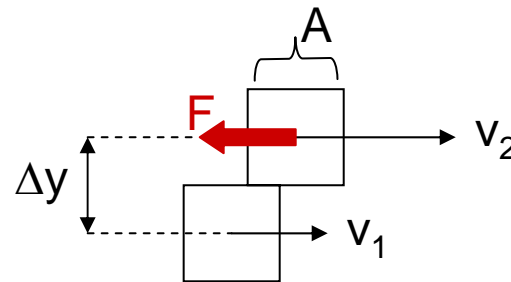
Flusso di shear. Misura della viscosità



shear rate imposta

$$\dot{\gamma}_{\text{EXT}} = \frac{V}{h}$$

Newton (1684):
“The lack of slipperyness”



$$\frac{F}{A} = \eta \frac{v_2 - v_1}{\Delta y}$$

profilo lineare di velocità $\Rightarrow \frac{dv_x}{dy} \equiv \dot{\gamma}_{\text{LOC}} = \cos t = \dot{\gamma}_{\text{EXT}}$

T_{xy} componente x della forza sulla superficie di normale y
(si misura su uno dei piatti)

$$\eta = \frac{T_{xy}}{\dot{\gamma}_{\text{EXT}}}$$

Viscosità tipiche

$$\eta = \frac{T_{xy}}{\frac{dv_x}{dy}}$$

$$[\eta] = \frac{\frac{N}{m^2}}{\frac{ms^{-1}}{m}} = Nm^{-2}s \equiv Pa\ s = m^{-1} kg\ s^{-1}$$

GAS ~ 10^{-5} Pa s


ACQUA ~ 10^{-3} Pa s

MIELE ~ 10 Pa s

PLASTICA ~ 10^3 Pa s

ASFALTO ~ 10^8 Pa s

VETRO ~ 10^{12} Pa s

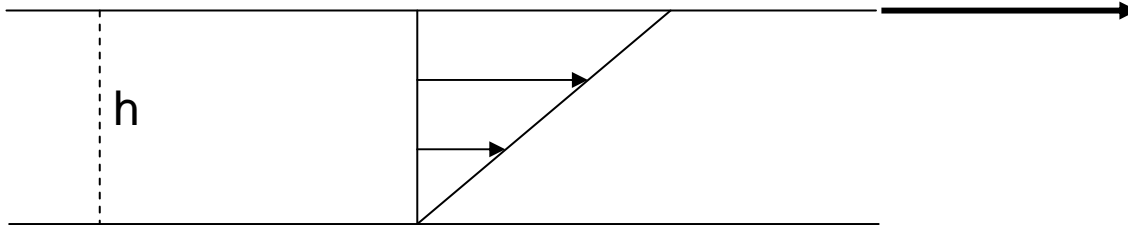
17 ordini
di grandezza !!!!


Sull'ipotesi di Newton... (1)

$$T_{xy} = \eta \frac{dv_x}{dy}$$

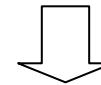
e' una equazione costitutiva (ma non troppo!)

WARNING:



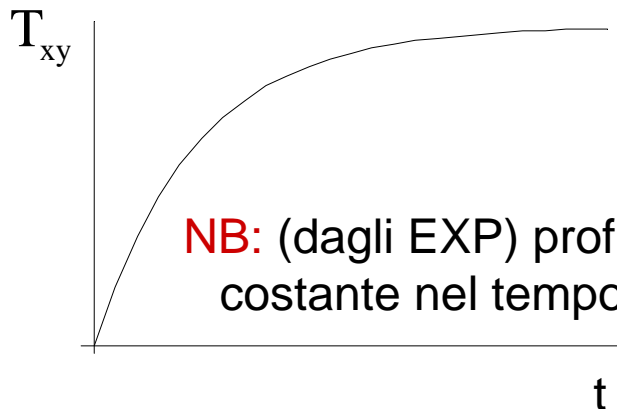
shear rate costante

$$\dot{\gamma}_{EXT} = \frac{V}{h} = \cos t$$



sforzo costante

**non sempre e' cosi'
(ad es., PLASTICA)**



NB: (dagli EXP) profilo lineare di v,
costante nel tempo, **istantaneo**

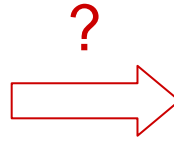
**altra equazione costitutiva
(fluido non-Newtoniano)**

un tempo!

$$\tau \frac{dT_{xy}}{dt} + T_{xy} = \eta \frac{dv_x}{dy}$$

Sull'ipotesi di Newton... (2)

$$T_{xy} = \eta \frac{dv_x}{dy}$$

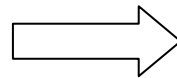


$$\underline{\underline{T}} = \eta \underline{\underline{\nabla v}} \quad (0)$$

Proviamo:

$$\underline{\underline{\nabla v}} = \begin{pmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$[a] = s^{-1}$$



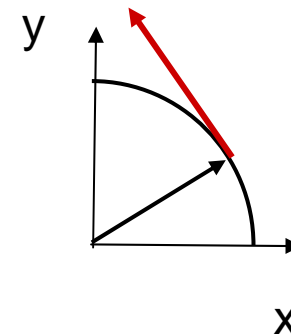
c'e' uno sforzo

!! ERRORE !!

