GMA08

Riunione del Gruppo Materiali
dell'Associazione Italiana di Meccanica Teorica e Applicata
(AIMETA)

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COSSERAT MATERIALS?
No thanks

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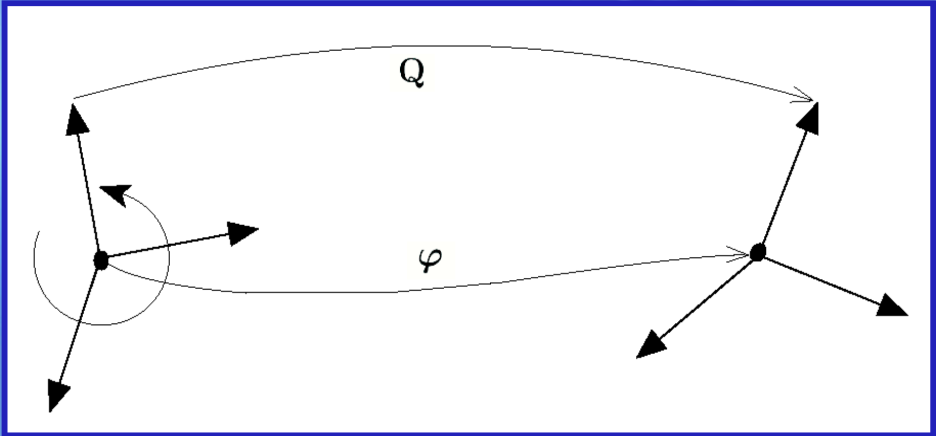
COSSERAT (POLAR)

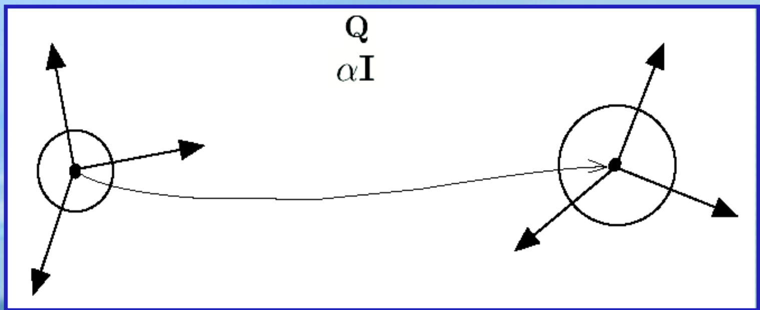


$$\begin{cases} \mathbf{Q}^T d\mathbf{Q}\,, & \text{curvature change} \\ \mathbf{Q}^T d\boldsymbol{\varphi} - \mathbf{I}\,, & \text{gap} \end{cases}$$

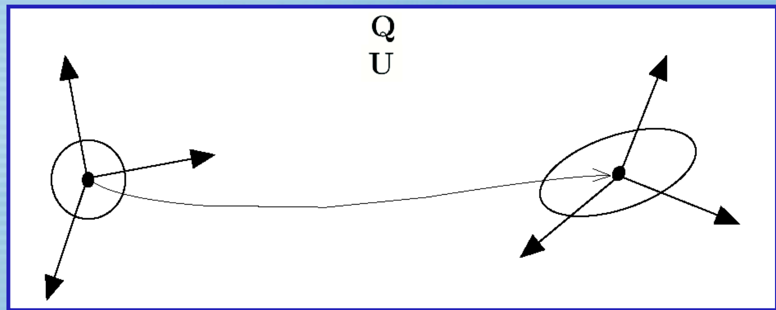
$$\begin{cases} d(axial\,\mathbf{W}) \\ d\mathbf{v} - \mathbf{W} \end{cases} \longrightarrow$$ *tangent strain measure*

Cosserat E. and F.: Théorie des Corps déformables. Hermann, Paris, (1909).

MICROSTRETCH



MICROMORPHIC



Eringen A.C.: Mechanics of Micromorphic Continua, in Mechanics of Generalized Continua, Ed. Kröner, Springer-Verlag, Berlin, pp. 18-35, (1968).

$$\mathbf{A} \in \mathrm{C}^2(\mathbb{M}\,;\mathrm{C}^0(\mathbb{B}_s\,;D))$$ → *Finite strain measure*

Essential requirements

$$\mathbf{u}_{t,s} \in \mathcal{R} \iff \mathbf{A}(\mathbf{u}_{t,s}) = 0 \in \mathrm{C}^0(\mathbb{B}_s\,;D)$$

$$\mathbf{A}(\mathbf{u}_{\tau,s}) = \mathbf{A}(\mathbf{u}_{t,s}) + \mathbf{S}(\mathbf{A}(\mathbf{u}_{\tau,t}), \mathbf{u}_{t,s})$$ → *consistency*

nonredundancy

A deformation measure $\mathbf{A} \in \mathrm{C}^2(\mathbb{M}\,;\mathrm{C}^0(\mathbb{B}_s\,;D))$ is said to be *redundant* if there exists a nontrivial decomposition $D = D_1 \oplus D_2$ such that

$$(\boldsymbol{\Pi}_1 \circ \mathbf{A})(\mathbf{u}_{t,s}) = 0 \implies \mathbf{A}(\mathbf{u}_{t,s}) = 0\,.$$

$$\boldsymbol{\varphi}\!\downarrow\!\mathbf{g} = cost \quad \text{in} \quad \mathbb{B}_s \implies d\boldsymbol{\varphi} = cost \quad \text{in} \quad \mathbb{B}_s$$

$$\mathcal{L}_{\mathbf{v}}\mathbf{g} = cost \quad \text{in} \quad \mathbb{B}_s \implies d\mathbf{v} = cost \quad \text{in} \quad \mathbb{B}_s$$

→ *Kinematic theorems*

The strain measure in Cosserat materials is REDUNDANT!

$$\begin{cases} \mathbf{Q}^T d\mathbf{Q} = \mathbf{O} \\ \mathbf{Q}^T d\boldsymbol{\varphi} - \mathbf{I} = \mathbf{O} \end{cases} \iff \begin{cases} d\mathbf{Q} = \mathbf{O} \\ d\boldsymbol{\varphi} = \mathbf{Q} \end{cases}$$

$$d\boldsymbol{\varphi} = \mathbf{Q} \implies d\mathbf{Q} = \mathbf{O}$$

Future Developments

The strain measure for Cosserat materials is redundant. Moreover any attempt to eliminate this unsound feature causes the model to collapse into the standard Cauchy material.

Applications of Cosserat-type materials:

- *Cholesteric liquid cristals (inextensible directed rodlike molecules);*
- *Nematic liquid cristals (inextensible undirected rodlike molecules);*
- *Void elasticity: change of volume fraction as homothetic strain (Cowin 1983). The change of volume fraction can be interpreted as a dilatation of the points in the continuum;*
- *Shell models with drilling rotations;*
- *etc…*