An Auction Algorithm for Procuring Wireless Channel in A Heterogenous Wireless Network

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Abstract— **In this paper, we develop an auction algorithm for procuring wireless channel by a wireless node in a heterogeneous wireless network. We assume that the service providers of the heterogeneous wireless network are selfish and non-cooperative in the sense that they are interested in maximizing their own utilities. The wireless user is in need of procuring wireless channel to execute multiple jobs. To solve the problem of wireless user, we propose a *reverse optimal (REVOPT)* auction. We characterize the expression for the expected payment by the wireless user. Our proposed auction mechanism *REVOPT* satisfies important game theoretic properties like Bayesian Incentive Compatibility and Individual Rationality.**

Keywords

Heterogeneous wireless network, game theory, mechanism design, selfishness, rationality, individual rationality, Bayesian incentive compatibility, optimal auction mechanisms.

I. INTRODUCTION

The primary goal in the wireless communication world can be briefly summarized as providing service for communication anywhere, anytime, any-media and principally at high-data rates. However, this goal is in conflict with the existence of different running and emerging wireless systems covering almost the whole world, each one following its own architecture.

The development of wireless systems evolved in an unimaginable way during the last two decades. For example, in cellular wireless systems, the so-called First Generation (1G) is no longer in use. Currently the dominant generations, which are nowadays attracting much attention, are 2G, 2.5G and 3G. In Europe their representatives are GSM (Global System for Mobile Communication), GPRS (General Packet Radio Service) and UMTS (Universal Mobile Telecommunications System) respectively and belong to the terrestrial wide area cellular systems. The circuit-switched GSM provides very slow data rates (9.6 14.4 kbps) to satisfy the burst applications, even after the appliance of High Speed Circuit Switched Data (HSCSD), it does not overcome the limit of 40 kbps. Packet-switched networks, based on the access network of GSM with actual changes only in the core network (GPRS), appeared with the promise of higher bit rates (theoretically 172 kbps), but in practice the maximum bit rate achieved is about 45 kbps. The UMTS access network follows a different approach, in comparison to GSM and GPRS, making the achievement of higher data rates more feasible. UMTS offers data rates up to 384 kbps, even if in theory the 2 Mbps transfer rate is possible. Nevertheless, the actual performance of UMTS has still to be verified during real operation conditions with heavy network loads.

On the other hand, development in new radio access technologies and increase in user demand for ubiquitous high speed access are driving the deployment of a wide array of wireless networks, ranging from wireless WAN to wireless MAN, Wireless LAN and Wireless PAN. These kind of networks provide incomparably high data rates. For example the 802.11b WLAN provides throughput up to 5 Mbps, while the data rates in 802.11a can be up to over 25 Mbps, with the perspective to reach in the future the inconceivable limit of 155 Mbps.

With complementary characteristics especially in terms of data rate and coverage of the various wireless communication technologies, the co-existence of these technologies results in a heterogeneous set of wireless communications systems that can provide better communication and service facilities to the mobile/wireless nodes. Such heterogenous set of wireless communication systems is called *Heterogeneous Wireless Networks*. There is another important reason for going towards heterogeneous wireless networks. In some type of wireless networks where there is no access point, such as wireless ad hoc networks, protocols suffer in network performance that includes large routing overhead, low throughput, and large end-to-end delay. In such networks, the issues of quality of service (QoS) are even more complicated because of the lack of reliable methods to distribute information in the entire network. The integration of heterogeneous wireless technologies can improve the network performance, thereby meeting the demands for different quality of service (QoS). Heterogeneous wireless networks give satisfiable solutions to the problems we mentioned above.

Heterogeneous wireless networks provide overlapping coverage to mobile users. Its active components are based on different theoretical backgrounds and are optimized for different ranges. Heterogeneous wireless networks pose many interesting research challenges. Among which, resource management in such a hybrid environment is still an open problem.

A. Problem Statement

In this paper, we attempt to provide solution to the following problem. We consider the situation where a mobile/wireless user has to perform a set of jobs, where each job is a parallel application and can be split over different wireless networks for execution. The mobile user can access heterogeneous wireless networks provided by a set of selfish, intelligent, and non-cooperative service providers. These service providers charge for allocating the wireless channel to the mobile/wireless users. Ultimately, the problem for a mobile/wireless user is to procure wireless channel to perform the jobs while minimizing the total amount to pay to the network service providers. So, there is a need for a mechanism that should be optimal (i.e. in minimal sense) for the mobile user and satisfy important game theoretic properties, say Bayesian incentive compatibility and individual rationality, so that the selfish and non-cooperative service providers participate in bidding for the time slots which are announced by the wireless user.