Infatti:

$$\underline{\underline{\nabla v}} \cdot \underline{r} = \begin{pmatrix} 0 & a & 0 \\ -a & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ay \\ -ax \\ 0 \end{pmatrix}$$

e' una **velocita'**
nel piano xy



**rotazione
rigida !!**

Una rotazione rigida NON PUO' generare sforzo.

La (0) non e' **INVARIANTE PER ROTAZIONE**.

Tensore degli sforzi Newtoniano

$$\underline{\underline{T}} = -p\underline{\underline{I}} + \eta(\underline{\nabla}\underline{v} + \underline{\nabla}\underline{v}^T)$$

Il bilancio di quantità di moto diviene:

$$\rho \frac{D\underline{v}}{Dt} = -\underline{\nabla}p + \eta \nabla^2 \underline{v}$$

Equazione di
Navier – Stokes
(1850)

osservazioni:

i) La pressione, **in generale**, non si può misurare (!)

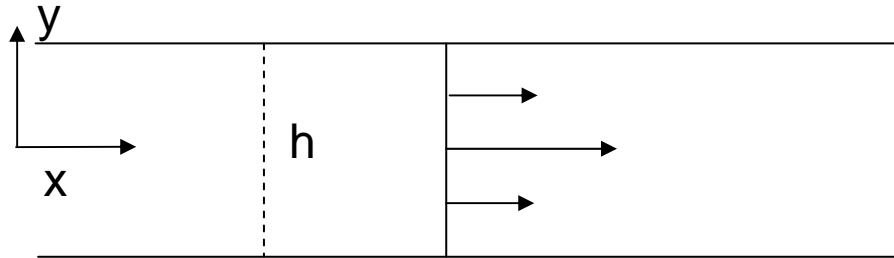
ii) Nonlinearità dai soli **termini inerziali**: $\rho \left(\frac{\partial v_i}{\partial t} + v_j \frac{\partial v_i}{\partial x_j} \right)$

iii) $\exists! v_x(y,t)$ $\rho \frac{\partial v_x}{\partial t} = -\frac{\partial p}{\partial x} + \eta \frac{\partial^2 v_x}{\partial y^2}$

iv) Lo shear stazionario: $\frac{d^2 v_x}{dy^2} = 0$ BD CD: $\begin{matrix} v_x(y=h) = V \\ v_x(y=0) = 0 \end{matrix} \Rightarrow$

$$v_x = \frac{V}{h} y = \dot{\gamma}_{EXT} y$$

Flusso in “tubi”



* **pressione** imposta

** **portata** Q imposta

$$Q = \rho \langle v \rangle A$$

Navier – Stokes (stazionario): $\frac{\partial p}{\partial x} = \eta \frac{\partial^2 v_x}{\partial y^2}$

BD CD: $v_x(y = h) = 0$
 $v_x(y = 0) = 0$

{	profilo lineare di pressione	⇒	$p = cx + p_0$
	profilo parabolico di velocità	⇒	$v_x = \frac{1}{2\eta} \frac{dp}{dx} y^2 + ay + b$

Misura della viscosità

Il numero di Reynolds

$$\rho \frac{D\underline{v}}{Dt} = -\underline{\nabla} p + \eta \nabla^2 \underline{v}$$

Nell' equazione di Navier - Stokes si devono assegnare la geometria e le condizioni al contorno

$$L_c, V_c$$

$$\underline{\nabla} = \frac{1}{L_c} \underline{\nabla}^*$$

$$\underline{v} = V_c \underline{v}^*$$

$$t = \frac{L_c}{V_c} t^*$$

$$p = \eta \frac{V_c}{L_c} p^*$$

$$\underline{\nabla}^* \cdot \underline{v}^* = 0$$

$$Re \frac{D\underline{v}^*}{Dt^*} = -\underline{\nabla}^* p^* + \nabla^{2*} \underline{v}^*$$

con :

$$Re = \frac{\rho}{\eta} L_c V_c$$

Proprieta' di SIMILARITA'

Proprieta' di STABILITA'

Reynolds (1883)



Figure 8: This picture shows laminar flow

Reynolds (1883)



Figure 6: This picture shows flow in the turbulent regime

Re alti implicano instabilità' (e turbolenza)

Osservazione sull'adimensionalizzazione

$$\rho \frac{D\underline{v}}{Dt} = -\underline{\nabla} p + \eta \nabla^2 \underline{v}$$

Nell' equazione di Navier - Stokes si devono assegnare la geometria e le condizioni al contorno

$$L_c, V_c$$

$$\underline{\nabla} = \frac{1}{L_c} \underline{\nabla}^*$$

$$\underline{v} = V_c \underline{v}^*$$

$$t = \frac{L_c}{V_c} t^*$$

$$p = \rho V_c^2 p^*$$

$$\underline{\nabla}^* \cdot \underline{v}^* = 0$$

$$Re \frac{D\underline{v}^*}{Dt^*} = -Re \underline{\nabla}^* p^* + \nabla^{2*} \underline{v}^*$$

