"Aerodynamics of the oscillating airfoils"

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OUTLINE

- Introduction
- Theodorsen solution
- CFD of the oscillating airfoils

References

1. Von Karman, T., and Burger J. M., 1935, “General Aerodynamic Theory. Perfect Fluids”, Peter Smith Publisher, Inc., 1976, pp. 280-310.
2. Theodorsen T., “General Theory of Aerodynamic Instability and the Mechanics of Flutter”, National Advisory Committee for Aeronautics, NACA Report. 496 (1935).
3. Bisplinghoff R. L., Ashley H. and Halfman R. L., “Aeroelasticity”, Dover Publications, Inc., New York
4. Saffman P. G. , “Vortex Dynamics”, Cambridge University Press, 1992
5. McCroskey W. J., “The Phenomenon of Dynamic Stall”, Lecture Notes presented at Von Karman Institute Lecture Series on Unsteady Airloads and Aeroelasticity Problems in Separated and Transonic Flows, 9-13 March 1981.
6. Leishman J. G., (2000) “Principles of Helicopter Aerodynamics ”. Cambridge University Press

Most of the material contained in these slides comes from my Ph.D. thesis made between 2007 and 2010 and related publications in collaboration with Prof. Tognaccini, (DIAS), University of Naples.

Introduction

Introduction

- Helicopter
- Turbomachinery
- Manoeuvring Aircrafts
- Wind Energy
- Biological flows (Insect flight)
- ...

Introduction

Examples: Helicopter blade motion.

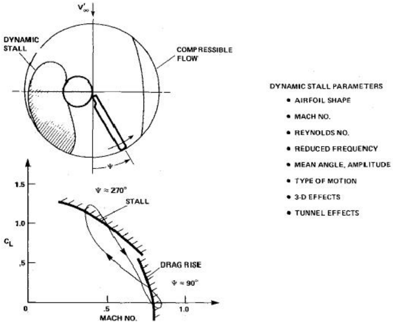


Fig. 1 Helicopter rotor airfoil requirements.

Carr L. W., Progress in Analysis and Prediction of Dynamic Stall, J. of Aircraft, Vol. 25, No. 1

- By combining the rotation and the advancing motion a time varying angle of attack is seen by each blade section.
- When the blade is in the advance phase ($0^\circ \leq \Psi \leq 180^\circ$) the local angles of attack are lower (marked compressible effects).
- For ($180^\circ \leq \Psi \leq 360^\circ$) the local angles of attack increase and the blades can stall (dynamic stall).

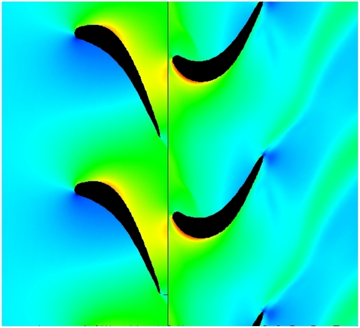
Introduction

Examples: Helicopter blade motion.

- Ex: main rotor of AW119, diameter = 10.83m, 400rpm, $\Rightarrow$ $M_t \sim 0.8$
- The dynamic stall causes significant vibrations and torsional loads.
- It is a strongly time dependent phenomenon.
- In the 60's, it was discovered that the dynamic stall could be investigated similarly on a two-dimensional airfoil under pitching conditions.
- The combination of the asymptotic free stream and the airfoil motion produces a change in the angle of attack.

Introduction

Examples: Turbomachinery, rotor-stator interaction.

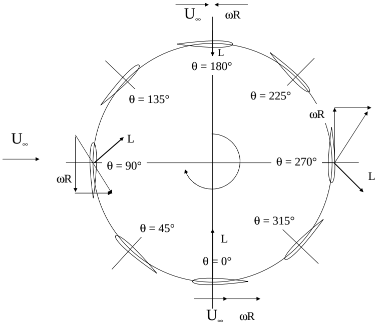


Rotor-stator interaction. Mach contours. Two-dimensional simulations with ZEN at CIRA (2006), in collaboration with P. L. Vitagliano. Sliding mesh technique.

- The fluid dynamics of turbomachinery is another wide sector in which the aerodynamics of oscillating airfoil is extensively applied.

Introduction

Examples: Wind Energy



- Similar phenomena occur on the blades of the wind turbines.
- The blades work in a wide range of angles of attack.
- The exact knowledge of the aerodynamic loads can improve the design and the structural life of the plant in terms of fatigue limits.
- Other requirements, such as the low noise emission, must be respected in order to reduce the environment impact.