B. Contributions of the Paper

As far as our knowledge is concerned, research is currently going on developing auction algorithms for wireless channel allocation in a single wireless network environment. The work presented in this paper is perhaps the earliest works in developing auction based algorithms in heterogeneous wireless network environment. In this paper, we develop a non-cooperative game theoretic based mechanism to solve the wireless channel procurement problem of a wireless/mobile user having access to a heterogeneous wireless network. Our work can be organized in the following way.

- We first define what we call *Resource Procurement Auction* to explain the context of the problem.
- Then, we formulate Resource Procurement Auction as a mechanism design problem in a quasi-linear environment.
- We next design a *reverse optimal (REVOPT)* mechanism for Resource Procurement Auction problem. In this design, we characterize both the allocation rule and payment rule of *REVOPT* auction mechanism. Finally, we compute an expression for the optimal (in minimal sense) total payment by the wireless/mobile user using our approach.

C. Organization of the Paper

The paper is organized in the following. Section 2 provides a review of the related literature. In Section 3, we define Resource Procurement Auction and also formulate it as a mechanism design problem in a quasi-linear environments. We present the *REVOPT* auction mechanism in Section 4. We conclude the paper in Section 5.

II. RELATED WORK

In this section, we present the related work from the literature. As we are concerned with developing auction based mechanism to the resource procurement problem in heterogeneous wireless networks, we present the review of the literature in two parts. In Section II(A), we present the research work in the area of heterogeneous wireless networks and resource management. In Section II(B), we deal with the review of related literature work on reverse auctions.

A. Research Work on Heterogeneous Wireless Networks

Examples of integrated heterogeneous wireless networks include ad hoc/cellular integrated networks. Wu, Mukherjee, and Chan proposed mobile-assisted connection-admission (MACA) channel allocation scheme to achieve load balancing in a cellular network [1]. In MACA, some special channels are used to connect mobile units from different cells. When a mobile unit cannot connect to its own base station due to heavy load, it may be able to get connected to its neighboring cells base station through a two-hop link. A similar approach, integrated cellular and ad hoc relaying systems (iCAR), is proposed by Wu et al. in [2]. It addresses the congestion problem due to unbalanced traffic in a cellular system and provides interoperability for heterogeneous networks. The basic idea is to place a number of ad hoc repaying stations at strategic locations, which can be used to relay signals between mobile hosts and base stations.

In [3], Brewer, et al. present the results of the BARWAN project, which focused on enabling truly useful mobile networking across an extremely wide variety of real-world networks and mobile devices. The authors present the overall architecture that enables seamless roaming in a single logical overlay network composed of many heterogeneous (mostly wireless) physical networks, and provides significantly better TCP performance for these networks. It also provides complex scalable and highly available services to enable powerful capabilities across a very wide range of mobile devices, and mechanisms for automated discovery and configuration of localized services.

Topology Control in Heterogeneous Wireless Networks is addressed for the first time in [4]. This presents the possible topology control problems in these networks and provides solutions to these problems. It proposes two localized topology control algorithms for heterogeneous wireless multi-hop networks with non-uniform transmission ranges: Directed Relative Neighborhood Graph (DRNG) and Directed Local Minimum Spanning Tree (DLMST). In both algorithms, each node selects a set of neighbors based on the locally collected information.

In [5], the authors present novel network scenarios where wired and wireless connections are melted together, a real measure of these parameters is fundamental in a planning process of new services over novel network infrastructures. Nowadays networks are heterogeneous in terms of access network technologies (wired LAN Ethernet 10/100/1000, Wireless LAN - 802.11a, 802.11b, 802.11g -, GPRS, UMTS, GSM, Bluetooth, ...), end-users devices (workstation, PC desktop, Laptop/Notebook, PDA, Advanced Mobile Phone, ...) and finally operating systems (Unix, Linux, Win 98/NT/2000/XP, Win CE, Linux Familiar, OS Embedded, ...). The authors also provide a heterogeneous network performance characterization with respect to delay and throughput in UDP and TCP environments.