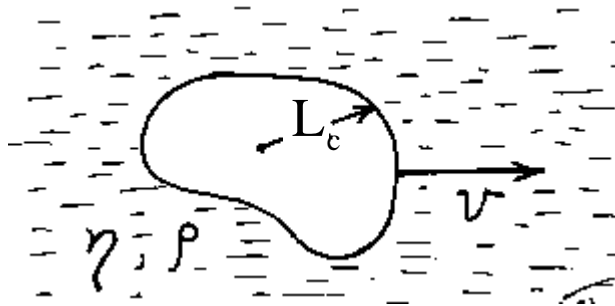
con :

$$Re = \frac{\rho}{\eta} L_c V_c$$

Re alti o bassi dettano l'adim

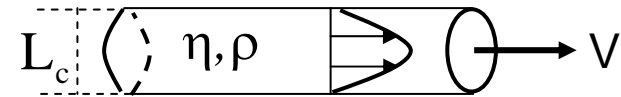
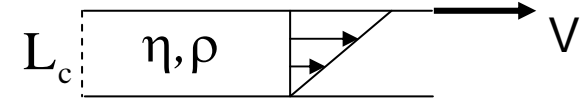
Numeri di Reynolds bassi

moto attorno



$$Re = \frac{L_c V_c}{\frac{\eta}{\rho}}$$

moto dentro



sospensioni

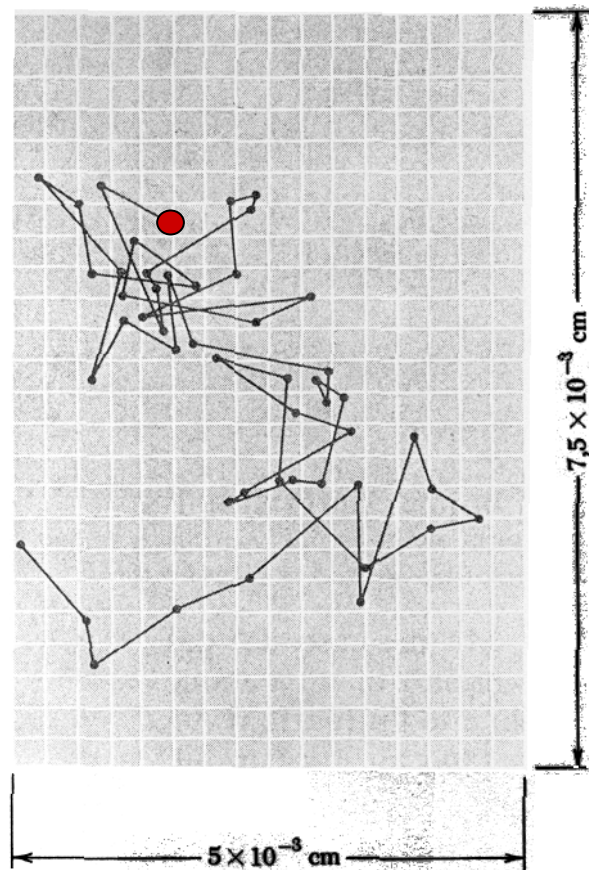
“viscosità cinematica”: $\nu = \frac{\eta}{\rho}$ $[\nu] = \frac{\text{Pa s}}{\frac{\text{kg}}{\text{m}^3}} = \frac{\text{m}^{-1} \text{kg s}^{-1}}{\frac{\text{kg}}{\text{m}^3}} = \frac{\text{m}^2}{\text{s}}$ (diffusività)

tabella

per ottenere
bassi Re

	η	ρ	ν	
GAS	10^{-5} Pa s	1 kg m^{-3}	$10^{-5} \text{ m}^2 \text{ s}^{-1}$	$\nearrow L_c V_c \sim 10^{-10} \frac{\text{m}^2}{\text{s}}$
ACQUA	10^{-3} Pa s	10^3 kg m^{-3}	$10^{-6} \text{ m}^2 \text{ s}^{-1}$	
MIELE	10 Pa s	$\sim 10^3 \text{ kg m}^{-3}$	$10^{-2} \text{ m}^2 \text{ s}^{-1}$	$\longrightarrow L_c V_c \sim 10^{-4} \frac{\text{m}^2}{\text{s}}$
POLIMERO	10^3 Pa s	$\sim 10^3 \text{ kg m}^{-3}$	$1 \text{ m}^2 \text{ s}^{-1}$	

SOSPENSIONI: Moti Browniani



ESPERIMENTI

Brown (1837)

