Una teoria consistente dell'induzione elettromagnetica

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Accademia di Scienze Fisiche e Matematiche in Napoli

1 febbraio 2013

ED=Electrodynamics
and
DG=Differential Geometry

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DG=Differential Geometry

ED is an important source of inspiration for DG and DG is the natural tool to develop a mathematical modeling of ED

ED=Electrodynamics and DG=Differential Geometry

ED is an important source of inspiration for DG and DG is the natural tool to develop a mathematical modeling of ED

Founders of DG

Johann Carl Friederich Gauss (1777 - 1855)

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Founders of DG

Hermann Günther Grassmann (1809 - 1877)

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Founders of DG

Bernhard Riemann (1826 - 1866)

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Founders of DG

Marius Sophus Lie (1842 - 1899)

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Founders of DG

Felix Klein (1849 - 1925)

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Founders of DG

Gregorio Ricci-Curbastro (1853 - 1925)

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Founders of DG

Tullio Levi-Civita (1873 - 1941)

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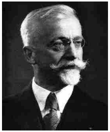
Founders of DG



Elie Cartan (1869 - 1951)

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Founders of DG

Henri Cartan (1904 - 2008)

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Founders of DG

Charles Ehresmann (1905 - 1979)

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Founders of DG

Hassler Whitney (1907 - 1989)

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Founders of DG

Jean-Louis Koszul (1921 -)

Electromagnetic Story Starring

Electromagnetic Story Starring

Luigi Galvani (1737-1798)

Electromagnetic Story Starring

Alessandro Volta (1745-1827)

Electromagnetic Story Starring

Gian Domenico Romagnosi (1761-1835)

Electromagnetic Story Starring

Hans Christian Ørsted (1777-1851)

Electromagnetic Story Starring

André-Marie Ampère (1775 - 1836)

Electromagnetic Story Starring

Johann Carl Friedrich Gauss (1777 - 1855)

Electromagnetic Story Starring

Francesco Zantedeschi (1797-1873)

Electromagnetic Story Starring

Joseph Henry (1797 - 1878)

Electromagnetic Story Starring

Michael Faraday (1791 - 1867)

Electromagnetic Story Starring

James Clerk-Maxwell (1831 - 1879)

Electromagnetic Story
Starring

Joseph John Thomson (1856 - 1940)

Electromagnetic Story Starring

NOTES

ON

RECENT RESEARCHES IN

ELECTRICITY AND MAGNETISM

INTENDED AS A SEQUEL TO

PROFESSOR CLERK-MAXWELL'S TREATISE
ON ELECTRICITY AND MAGNETISM

BY

J. J. THOMSON, M.A., F.R.S.
HON. SC. D. DUBLIN
FELLOW OF TRINITY COLLEGE
PROFESSOR OF EXPERIMENTAL PHYSICS IN THE UNIVERSITY OF CAMBRIDGE

Oxford
AT THE CLARENDON PRESS
1893

In the course of Maxwell's investigation of the values of X, Y, Z due to induction, the terms

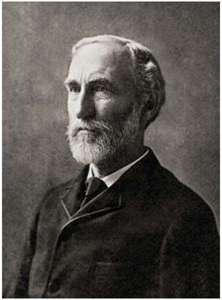
$$-\frac{d}{dx}(Fu+Gv+Hw), \quad -\frac{d}{dy}(Fu+Gv+Hw), \quad -\frac{d}{dz}(Fu+Gv+Hw)$$

respectively in the final expressions for X, Y, Z are included under the Ψ terms. We shall find it clearer to keep these terms separate and write the expressions for X, Y, Z as

$$\left.\begin{aligned} X &= cv - bw - \frac{dF}{dt} - \frac{d}{dx}(Fu+Gv+Hw) - \frac{d\phi}{dx}, \\ Y &= aw - cu - \frac{dG}{dt} - \frac{d}{dy}(Fu+Gv+Hw) - \frac{d\phi}{dy}, \\ Z &= bu - av - \frac{dH}{dt} - \frac{d}{dz}(Fu+Gv+Hw) - \frac{d\phi}{dz}. \end{aligned}\right\} \qquad (1)$$

Vector fields in Electromagnetics

Vector fields in Electromagnetics



Josiah Willard Gibbs (1839 - 1903)

Vector fields in Electromagnetics

Oliver Heaviside (1850 - 1925)

Vector fields in Electromagnetics

Heinrich Rudolf Hertz (1857 - 1894)