Kyriazakos, et al. investigated the real-time radio resource management in heterogeneous wireless networking environments in [6]. The authors presented a methodology and an approach for designing a hierarchical system that is augmenting the functionality of wireless network architectures by enforcing smooth co-operation and is capable to react when resource shortcomings appear.

Qadeer, et al. in [7] presented an approach for power management of the wireless network interfaces (WNIC) for heterogeneous wireless networks. The authors develop an integrated approach for the management of power and performance of mobile devices for these environments. Their policy decides which WNIC to employ for a given application and optimizes its usage based on the current power and performance needs of the system. The policy dynamically switches between WNICs during program execution if data communication requirements and/or network conditions change.

Suliman et al. for the first time introduced cooperative game theoretic concepts for resource allocation in heterogeneous wireless networks in [8]. But, this paper does not provide the mathematical modelling of the cooperative game associated with the resource allocation problem in which different wireless network service providers cooperate among themselves. In contrast to this approach, we consider the situation where the service providers of the heterogeneous wireless network are selfish, intelligent, and *non-cooperative* in the sense that they are interested in maximizing their own utilities. We provide rigorous mathematical modelling of our model.

B. *Research Work on Reverse Auctions*

Auctions are concerned with the design of certain *rules of interaction* using the tools of game theory and mechanism design [9], [10], for electronic transactions that will, in principle, yield some desired outcome. In the context of negotiations for procurement we require rules governing: (1)bidding for contracts, (2)the issues and attributes that will be considered to determine winner(s) of the contract, (3)determination of winning suppliers, and (4)payments that will be made. English auctions and Dutch auctions, and sealed bid contracts are well understood, widely used economic mechanisms in the context of reverse auctions. Since the *rules of interaction* in these auctions are well laid out, they have been a natural target for automation.

A comprehensive survey on reverse auction based mechanisms appears in [11]. Other recent surveys can be found in [12],[13].

III. RESOURCE PROCUREMENT AUCTION

We consider the resource procurement problem for a mobile user having access to heterogeneous wireless networks provided by a set of selfish and non-cooperative service providers. We assume that the mobile user is equipped with multiple network interfaces in order to access heterogeneous wireless networks. The mobile user has a set of different jobs to be performed. The worth to the mobile user by different jobs is also different. So, the time slots for getting access to wireless channel to perform these jobs also worth differently to the wireless/mobile user. Without loss of generality, we assume that the mobile user announces the time slots in decreasing order of the preference. It means that the value of the mobile user for the first time slot is more when compared to the second time slot, the value for the second time slot is more when compared to the third time slot, so on. Once the time slots are announced, then the service providers will submit bids on them. Having received the bids, the mobile user uses a mechanism for selecting the winning bids and deciding the payment to the winning bidders. We call this auction mechanism *Resource Procurement Auction*. The payment by the mobile user to the winning service providers depends on the bids submitted by the service providers (bidders).

Now, let us assume there are n service providers and $N = \{1, 2, ..., n\}$ represents the set of service providers. Let there are m time slots announced by the mobile user and $M = \{1, 2, ..., m\}$ represents the set of time slots to be auctioned. Let $\theta = (\theta_1, \theta_2, ..., \theta_n)$ be the vector of bids received from these n service providers. Let $p_i(\theta)$ be the payment to the service provider i by the mobile user, when the vector of bids of the service providers is θ.

A. *Resource Procurement Auction as Mechanism Design Problem*

To model Resource Procurement Auction as mechanism design problem, we will make the following four assumptions. These four assumptions are treated as bench mark assumptions for the design of auctions from the literature point of view.

1) *Risk Neutral:* The heterogeneous wireless network service providers are risk neutral.

2) *Independent Private Value (IPV) Model:* Each wireless network service provider knows the value of a particular time slot that he is going to bid and does not know the value of the other wireless network service providers. Each service provider perceives any other service provider's valuation as a draw from some probability distribution. In the same fashion, he knows that the other service providers regard his own valuation as a draw from some probability distribution. More precisely, for service provider i, $i = 1, 2, ..., n$, there is some probability distribution $\Phi_i(.)$ from which he draws his valuation θ_i for the time slot. Any service provider's valuation is statistically independent from any other service provider's valuation. The valuation θ_i can be viewed as his private value. Let Θ_i, $i = 1, 2, ..., n$ denote the set of all possible types of service provider i and assume that Θ_i is a closed interval of the real line, that is $\Theta_i = [\theta_i^l, \theta_i^u]$. This implies that $\Phi_i(.)$, $i = 1, 2, ..., n$ are probability distribution functions of

the random variables Θ_i, $i = 1, 2, ..., n$.