Perrin (1916):

$$\langle r^2 \rangle \propto t$$

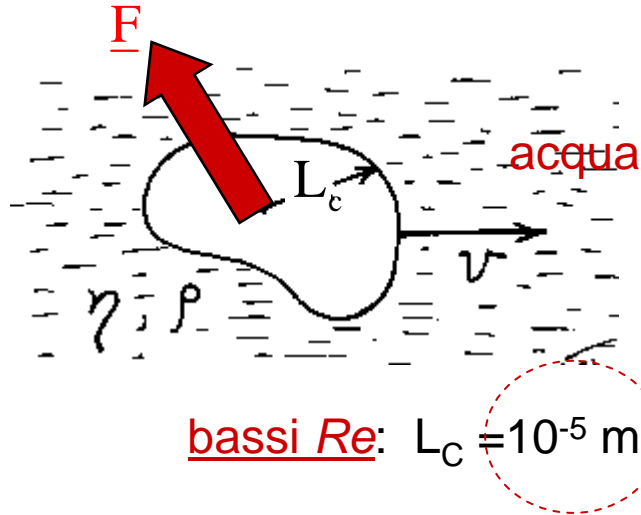
FIGURA 1.25 Moto browniano di una particella solida, avente un diametro $< 10^{-4}$ cm, che è stata messa in sospensione nell'acqua e osservata al microscopio. In questo disegno si mostra la proiezione del moto tridimensionale di tale particella sul piano orizzontale del campo visivo di un microscopio; le linee congiungono le posizioni successive della particella osservata a intervalli di 30 secondi. [I dati sono forniti da J. Perrin, *Atoms*, (Gli Atomi), p. 115. (D. Van Nostrand Company, Inc., Princeton, N.J., 1916)].

TEORIA

Einstein (1905)

Langevin (1907)

SOSPENSIONI: Effetti inerziali



moto della
particella:

$$m_p \underline{a} = \sum_i \underline{F}_i = \underline{F}_{\text{attrito}} + \underline{F}$$

Cosa succede se spengo \underline{F} ?

$$\rho_P v_P \frac{\Delta v}{\Delta t} = \zeta v$$

ζ coefficiente di attrito

$$\zeta = L_c \eta$$

$$1) \quad \Delta t_{\text{STOP}} = \frac{\rho_P v_P}{\zeta} = \frac{\rho}{\eta} \frac{v_P}{L_c} = \frac{1}{v} L_c^2 = \frac{1}{10^{-6} \text{ m}^2 \text{ s}^{-1}} (10^{-5} \text{ m})^2 = 10^{-4} \text{ s}$$

indipendentemente dalla velocita' !

$$2) \quad \Delta x_{\text{STOP}} = v \Delta t_{\text{STOP}} = 10^{-3} \text{ ms}^{-1} 10^{-4} \text{ s} = 10^{-7} \text{ m}$$

un mondo ARISTOTELICO !!!

Vita in acqua: $\nu = 10^{-6} \text{ m}^2 \text{ s}^{-1}$



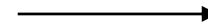
$$\underline{L = 1 \text{ m}}$$

$$\underline{V = 1 \text{ ms}^{-1}} \rightarrow$$

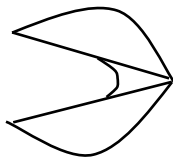
$$Re \sim 10^6$$



$$\underline{1 \text{ dm}}$$



$$Re \sim 10^4$$

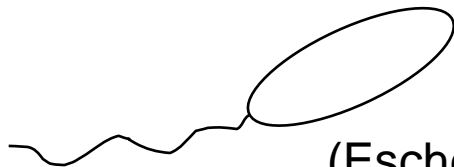


(conchiglia)

$$\underline{1 \text{ cm}}$$



$$Re \sim 10^2$$



(Escherichia Coli)

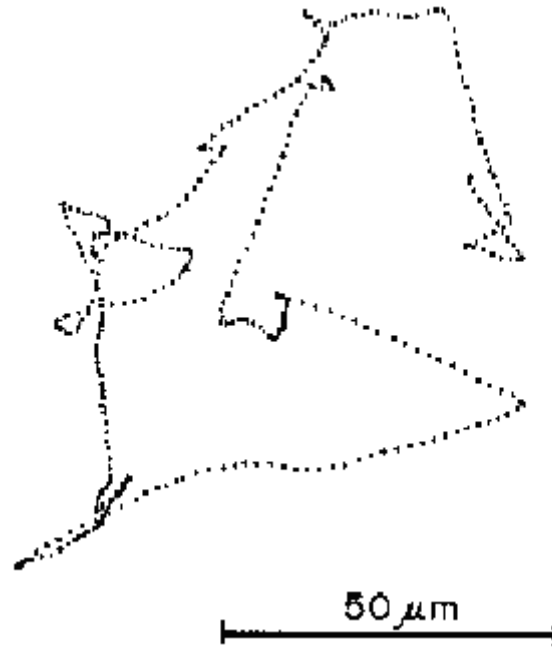
$$\underline{1 \mu\text{m}}$$



$$Re \sim 10^{-6} \text{ !!!!}$$

Un problema di vita a bassi Reynolds: Serve nuotare? (E. Purcell, 1976)