Zero spatial velocity

$$\oint_{\partial\Sigma_{\text{IN}}} \mathbf{g} \cdot \mathbf{E} = -\int_{\Sigma_{\text{IN}}} \boldsymbol{\mu} \cdot (\partial_{\theta=0}\, \mathbf{B}) \quad \text{Henry} - \text{Faraday}(1831)$$

$$\oint_{\partial\Sigma_{\text{OUT}}} \boldsymbol{\mu} \cdot \mathbf{B} = 0 \qquad \text{Gauss}(1831)$$

$$\oint_{\partial\Sigma_{\text{OUT}}} \mathbf{g} \cdot \mathbf{H} = \int_{\Sigma_{\text{OUT}}} \boldsymbol{\mu} \cdot (\partial_{\theta=0}\, \mathbf{D} + \mathbf{J}) \qquad \text{Ampère}(1826)$$

$$\text{Maxwell}(1861)$$

$$\oint_{\partial\Sigma_{\text{OUT}}} \boldsymbol{\mu} \cdot \mathbf{D} = \int_{\Sigma_{\text{OUT}}} \rho\, \boldsymbol{\mu} \qquad \text{Gauss}(1835)$$

with Σ_{OUT} a bounded connected surface and Σ_{OUT} bounded connected domain in $\mathcal{S}$. Applying Ampère law to closed surfaces $\Sigma_{\text{OUT}} = \partial\boldsymbol{\Omega}$, we get

$$\partial_{\theta=0}\, \rho + \operatorname{div} \mathbf{J} = 0$$

the *equation of continuity*.

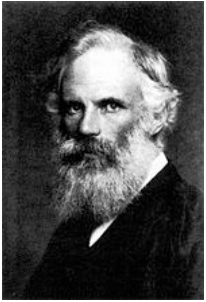
Relativity Story

Length contraction and time dilation effects

Woldemar Voigt (1850 - 1919)

Relativity Story

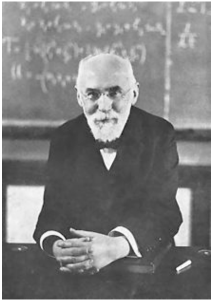
Length contraction and time dilation effects



George Francis FitzGerald (1851 - 1901)

Relativity Story

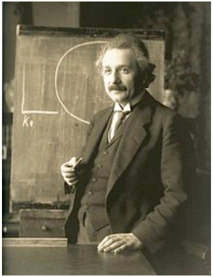
Length contraction and time dilation effects



Hendrik Antoon Lorentz (1853 - 1928)

Relativity Story

Length contraction and time dilation effects



Albert Einstein (1879 - 1955)

Relativity Story

Space-time metric

Henri Poincaré (1854 - 1912)

Relativity Story

Space-time metric

Hermann Minkowski (1864 - 1909)

Relativity Story

General relativity

Christian Felix Klein (1849 - 1925)

Relativity Story

General relativity

David Hilbert (1862 - 1943)

3. *Zur Elektrodynamik bewegter Körper;* *von A. Einstein.*

Daß die Elektrodynamik Maxwells — wie dieselbe gegenwärtig aufgefaßt zu werden pflegt — in ihrer Anwendung auf bewegte Körper zu Asymmetrien führt, welche den Phänomenen nicht anzuhaften scheinen, ist bekannt. Man denke z. B. an die elektrodynamische Wechselwirkung zwischen einem Magneten und einem Leiter. Das beobachtbare Phänomen hängt hier nur ab von der Relativbewegung von Leiter und Magnet, während nach der üblichen Auffassung die beiden Fälle, daß der eine oder der andere dieser Körper der bewegte sei, streng voneinander zu trennen sind. Bewegt sich nämlich der Magnet und ruht der Leiter, so entsteht in der Umgebung des Magneten ein elektrisches Feld von gewissem Energiewerte, welches an den Orten, wo sich Teile des Leiters befinden, einen Strom erzeugt. Ruht aber der Magnet und bewegt sich der Leiter, so entsteht in der Umgebung des Magneten kein elektrisches Feld, dagegen im Leiter eine elektromotorische Kraft, welcher an sich keine Energie entspricht, die aber — Gleichheit der Relativbewegung bei den beiden ins Auge gefaßten Fällen vorausgesetzt — zu elektrischen Strömen von derselben Größe und demselben Verlaufe Veranlassung gibt, wie im ersten Falle die elektrischen Kräfte.