3) *Symmetry among Service Providers:* The service providers are symmetric in the following sense:
   - Θ_1=Θ_2= ... = Θ_n= Θ
   - $\Phi_1(.)$ = $\Phi_2(.)$= ... = $\Phi_n(.)$ = $\Phi(.)$
4) *Properties of $\Phi(.)$ and Θ:* We assume that $\Phi(.)$ satisfy the following properties:
   - $\Theta = [\theta^l, \theta^u]$
   - $\theta^l > 0$
   - $\phi(\theta) = \Phi^{'}(\theta) > 0$; $\forall \theta^l \leq \theta \leq \theta^u$

Under this setting, we can formulate the resource procurement auction as mechanism design problem. The following are the main components of the Resource Procurement Auction mechanism design problem.

1) *Outcome Set X:* An outcome in our mechanism design problem can be represented by a vector $x = ((y_{ij})_{i \in N, j \in M}, (p_i)_{i \in N})$, where y_{ij} is the probability that service provider i is the winner for the time slot j and p_i is the payment received by i^{th} service provider. The set of all feasible alternatives is represented in the following way.
$$\begin{aligned} X \quad = \quad & \{((y_{ij})_{i \in N, j \in M}, (p_i)_{i \in N}) | y_{ij} \in [0, 1], \\ & \textstyle\sum_{i=1}^{n} y_{ij} \leq 1, \sum_{j=1}^{m} y_{ij} \leq 1, p_i \geq 0, \\ & \forall i \in N, j \in M\} \end{aligned}$$
2) *Utility Function of Service Providers $(u_i(.))$:* The Bernoulli utility function of the i^{th} service provider is given by,
$$u_i(x, \theta_i) = (\textstyle\sum_{j=1}^{m} y_{ij})(p_i - \theta_i)$$
3) *Social Choice Function $(f(.))$:* The general structure of the social choice function for this type of problems is,
$$f(\theta) = ((y_{ij})_{i \in N, j \in M}, (p_i)_{i \in N})$$
Note that $y_{ij}(\theta)$ depends on allocation rule and $p_i(\theta)$ depends on the payment rule.

Now under these four benchmark assumptions, a resource procurement auction can be viewed as a direct revelation mechanism $\Lambda = ((\Theta_i)_{i \in N}, f(.))$ in quasi linear environment, where Θ_i is the type set of an service provider i and $f(.)$ is the social choice function.

A mechanism Λ combined with possible types of the service providers $(\Theta_1, \Theta_2, ..., \Theta_n)$, probability density $\phi(.)$, and Bernoulli utility functions $(u_1(.), u_2(.), ..., u_n(.))$ defines a Beyesian game of incomplete information which gets induced among the service providers after the mobile user announces the time slots. The induced Bayesian Game Γ^b can be given in the following manner

$$\Gamma^b = (N, (\Theta_i)_{i \in N}, (\Theta_i)_{i \in N}, \phi(.), (u_i^{'})_{i \in N})$$

where $u_i^{'} : \Theta \times \Theta \mapsto \mathbb{R}$ is the utility function of agent i and is defined in the following manner

$$u_i^{'}(\theta^{'}, \theta) = u_i(f(\theta^{'}), \theta_i)$$

where $\Theta = \times_{i \in N} \Theta_i$.

A strategy for the service provider i in the above game Γ^b is a function $s_i : \Theta_i \mapsto \Theta_i$ giving service provider i's contingent bidding plan of revealing his own type based on his actual type θ_i. Depending on kind of mechanism the mobile user uses, the service providers decide their bidding strategy. The strategies selected by the service providers in turn decide the payment made by the mobile user. There comes *Game theory* into the situation to analyze the existing scenario. It says that each bidder will follow the bidding strategy $s_i^*(.)$ such that the strategy profile $(s_1^*(.), s_2^*(.), ..., s_n^*)$ is a *Bayesian Nash equilibrium* of the game Γ^b induced by the mechanism that the mobile user uses.

Some natural questions that arise in this context are the following. What kind of mechanism that the mobile user imposes to procure resources in an optimal way (i.e. minimizing the total payment)? What are the key game theoretic properties the mechanism has to satisfy? We will answer these questions in the rest of this paper. In general, the mechanism that the mobile user imposes should give minimum payment (to pay to the winning service providers) and satisfy the following two important properties of a social choice function.