osserviamo il moto
dell' E.Coli



cibo a distanza ℓ

Stirring vs. Diffusion

time for transport by stirring: $\frac{\ell}{v}$

time for transport by diffusion: $\frac{\ell^2}{D}$

stirring works if $\frac{\ell v}{D} > 1$

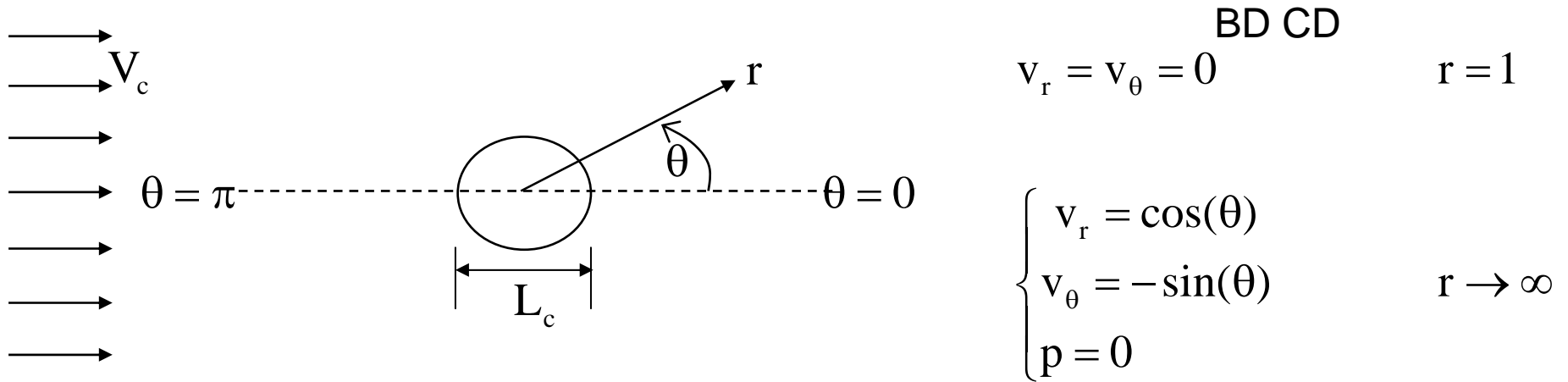
valori tipici: $V_c \sim 10^{-4} \text{ ms}^{-1}$
 $D \sim 10^{-9} \text{ m}^2 \text{ s}^{-1}$ $\Rightarrow \frac{D}{V_c} \sim 10^{-5} \text{ m} \sim 10 \mu\text{m}$

conclusione: perche' convenga muoversi, deve essere: $\ell > 10 \mu\text{m}$ (!!!)

local stirring accomplishes nothing

$$V_c > 10 \mu\text{ms}^{-1} \quad (!!!)$$

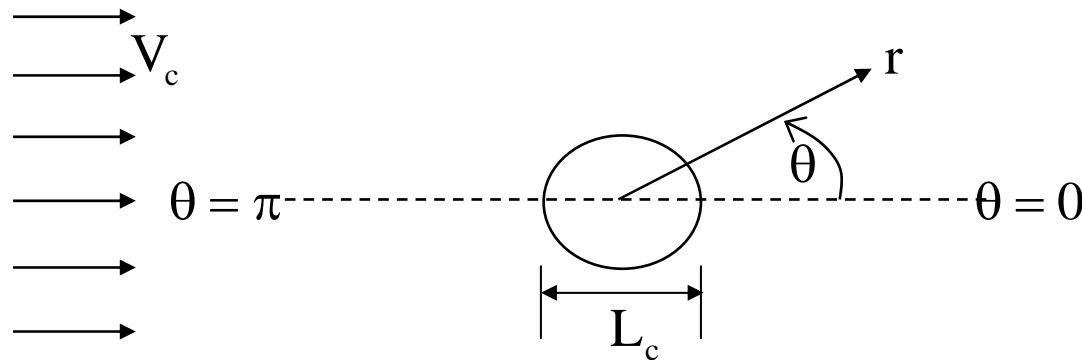
Creeping flow attorno ad una sfera



$$\begin{cases} v_r = \left(1 - \frac{3}{2r} + \frac{1}{2r^3}\right) \cos \theta \\ v_\theta = -\left(1 - \frac{3}{4r} - \frac{1}{4r^3}\right) \sin \theta \\ p = -\frac{3}{2r^2} \cos \theta \end{cases}$$

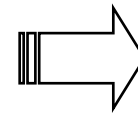
$$\mathbf{F}_{\text{DRAG}} = \int_{\text{sup}} (\underline{\mathbf{T}} \cdot \underline{\mathbf{i}}_r) \cdot \underline{\mathbf{i}}_z dA = 6\pi\eta R V_c \quad (\text{STOKES, 1845})$$

Altra procedura di soluzione (“tensoriale”)