Beispiele ähnlicher Art, sowie die mißlungenen Versuche, eine Bewegung der Erde relativ zum „Lichtmedium" zu konstatieren, führen zu der Vermutung, daß dem Begriffe der absoluten Ruhe nicht nur in der Mechanik, sondern auch in der Elektrodynamik keine Eigenschaften der Erscheinungen entsprechen, sondern daß vielmehr für alle Koordinatensysteme, für welche die mechanischen Gleichungen gelten, auch die gleichen elektrodynamischen und optischen Gesetze gelten, wie dies für die Größen erster Ordnung bereits erwiesen ist. Wir wollen diese Vermutung (deren Inhalt im folgenden „Prinzip der Relativität" genannt werden wird) zur Voraussetzung erheben und außerdem die mit ihm nur scheinbar unverträgliche

Albert Einstein (1905)

ON THE ELECTRODYNAMICS OF MOVING BODIES

By A. EINSTEIN

June 30, 1905

It is known that Maxwell's electrodynamics—as usually understood at the present time—when applied to moving bodies, leads to asymmetries which do not appear to be inherent in the phenomena. Take, for example, the reciprocal electrodynamic action of a magnet and a conductor. The observable phenomenon here depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion. For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated. But if the magnet is stationary and the conductor in motion, no electric field arises in the neighbourhood of the magnet. In the conductor, however, we find an electromotive force, to which in itself there is no corresponding energy, but which gives rise—assuming equality of relative motion in the two cases discussed—to electric currents of the same path and intensity as those produced by the electric forces in the former case.

Examples of this sort, together with the unsuccessful attempts to discover any motion of the earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good.[1] We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that light is always propagated in empty space with a definite velocity c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. The introduction of a "luminiferous ether" will

[1] The preceding memoir by Lorentz was not at this time known to the author.

Maxwell-Hertz equations - (Einstein 1905)

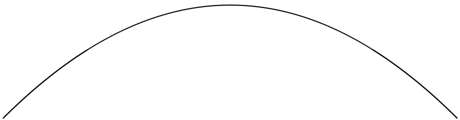
$$
\begin{aligned}
\frac{1}{c}\frac{\partial \mathrm{X}}{\partial t} &= \frac{\partial \mathrm{N}}{\partial y} - \frac{\partial \mathrm{M}}{\partial z} & \qquad \frac{1}{c}\frac{\partial \mathrm{L}}{\partial t} &= \frac{\partial \mathrm{Y}}{\partial z} - \frac{\partial \mathrm{Z}}{\partial y} \\
\frac{1}{c}\frac{\partial \mathrm{Y}}{\partial t} &= \frac{\partial \mathrm{L}}{\partial z} - \frac{\partial \mathrm{N}}{\partial x} & \qquad \frac{1}{c}\frac{\partial \mathrm{M}}{\partial t} &= \frac{\partial \mathrm{Z}}{\partial x} - \frac{\partial \mathrm{X}}{\partial z} \\
\frac{1}{c}\frac{\partial \mathrm{Z}}{\partial t} &= \frac{\partial \mathrm{M}}{\partial x} - \frac{\partial \mathrm{L}}{\partial y} & \qquad \frac{1}{c}\frac{\partial \mathrm{N}}{\partial t} &= \frac{\partial \mathrm{X}}{\partial y} - \frac{\partial \mathrm{Y}}{\partial x}
\end{aligned}
$$

Differential forms vs. vector fields

$\boldsymbol{\omega}^1_{\mathbf{E}} = \mathbf{g} \cdot \mathbf{E}$	electric field
$\boldsymbol{\omega}^2_{\mathbf{B}} = \boldsymbol{\omega}^3_{\boldsymbol{\mu}} \cdot \mathbf{B}$	magnetic vortex
$\boldsymbol{\omega}^1_{\mathbf{H}} = \mathbf{g} \cdot \mathbf{H}$	magnetic field
$\boldsymbol{\omega}^2_{\mathbf{D}} = \boldsymbol{\omega}^3_{\boldsymbol{\mu}} \cdot \mathbf{D}$	electric flux
$\boldsymbol{\omega}^1_{\mathbf{B}} = \mathbf{g} \cdot \mathbf{A}$	magnetic potential
$\boldsymbol{\omega}^2_{\mathbf{J}} = \boldsymbol{\omega}^3_{\boldsymbol{\mu}} \cdot \mathbf{J}$	electric current
$\boldsymbol{\omega}^3_{\boldsymbol{\rho}} = \rho\, \boldsymbol{\omega}^3_{\boldsymbol{\mu}}$	electric charge
$\boldsymbol{\omega}^0_{\mathbf{E}} = V_{\mathbf{E}}$	electric potential