Individual Rationality:

The service providers participation in the auction is *voluntary* in the sense that the mobile user should not force them to participate in the auction mechanism. As a result, the service provider will choose to participate in the mechanism only if he loses nothing out of participating in the auction. This is known as *individual rationality constraints*. So, in order to ensure the service provider i's participation in the mechanism, after he has learned his actual type as θ_i, the following *interim individual rationality constraints* must be satisfied

$$U_i(\theta_i|f) = E_{\theta_{-i}}[u_i(f(\theta_i, \theta_{-i})|\theta_i)] \geq 0, \; \forall \theta_i \in \Theta_i$$

where $U_i(\theta_i|f)$ is service provider i's *interim expected utility* under social choice function $f(.)$ when his type is θ_i.

Incentive Compatibility:

The service providers prefer to have an auction mechanism Λ for which truth telling is the equilibrium strategy for all the service providers. The reason being for this is that some times it is very difficult to compute the optimal strategy in closed form given the bidders are bounded rational. These constraints are called *Incentive Compatibility* constraints. Depending on the type of the equilibrium concept in hand, there are two types of incentive compatibility constraints.

- *Dominant Strategy Incentive Compatibility:* The social choice function $f(.)$ is said to be dominant strategy incentive compatible if the direct revelation mechanism $\Lambda = ((\Theta_i)_{i \in N}, f(.))$ has a dominant strategy equilibrium $s^{'} = (s_1^{'}(.), s_2^{'}(.), ..., s_n^{'}(.))$, where
$$s_i^{'}(\theta_i) = \theta_i, \; \forall \theta_i \in \Theta_i,, \; \forall i \in N$$

- *Bayesian Incentive Compatibility:* The social choice function $f(.)$ is said to be Bayesian incentive compatible if the direct revelation mechanism $\Lambda = ((\Theta_i)_{i \in N}, f(.))$ has a Bayesian Nash equilibrium $s^* = (s_1^*(.), s_2^*(.), ..., s_n^*(.))$, where

$$s_i^*(\theta_i) = \theta_i,\ \forall \theta_i \in \Theta_i,\ \forall i \in N$$

In order to make the work of the service providers simpler, it is better to choose a mechanism $\Lambda = ((\Theta_i)_{i \in N}, f(.))$ such that the social choice function $f(.)$ is ideally dominant strategy incentive compatible.

IV. REVERSE OPTIMAL MECHANISM (*REVOPT*)

From the previous section, it is desirable for the mobile user to have a mechanism which minimizes the expected payment along with satisfying the properties of individual rationality and Bayesian incentive compatibility. Myerson studied such type of auction mechanisms in the context of selling (i.e. forward auction settings) a single individual item [14]. Myerson called such an auction mechanism as *optimal auction*. In this paper, we are considering the case where multiple items (i.e. time slots) are procured (i.e. reverse auction settings) by the mobile user from the non-cooperative and selfish service providers. So, we call our proposed approach *Reverse Optimal (REVOPT)* auction.

As mentioned in Section III, we assume there are n service providers and $N = \{1, 2, ..., n\}$ represents the set of service providers. Let there are m time slots announced by the mobile user and $M = \{1, 2, ..., m\}$ represents the set of time slots to be auctioned. Let $\theta = (\theta_1, \theta_2, ..., \theta_n)$ be the vector of bids received from these n service providers. Let $p_i(\theta)$ be the payment to the service provider i by the mobile user, when the vector of bids of the service providers is θ. The Bernoulli utility function of service provider i is given by

$$\begin{aligned} u_i(f(\theta), \theta_i) &= (\textstyle\sum_{j=1}^{m} y_{ij}(\theta))(p_i(\theta) - \theta_i) \\ &= v_i(y(\theta))(p_i(\theta) - \theta_i) \\ &= t(\theta_i) - v_i(y(\theta))\theta_i \end{aligned}$$

where $v_i(y(\theta)) = (\sum_{j=1}^{m} y_{ij}(\theta))$ is known as value function of the service provider i. $t_i(\theta) = v_i(y(\theta))p_i(\theta)$ denotes the payment to the service provider by the mobile user. Now expected payment, $\bar{t}_i(\hat{\theta}_i)$, to the service provider i when he announces his type to be $\hat{\theta}_i$ and all the service providers $j \neq i$ truthfully reveal their type is given by

$$\bar{t}_i(\hat{\theta}_i) = E_{\theta_{-i}}[t_i(\hat{\theta}_i, \theta_{-i})]$$