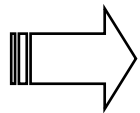
Da determinare:
 uno scalare p
 un vettore \underline{v}

i) Vettori (o tensori) disponibili: \underline{V}_c , \underline{r}



ii) Linearita' creeping \Rightarrow p e \underline{v} lineari in \underline{V}_c

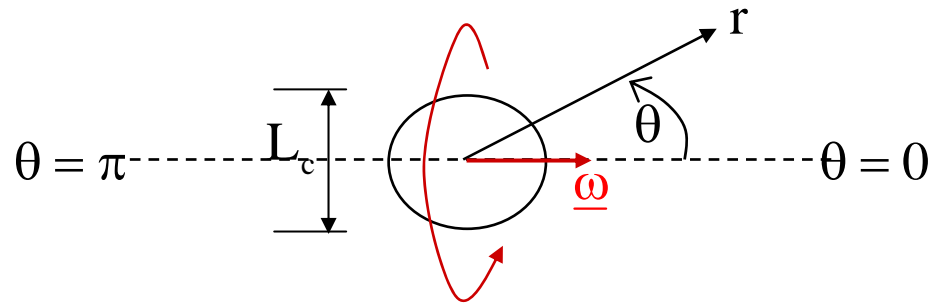
scalare $(\underline{V}_c \cdot \underline{r})$
 vettori \underline{V}_c , $(\underline{V}_c \cdot \underline{r})\underline{r}$



$$p = P(r)(\underline{V}_c \cdot \underline{r})$$

$$\underline{v} = F(r)\underline{V}_c + G(r)(\underline{V}_c \cdot \underline{r})\underline{r}$$

Sfera in rotazione

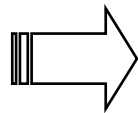


BD CD

$$\begin{cases} v_\phi = \omega R \sin \theta \\ v_r = v_\theta = 0 \end{cases} \quad r = R$$

$$\begin{cases} v_\phi = v_r = v_\theta = 0 \\ p = 0 \end{cases} \quad r \rightarrow \infty$$

Vettori (o tensori) disponibili: $\underline{\omega}$, \underline{r} \Rightarrow NO scalari, vettore $\underline{\omega} \times \underline{r}$

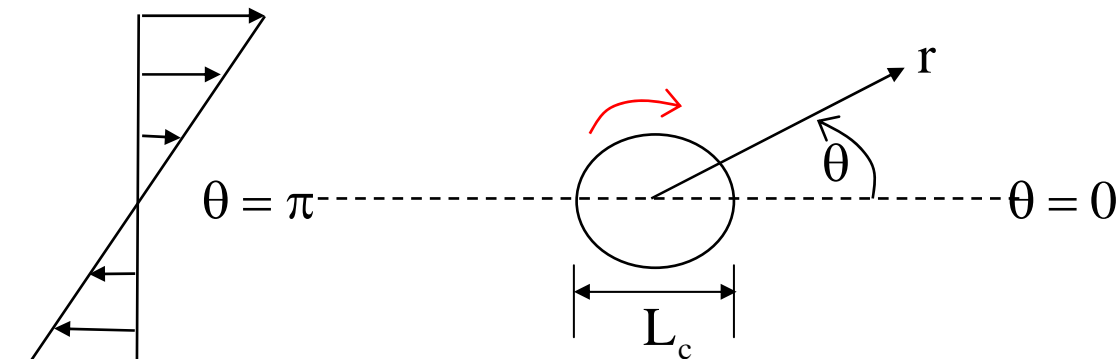


$$p = 0$$

$$\underline{v} = F(r) \underline{\omega} \times \underline{r} = \frac{1}{r^3} \underline{\omega} \times \underline{r}$$

Sfera in shear (Einstein, 1905)

BD CD



non dico nulla ad $r = 1$

$$\begin{cases} \underline{v}_x = \dot{\gamma} y \\ \underline{v}_y = \underline{v}_z = 0 \\ p = 0 \end{cases} \quad r \rightarrow \infty$$

Invarianza rotazionale

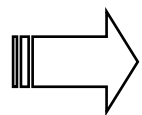


Vettori (o tensori) disponibili: $\underline{\nabla} \underline{v}^{(\infty)} = \begin{pmatrix} 0 & \dot{\gamma} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \underline{r}$

$$\underline{\underline{D}}^{(\infty)} = \underline{\nabla} \underline{v}^{(\infty)} + \underline{\nabla} \underline{v}^{(\infty)T}$$

$$\underline{\underline{\Omega}}^{(\infty)} = \underline{\nabla} \underline{v}^{(\infty)} - \underline{\nabla} \underline{v}^{(\infty)T}$$

⇒ scalare $(\underline{\underline{D}}^{(\infty)} \cdot \underline{r} \cdot \underline{r})$ vettori $\underline{\underline{D}}^{(\infty)} \cdot \underline{r}, \quad \underline{\underline{\Omega}}^{(\infty)} \cdot \underline{r}, \quad (\underline{\underline{D}}^{(\infty)} \cdot \underline{r} \cdot \underline{r}) \underline{r}$