Introduction

Airfoil unsteady aerodynamics

- The steady aerodynamics provides relations of kind

$$C_l = C_l(\alpha, Re_\infty, M_\infty)$$

- In the unsteady aerodynamics, the dependency must be of kind

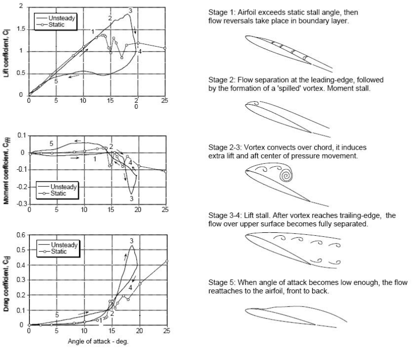
$$C_l = C_l(\alpha, \dot{\alpha}, \ddot{\alpha}, \ddot{h}, Re_\infty, M_\infty)$$

  where h is the vertical displacement.

- Namely, the aerodynamic characteristics must include the dependency upon the airfoil motion
- The flow exhibits a memory of the past history.
- Each flow phenomenon typical of the airfoil aerodynamics (transition, separation, stall, buffet, ...) must be revisited under this perspective.

Introduction

Airfoil unsteady aerodynamics. Dynamic Stall.



From Leishman (2000). Principles of Helicopter Aerodynamics.

- The stage (1) is in correspondance of the static stall angle of the airfoil. Under dynamic conditions the lift curve continues to grow.
- An extrapolation of the linear slope is observed up to the points (2) or (3). The excess of lift is due to a production of vorticity in the boundary layer.
- At stage (2) the dynamic stall vortex is formed changing the lift curve slope.
- The moment coefficient stalls between stages (2) and (3).
- The maximum peak of lift is achieved when the dynamic stall vortex reaches the trailing edge.
- During stages (3) and (4), the flow is separated.
- The reattachment point advances at a velocity less than U_∞. After, the flow acquires again a linear behaviour (stage 5).

Introduction

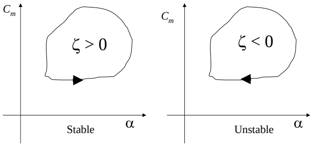
Airfoil unsteady aerodynamics. Dynamic Stall.

- An important quantity is the aerodynamic damping

$$\zeta = -\int C_m \, d\alpha$$

It represents the work done by the aerodynamic forces acting on the airfoil.

- If $\zeta > 0$ the fluid receives energy from the airfoil (stable).
- If $\zeta < 0$ the fluid transfers energy to the airfoil (unstable).
- Geometrically, ζ is the measure of the area enclosed by the C_m curve in the plane $C_m - \alpha$.



Introduction

Some concepts for ideal flows. Virtual or apparent mass

- The aerodynamic force is related to the *virtual* or *apparent mass* contribution, defined as (see Saffman):

$$\underline{\mathbf{I}}_B = \int_{\partial B} \phi \, \underline{n} \, dS$$

  ϕ is the velocity potential.

- It is possible to show that:

$$\underline{\mathbf{F}} = -\frac{d}{dt}\underline{\mathbf{I}}_B$$

- If $M\underline{\mathbf{U}}$ is the momentum of the solid body and $\underline{\mathbf{f}}$ the resultant of the external forces applied on the solid body, Newton's second law is:

$$\frac{d}{dt}M\underline{\mathbf{U}} = \underline{\mathbf{F}} + \underline{\mathbf{f}}$$

- Then we have:

$$\frac{d}{dt}(M\underline{\mathbf{U}} + \underline{\mathbf{I}}_B) = \underline{\mathbf{f}}$$

Introduction

Some concepts for ideal flows. Virtual or apparent mass

- The presence of the *virtual mass* $\underline{\mathbf{I}}_B$ alters the solid body inertia.
- The term $\underline{\mathbf{I}}_B$ accounts for the effects of the fluid surrounding B.
- The external force $\underline{\mathbf{f}}$ applied on B is balanced by the real mass of the body and the fluid virtual mass.
- These effects appear in case of unsteady equilibrium only.

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Theodorsen solution.

Theodorsen Solution

Hypothesis.

In 1935 Theodorsen obtained the unsteady flow solution for a thin oscillating airfoil.

- Inviscid flow
- Incompressible flow
- thin airfoil
- small disturbances

Suppose the free stream velocity parallel to the x axis and the wall normal directed as z. We have

$$u = U_\infty + u' \tag{1}$$

$$u', w \ll V_\infty \tag{2}$$

Theodorsen Solution

Problem equations.