A similar expression for the valuation function of i^{th} service provider is

$$\bar{v}_i(\hat{\theta}_i) = E_{\theta_{-i}}[v_i(y(\hat{\theta}_i, \theta_{-i}))]$$

Now, the expected utility of i^{th} service provider when his type θ_i is given by,

$$U_i(\theta_i) = \bar{t}_i(\theta_i) - \bar{v}_i(\theta_i)\theta_i \tag{1}$$

A social choice function that the mobile user chooses in this environment is in the form $f(.) = ((y_{ij})_{i \in N, j \in M}, (t_i)_{i \in N})$. We want this social choice function to satisfy individual rationality and Bayesian incentive compatibility, which are mentioned in the previous section.

For the above mentioned social choice function to satisfy the interim individual rationality constraints, the following equation must be hold. $\forall i \in N$, $\forall \theta_i \in \Theta_i$

$$\begin{aligned} U_i(\theta_i) &= E_{\theta_{-i}}[u_i(f(\theta_i, \theta_{-i}), \theta_i)|\theta_i] \\ &= \bar{t}_i(\theta_i) - \bar{v}_i(\theta_i)\theta_i \\ &\geq 0 \end{aligned}$$

because the service providers are always free to participate in the bidding of time slots.

Now, we need to characterize the conditions under which the above social choice function satisfies the Bayesian Incentive compatibility. The following Proposition serves the requirement.

Proposition-1: The social choice function, $f(.) = ((y_{ij})_{i \in N, j \in M}, (t_i)_{i \in N})$, chosen by the mobile user is Bayesian incentive compatible if and only if, $\forall i \in N$,

(i) $\bar{v}_i(.)$ is *non-increasing*,
(ii) $U_i(\theta_i) = U_i(\theta_i^l) - \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds$, $\forall \theta_i \in \Theta_i$.

Proof:(a) *Necessity:* Bayesian incentive compatibility implies that for each $\hat{\theta}_i > \theta_i$ we have,

$$\begin{aligned} U_i(\theta_i) &\geq t_i(\hat{\theta}_i) - \theta_i\bar{v}_i(\hat{\theta}_i) \\ &= U_i(\hat{\theta}_i) + (\hat{\theta}_i - \theta_i)\bar{v}_i(\hat{\theta}_i), \end{aligned}$$

and

$$\begin{aligned} U_i(\hat{\theta}_i) &\geq t_i(\theta_i) - \hat{\theta}_i\bar{v}_i(\theta_i) \\ &= U_i(\theta_i) + (\theta_i - \hat{\theta}_i)\bar{v}_i(\theta_i). \end{aligned}$$

Thus,

$$\bar{v}_i(\theta_i) \geq \tfrac{U_i(\theta_i) - U_i(\hat{\theta}_i)}{\hat{\theta}_i - \theta_i} \geq \bar{v}_i(\hat{\theta}_i)$$

This expression immediately implies that $\bar{v}_i(.)$ must be non-increasing (since we have taken $\hat{\theta}_i > \theta_i$). In addition, letting $\hat{\theta}_i \rightarrow \theta_i$ and using the above expression, we have $\forall \theta_i$

$$U_i^{'}(\theta_i) = -\bar{v}_i(\theta_i)$$

and so

$$U_i(\theta_i) = U_i(\theta_i^l) - \textstyle\int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds,\ \forall \theta_i \in \Theta_i.$$

This completes the proof for the necessary conditions.

(b) Sufficiency: Consider any θ_i and $\hat{\theta}_i$ and suppose without loss of generality $\theta_i > \hat{\theta}_i$ holds. If the conditions (i) and (ii) in the statement of the proposition hold, then

$$\begin{aligned} U_i(\theta_i) - U_i(\hat{\theta}_i) &= -\textstyle\int_{\hat{\theta}_i}^{\theta_i} \bar{v}_i(s)ds \\ &\geq -\textstyle\int_{\hat{\theta}_i}^{\theta_i} \bar{v}_i(\hat{\theta}_i)ds \\ &= -(\theta_i - \hat{\theta}_i)\bar{v}_i(\bar{\theta}_i) \end{aligned}$$