Space-time split

$\mathbf{Z}$		field of time-arrows
$\mathbf{R}$	$= dt \otimes \mathbf{Z}$	projector on time-arrows
$\mathbf{P}$	$= \mathbf{I} - \mathbf{R}$	projector on space slices



Space-time theory of electromagnetics

$$\begin{aligned} \boldsymbol{\omega}^2_{\mathbf{B}} = \mathbf{i}{\downarrow}\boldsymbol{\Omega}^2_{\mathbf{B}} \in \boldsymbol{\Lambda}^2(\mathbb{V}\mathcal{S}\,;\mathcal{R}) \quad & \text{magnetic vortex} \\ \boldsymbol{\omega}^1_{\mathbf{E}} = \mathbf{i}{\downarrow}\boldsymbol{\Omega}^1_{\mathbf{E}} \in \boldsymbol{\Lambda}^1(\mathbb{V}\mathcal{S}\,;\mathcal{R}) \quad & \text{electric field} \end{aligned}$$

with $\mathbf{i}$ spatial immersion.

$$\begin{aligned} \boldsymbol{\Omega}^2_{\mathbf{B}} &:= \mathbf{P}{\downarrow}\boldsymbol{\Omega}^2_{\mathbf{F}} \\ -\boldsymbol{\Omega}^1_{\mathbf{E}} &:= \mathbf{P}{\downarrow}(\boldsymbol{\Omega}^2_{\mathbf{F}} \cdot \mathbf{V}) \end{aligned}$$

$\mathbf{P}$ projector on the spatial slices
$\mathbf{V} = \mathbf{Z} + \mathbf{PV}$ space-time velocity
$\mathbf{v} = \mathbf{i}{\downarrow}(\mathbf{PV})$ spatial velocity.
Representation formula

$$\boldsymbol{\Omega}^2_{\mathbf{F}} = \boldsymbol{\Omega}^2_{\mathbf{B}} - dt \wedge (\boldsymbol{\Omega}^1_{\mathbf{E}} + \boldsymbol{\Omega}^2_{\mathbf{B}} \cdot \mathbf{V})\,.$$

Faraday law in space-time

Closedness of FARADAY two-form in the trajectory manifold is equivalent to the spatial GAUSS law for the magnetic vortex and to the spatial HENRY-FARADAY induction law, i.e.

$$d\,\mathbf{\Omega}^2_{\mathsf{F}} = 0 \quad \Longleftrightarrow \quad \begin{cases} d_{\mathcal{S}}\,\boldsymbol{\omega}^2_{\mathsf{B}} = \mathbf{0} \\ \mathcal{L}^{\mathcal{S}}_{\mathsf{V}}\,\boldsymbol{\omega}^2_{\mathsf{B}} + d_{\mathcal{S}}\,\boldsymbol{\omega}^1_{\mathsf{E}} = \mathbf{0} \end{cases}$$

with the integral formulation

$$\partial_{\theta=0} \int_{\boldsymbol{\varphi}^{\mathcal{S}}_{\theta}(\Sigma_{\rm IN})} \boldsymbol{\omega}^2_{\mathsf{B}} = - \oint_{\partial\Sigma_{\rm IN}} \boldsymbol{\omega}^1_{\mathsf{E}}$$

for any inner-oriented surface $\Sigma_{\rm IN}$ in a spatial slice.

Faraday law

$$
\begin{aligned}
-\omega^1_{\mathbf{E}} &= \mathcal{L}^{\mathcal{S}}_{\mathbf{V}}\,\omega^1_{\mathbf{B}} + d_{\mathcal{S}}\,\omega^0_{\mathbf{E}} \\
&= \mathcal{L}^{\mathcal{S}}_{\mathbf{Z}}\,\omega^1_{\mathbf{B}} + \mathcal{L}_{\mathbf{v}}\,\omega^1_{\mathbf{B}} + d_{\mathcal{S}}\,\omega^0_{\mathbf{E}} \\
&= \mathcal{L}^{\mathcal{S}}_{\mathbf{Z}}\,\omega^1_{\mathbf{B}} + (d_{\mathcal{S}}\,\omega^1_{\mathbf{B}})\cdot\mathbf{v} + d_{\mathcal{S}}\,(\omega^1_{\mathbf{B}}\cdot\mathbf{v}) + d_{\mathcal{S}}\,\omega^0_{\mathbf{E}} \\
&= \mathcal{L}^{\mathcal{S}}_{\mathbf{Z}}\,\omega^1_{\mathbf{B}} + \omega^2_{\mathbf{B}}\cdot\mathbf{v} + d_{\mathcal{S}}\,(\omega^1_{\mathbf{B}}\cdot\mathbf{v}) + d_{\mathcal{S}}\,\omega^0_{\mathbf{E}}
\end{aligned}
$$