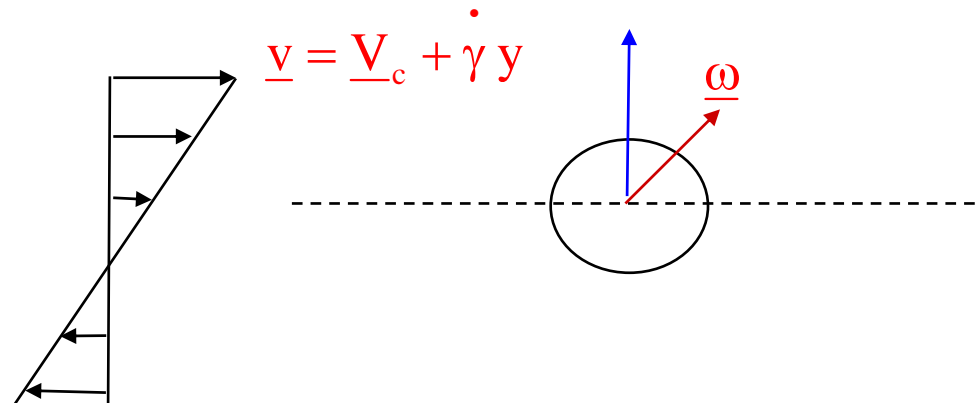
$$p = P(r) (\underline{\underline{D}}^{(\infty)} \cdot \underline{r} \cdot \underline{r})$$

$$\underline{v} = F(r) \underline{\underline{D}}^{(\infty)} \cdot \underline{r} + G(r) \underline{\underline{\Omega}}^{(\infty)} \cdot \underline{r} + H(r) (\underline{\underline{D}}^{(\infty)} \cdot \underline{r} \cdot \underline{r}) \underline{r}$$

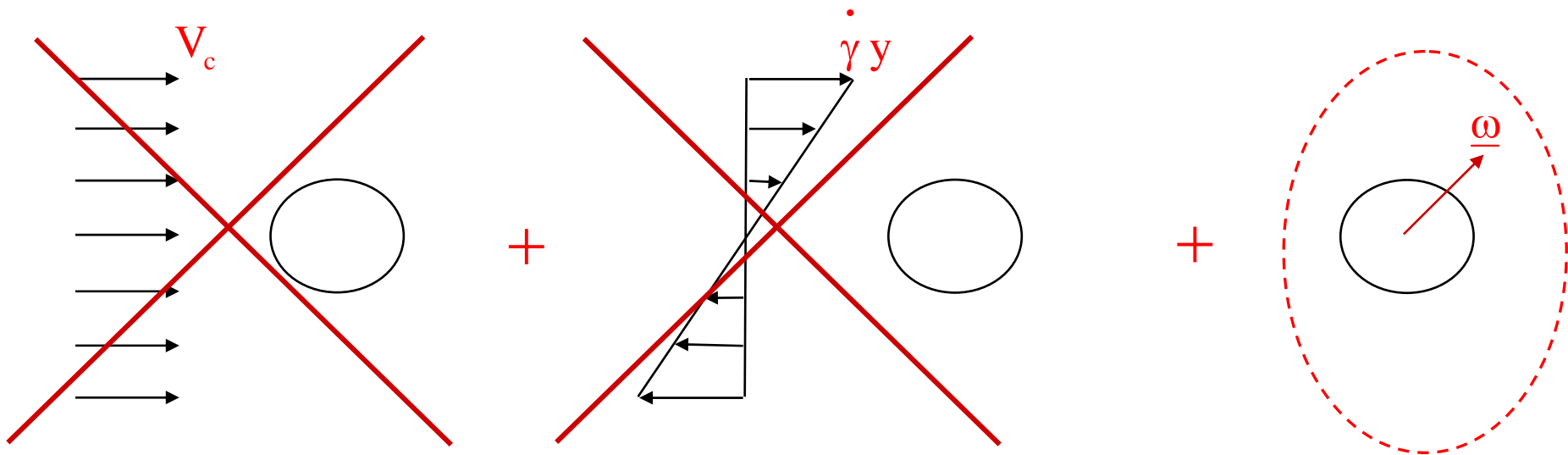
**MOLTO
GENERALE!!!**

etc

Sfera rotante e traslante in shear

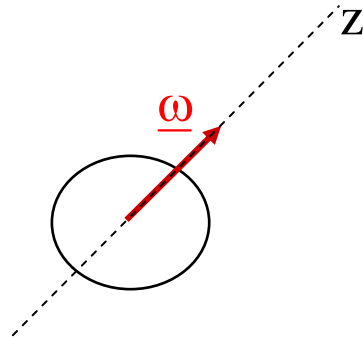


e' uguale a



DOMANDA: oltre al DRAG, ci puo' essere LIFT?