Hence,

$$\begin{aligned} U_i(\theta_i) &\geq U_i(\hat{\theta}_i) - (\theta_i - \hat{\theta}_i)\bar{v}_i(\bar{\theta}_i) \qquad (2) \\ &= \bar{t}_i(\hat{\theta}_i) - \theta_i\bar{v}_i(\hat{\theta}_i) \qquad (3) \end{aligned}$$

Similarly, we can derive the following expression for the case where $\hat{\theta}_i \geq \theta_i$,

$$\begin{aligned} U_i(\hat{\theta}_i) &\geq U_i(\theta_i) - (\hat{\theta}_i - \theta_i)\bar{v}_i(\theta_i) && (4)\\ &= \bar{t}_i(\theta_i) - \hat{\theta}_i\bar{v}_i(\theta_i) && (5)\end{aligned}$$

Equations (3) and (4) establish the required sufficiency conditions. So, $f(.)$ is Bayesian incentive compatible. *Q.E.D.*

A social choice function chosen by the mobile user in the environment is a function $f(.) = ((y_{ij})_{i\in N, j\in M}, (t_i)_{i\in N})$ having the properties that, $y_{ij}(\theta) \in [0,1]$, $\forall i \in N, \forall j \in M$, $\forall \theta \in \Theta$, $\sum_{j\in M} y_{ij}(\theta) \leq 1$, $\sum_{i\in N} y_{ij}(\theta) \leq 1$, and $\sum_{i\in N} t_i(\theta)$ being the total payment by the mobile user. The mobile user's reverse optimal mechanism can be written as one of choosing functions $(y_{ij})_{i\in N, j\in M}$ and $(U_i(.))_{i\in N}$ to solve

$$Minimize \quad \sum_{i=1}^{n} \int_{\theta_i^l}^{\theta_i^u} \{\bar{v}_i(\theta_i)\theta_i + U_i(\theta_i)\} \phi_i(\theta_i) d\theta_i$$

subject to

(i) $\bar{v}_i(.)$ is non-increasing, $\forall i \in N$
(ii) $y_{ij}(\theta) \in [0,1], \sum_{j=1}^{m} y_{ij}(\theta) \leq 1, \sum_{i=1}^{n} y_{ij}(\theta) \leq 1$, $\forall i \in N, \forall j \in M, \forall \theta \in \Theta$
(iii) $U_i(\theta_i) = U_i(\theta_i^l) - \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds$, $\forall i \in N$, $\forall \theta_i \in \Theta_i$
(iv) $U_i(\theta_i) \geq 0$, $\forall i \in N$, $\theta_i \in \Theta_i$

In this formulation, the objective function corresponds to the total expected payment by the mobile user to all service providers. Here note that constraint (iv) is service provider's individual rationality constraints, constraint (i)&(iii) are the necessary and sufficient conditions for the social choice function $f(\theta) = ((y_{ij}(\theta))_{i\in N, j\in M}, (t_i(\theta))_{i\in N})$ to be Bayesian incentive compatible from Proposition-1.

Note that if constraint (iii) is satisfied, then constraint (iv) will be satisfied if and only if

$$(\text{v}) \quad U_i(\theta_i^l) \geq \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds, \ \forall i \in N$$

So, we can replace constraint (iv) with constraint (v). Next, substituting for $U_i(\theta_i)$ in the objective function from constraint (iii), we get

Minimize

$$\sum_{i=1}^{n} \int_{\theta_i^l}^{\theta_i^u} \left\{\bar{v}_i(\theta_i)\theta_i + U_i(\theta_i^l) - \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds\right\} \phi_i(\theta_i)d\theta_i$$

Integrating by parts the above equation, the mobile user problem can be written as one of choosing the functions $y_{ij}(.)$ and the values $U_1(\theta_1^l), U_2(\theta_2^l), ..., U_n(\theta_n^l)$ such that

Minimize

$$\int_{\theta_1^l}^{\theta_1^u} \dots \int_{\theta_n^l}^{\theta_n^u} \{\sum_{i=1}^{n} v_i(\theta_i)J_i(\theta_i)\} \{\prod_{i=1}^{n} \phi_i(\theta_i)\} d\theta_n \dots d\theta_1 + \sum_{i=1}^{n} U_i(\theta_i^l)$$

subject to constraints (i), (ii), and (v), where

$$J_i(\theta_i) = \theta_i - \frac{1-\Phi_i(\theta_i)}{\phi_i(\theta_i)} = \theta_i - \frac{\bar{\Phi}_i(\theta_i)}{\phi_i(\theta_i)}.$$