Usual formulation with $d_{\mathcal{S}}\,(\omega^1_{\mathbf{B}}\cdot\mathbf{v})$ dropped

$$
-\omega^1_{\mathbf{E}} = \mathcal{L}^{\mathcal{S}}_{\mathbf{Z}}\,\omega^1_{\mathbf{B}} + \omega^2_{\mathbf{B}}\cdot\mathbf{v} + d_{\mathcal{S}}\,\omega^0_{\mathbf{E}}
$$

Synoptic table ($\mathbf{v} = 0$)		
new		old
$(\mathbf{E}^{\parallel}, \mathbf{E}^{\perp})$	$\to (\gamma\, \mathbf{E}^{\parallel}, \mathbf{E}^{\perp})$	$(\mathbf{E}^{\parallel}, \gamma\, (\mathbf{E}^{\perp} + \mathbf{w} \times \mathbf{B}))$
$(\mathbf{B}^{\parallel}, \mathbf{B}^{\perp})$	$\to (\mathbf{B}^{\parallel}, \gamma\, (\mathbf{B}^{\perp} - (\mathbf{w}/c^2) \times \mathbf{E}))$	*idem*
$(\mathbf{H}^{\parallel}, \mathbf{H}^{\perp})$	$\to (\gamma\, \mathbf{H}^{\parallel}, \mathbf{H}^{\perp})$	$(\mathbf{H}^{\parallel}, \gamma\, (\mathbf{H}^{\perp} - \mathbf{w} \times \mathbf{D}))$
$(\mathbf{D}^{\parallel}, \mathbf{D}^{\perp})$	$\to (\mathbf{D}^{\parallel}, \gamma\, (\mathbf{D}^{\perp} + (\mathbf{w}/c^2) \times \mathbf{H}))$	*idem*
$(\mathbf{J}^{\parallel}, \mathbf{J}^{\perp})$	$\to (\mathbf{J}^{\parallel}, \gamma\, \mathbf{J}^{\perp})$	$(\gamma\, (\mathbf{J}^{\parallel} - \rho\, \mathbf{w}), \mathbf{J}^{\perp})$
ρ	$\to \gamma\, (\rho - \mathbf{g}(\mathbf{w}/c^2, \mathbf{J}))$	*idem*
$V_{\mathbf{E}}$	$\to V_{\mathbf{E}}$	$\gamma\, (V_{\mathbf{E}} - \mathbf{g}(\mathbf{w}, \mathbf{A}))$
$(\mathbf{A}^{\parallel}, \mathbf{A}^{\perp})$	$\to (\gamma\, (\mathbf{A}^{\parallel} - (\mathbf{w}/c^2)\, V_{\mathbf{E}}), \mathbf{A}^{\perp})$	*idem*

Denoting by ∇ the EUCLID connection in spatial slices and assuming that the observer measures
1) a magnetic potential independent of time,

$$\mathcal{L}^{\mathcal{S}}_{\mathbf{Z}}\, \boldsymbol{\omega}^1_{\mathbf{B}} = \mathbf{0}$$

2) a spatially constant scalar electric potential,

$$d_{\mathcal{S}}\, \boldsymbol{\omega}^0_{\mathbf{E}} = \mathbf{0}$$

3) a spatially constant magnetic vortex field,

$$\nabla \boldsymbol{\omega}^2_{\mathbf{B}} = \mathbf{0}$$

the formula for the electric field may be evaluated to get

$$-\boldsymbol{\omega}^1_{\mathbf{E}} = (d_{\mathcal{S}}\, \boldsymbol{\omega}^1_{\mathbf{B}}) \cdot \mathbf{v} + d_{\mathcal{S}}\, (\boldsymbol{\omega}^1_{\mathbf{B}} \cdot \mathbf{v}) = \boldsymbol{\omega}^2_{\mathbf{B}} \cdot \mathbf{v} - \frac{1}{2}\, \boldsymbol{\omega}^2_{\mathbf{B}} \cdot \mathbf{v} = \frac{1}{2}\, \boldsymbol{\omega}^2_{\mathbf{B}} \cdot \mathbf{v}\,,$$

or in terms of vector fields $\mathbf{E} = \frac{1}{2}\, \mathbf{v} \times \mathbf{B}$ which is just one-half of what is improperly called the LORENTZ *force*.