The First Principle of Thermodynamics and the Virtual Temperatures Theorem

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Summary. The first principle of thermodynamics is reformulated as a variational principle, in which the test fields are piecewise constant virtual temperatures. This simple trick opens the way to get the proof of the existence of a square integrable covector field, the *cold flow* field, in duality with the vector field of thermal gradients. To build up a proper functional context which meets the principle of reproducibility, we define the *thermal space* of GREEN-regular temperature fields, which is the pre-HILBERT space of square integrable scalar fields with piecewise square integrable distributional gradients. This means that each field in the thermal space is associated with a regularity patchwork in whose elements the temperature field has a square integrable distributional gradient. The trial-constraints are assumed to define in the thermal space a closed linear subspace of conforming temperature fields, admitting a common regularity patchwork. The set of conforming temperature fields is then a HILBERT space. The existence proof is based on the closed range property of the distributional gradient and is a straightforward consequence of BANACH's closed range theorem. The square integrable *cold flow* covector fields are the LAGRANGE multipliers corresponding to the implicit form of the test-constraint imposing that virtual temperature fields are piecewise constant. The *cold flow* vector fields are associated with the covector fields by RIESZ isomorphism. The first principle of thermodynamics, expressed as a variational principle in terms of these fields of LAGRANGE multipliers, provides a variational equality which is called the *principle of virtual temperatures* to underline the perfect analogy with the *principle of virtual works* of mechanics, stemming from the isometry constraint in the variational condition of equilibrium. It follows that these statements, in spite of their traditional names, are rather theorems than principles since their proof, based on the variational equations, can be performed by standard results of functional analysis. By assuming that the trial-constraints define a boundary value problem, i.e. that zero boundary valued fields are conforming, and that the cold flow vector field is square integrable with piecewise square integrable divergence, an application of GREEN's formula shows that the generalized divergence and the boundary flux of the cold vector field, are respectively the sources and the outflows of the cold content in the body. This fact provides the explicit representation of the vector field as the cold-flow in the body. A standard localization argument provides the boundary and differential equations of thermodynamics. Another simple instance in which this reasoning can be applied,

is provided by the flow of a fluid across a control volume of a porous medium. Then conservation of mass leads to the proof of the existence of a mass-flow vector field. An analogous reasoning can be applied to any balance law in continuum mechanics and thermodynamics. Introducing maximal monotone and conservative constitutive relations between the fields of Lagrange multipliers and the fields expressing the implicit test-constraint, minimum or saddle point principles equivalent to problems in heat conduction, permeability, elastostatics , ect. are got in the various contexts.