It is evident that solution must have $U_i(\theta_i^l) = \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds$, $\forall i \in N$. Hence the mobile user's optimization problem reduces to choosing functions $y_{ij}(.)$ such that,

Minimize

$$\int_{\theta_1^l}^{\theta_1^u} \dots \int_{\theta_n^l}^{\theta_n^u} \{\sum_{i=1}^{n} v_i(\theta_i)J_i(\theta_i)\} \{\prod_{i=1}^{n} \phi_i(\theta_i)\} d\theta_n \dots d\theta_1 + \sum_{i=1}^{n} \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds$$

subject to

(i) $\bar{v}_i(.)$ is non-increasing, $\forall i \in N$
(ii) $y_{ij}(\theta) \in [0,1], \sum_{j=1}^{m} y_{ij}(\theta) \leq 1, \sum_{i=1}^{n} y_{ij}(\theta) \leq 1$, $\forall i \in N, \forall j \in M, \forall \theta \in \Theta$

Let us ignore the constraint (i) for the moment. Then the above optimization problem indicates that $y_{ij}(.)$ is a solution to this relaxed problem iff $\forall i \in N$, we have

$$y_{ij} = \begin{cases} 0 \quad \forall j = 1, 2, \dots, m & : \ if J_i(\theta_i) > 0 \\ 1 \quad \forall j = 1, 2, \dots, m < n & : \ if J_i(\theta_i) = J(j) \\ 1 \quad \forall j = 1, 2, \dots, m \geq n & : \ if J_i(\theta_i) = J(j) \\ 0 & : \ \text{otherwise} \end{cases}$$

where $J(j)$ is the j^{th} lowest among $J_i(\theta_i), \forall i \in N$. It says, if we ignore the constraint (i), then y_{ij} is a solution to the relaxed problem iff any the service provider for whom the value $J_i(\theta_i)$ is positive, no time slot is assigned and the rest of the service providers will be assigned to the time slots in the same order as the values of $J_i(\theta_i)$ starting from the lowest possible value. That is, the first time slot is allocated to the service provider who has *the highest negative value* for $J_i(\theta_i)$, the second time slot is assigned to the service provider who has *the second highest negative value* for $J_i(\theta_i)$, and so on. This completes the characterization of the allocation rule of the social choice function chosen by the mobile user.

Now, we will characterize the payment rule in the social choice function.

$$\begin{aligned} t_i(\theta) &= \bar{t}_i(\theta_i) \\ &= \theta_i\bar{v}_i(\theta_i) + U_i(\theta_i) \\ &= \theta_i\bar{v}_i(\theta_i) + U_i(\theta_i^l) - \int_{\theta_i^l}^{\theta_i} \bar{v}_i(s)ds \\ &= \theta_i\bar{v}_i(\theta_i) \end{aligned}$$

where $\bar{v}_i(\theta_i)$ is expected value of the i^{th} service provider and

$$\begin{aligned} \bar{v}_i(\theta_i) &= E_{\theta_{-i}}[v_i(y(\theta))] \\ &= E_{\theta_{-i}}[\sum_{j=0}^{m} y_{ij}(\theta)] \end{aligned}$$

This completes the analysis on the structure of *REVOPT* mechanism chosen by the mobile user. Using this auction mechanism the total payment, P_{REVOPT}, by the mobile user for procuring wireless channel is given by (under bench mark assumptions)

$$P_{REVOPT} = n \int_{\theta=\theta^l}^{\theta^u} \bar{t}(\theta)\phi(\theta)d\theta$$

V. CONCLUSIONS AND FUTURE WORK

In this paper, we addressed the problem of procuring resources by the wireless/mobile user in non-cooperative heterogenous wireless networks. We have proposed a reverse optimal auction *(REVOPT)* mechanism to solve this problem. We characterized the allocation rule and payment rule in our solution. This proposed auction mechanism satisfies important game theoretic properties like individual rationality and incentive compatibility.

In the above design, we assumed the service providers of heterogeneous wireless network are selfish and non-cooperative. But in real world situations, the service providers can collude with each other to improve their utilities rather than being individual. We are interested in looking into these aspects using cooperative game theory.

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