PRIMA RISPOSTA: se c'e' LIFT, e' lungo l'asse



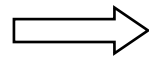
devo calcolare:

$$F_{\text{LIFT}} = \int_{\text{sup}} (\underline{T} \cdot \underline{i}_r) \cdot \underline{i}_z dA$$

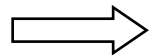
MA:

Ricorda:

$$\underline{p} = 0$$
$$\underline{v} = F(r) \underline{\omega} \times \underline{r}$$



NON c'e' contributo

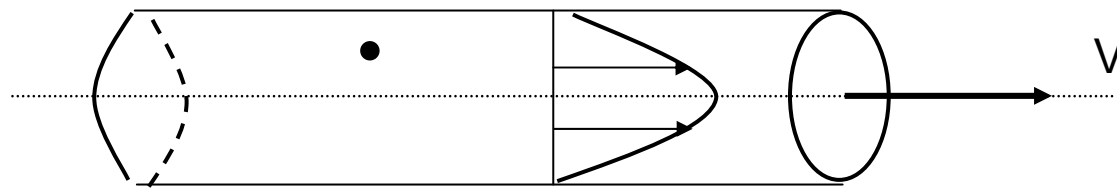


NON c'e' v_z
ne' v_r $\Rightarrow T_{rz}(v_r, v_z) = 0$



QUINDI: NON c'e' LIFT

**ATTENZIONE: in alcuni EXPS ci puo' essere LIFT
anche a velocita' molto basse**



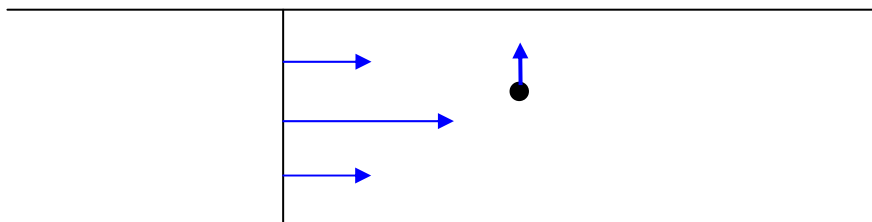
Segre'-Silberberg 1962

$$r_{ss} \rightarrow 0.6R$$

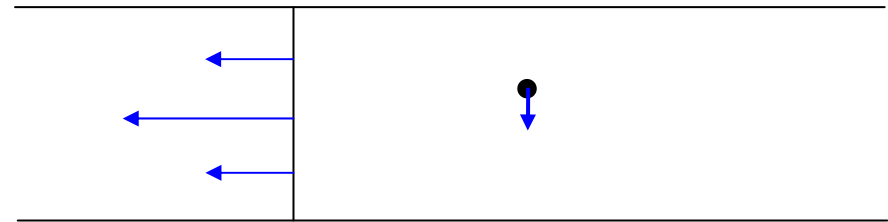
? creeping flow ?

$$\nabla p = \nabla^2 \underline{v}$$

soluzione p , \underline{v} , v_p



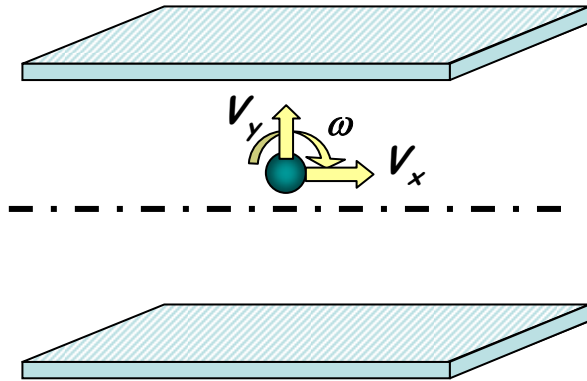
soluzione $-p$, $-\underline{v}$, $-v_p$



e' un effetto di Re .

SOLUZIONE ANALITICA 1980!! SOLUZIONE NUMERICA 2001!!!!!!

Equazioni



Assumiamo:

- Fluido Newtoniano, Re finito
- Δp assegnato
- no-slip

Governing equations

$$\nabla \cdot \mathbf{u} = 0$$

$$-\nabla p + \eta \nabla^2 \mathbf{u} = \rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right)$$

Boundary conditions

$$\mathbf{u} = (0, 0, 0) \quad \text{on } \partial\Omega_{UP, DOWN}$$

$$\mathbf{u} = \mathbf{V} + \boldsymbol{\omega} \times \mathbf{r} \quad \text{on } \partial P(t)$$

Particle motion

$$\frac{d\mathbf{V}}{dt} = \mathbf{a}$$

$$\mathbf{F} = m\mathbf{a}$$

$$\frac{d\boldsymbol{\omega}}{dt} = \boldsymbol{\alpha}$$

$$\boldsymbol{\tau} = I\boldsymbol{\alpha}$$

Force and Torque

$$\mathbf{F}_{TOT} = \int_{\partial P(t)} \mathbf{T} \cdot \mathbf{n} dA$$

$$\boldsymbol{\tau}_{TOT} = \int_{\partial P(t)} \mathbf{r} \times (\mathbf{T} \cdot \mathbf{n}) dA$$