Suppose it is possible to introduce a potential perturbation ϕ' such that

$$\frac{\partial \phi'}{\partial x} = u', \quad \frac{\partial \phi'}{\partial z} = w$$

The potential perturbation satisfies the equation of Laplace:

$$\nabla^2 \phi' = 0 \qquad (3)$$

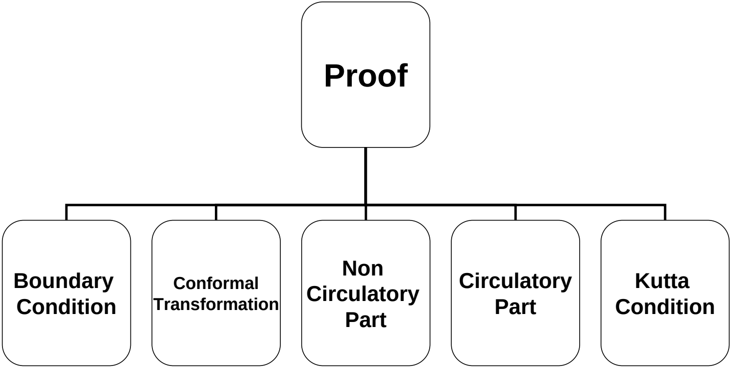
The linearized Bernoulli equation is also written as

$$p - p_\infty = -\rho U_\infty u' - \rho \frac{\partial \phi'}{\partial t} \qquad (4)$$

The problem is defined by setting the initial and boundary conditions.

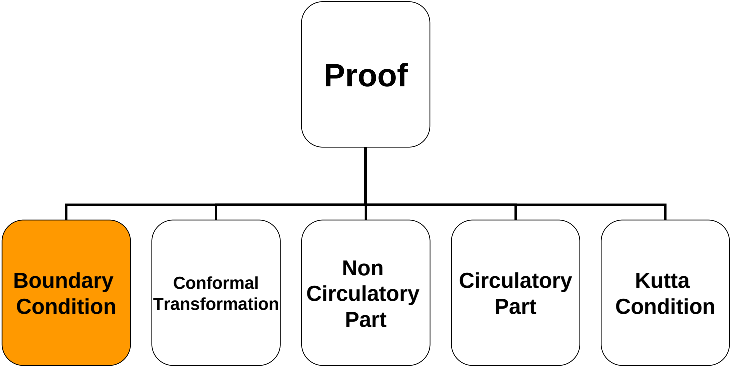
Theodorsen Solution

Road Map.



Theodorsen Solution

Road Map.



Theodorsen Solution

Part 1. Boundary conditions.

- On the solid body, the wall normal component of the velocity must be specified.
- The airfoil surface can be expressed in this form

$$F_u = z - z_u(x, z, t) = 0$$

$$F_L = z - z_L(x, z, t) = 0$$

- The boundary condition requires for the upper and lower sides that

$$\frac{DF}{Dt} = \frac{\partial z}{\partial t} + u\frac{\partial z}{\partial x} + w = 0$$

- Since of the hypothesis of small disturbances the upper and lower surfaces are mathematically approximated as a plane surface at $z = 0$. Besides,

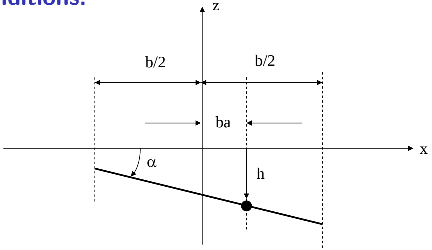
$$u\frac{\partial z}{\partial x} \approx V_\infty \frac{\partial z}{\partial x}$$

- Then we have the following condition

$$\boxed{w\frac{\partial z}{\partial t} + U_\infty \frac{\partial z}{\partial x} = w_a(x, t)} \tag{5}$$

Theodorsen Solution

Part 1. Boundary conditions.



Scheme of the velocity components on the circumference in the complex plane

- The plate oscillates at *ba* from the half chord location.
- The instantaneous position is:

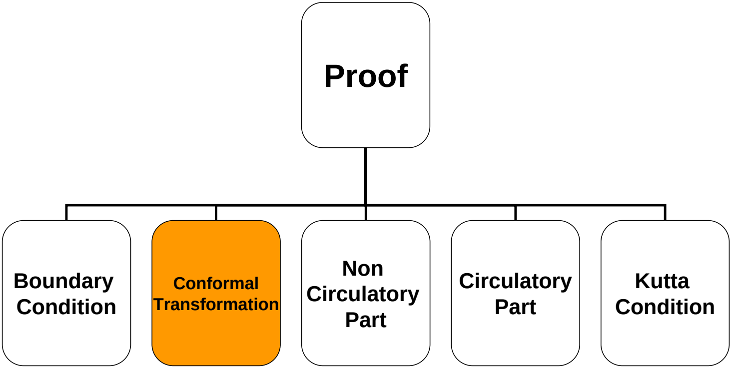
$$z_a(x,t) = -h - \alpha(x - ba)$$

from which

$$w_a(x,t) = -\dot{h} - \dot{\alpha}(x - ba) - U_\infty \alpha \tag{6}$$

Theodorsen Solution

Road Map.



Theodorsen Solution

Part 2. Conformal transformation.

- The proof of Theodorsen's solution is achieved by transforming the flat plate from the physical plane xz in a circle in the complex plane XZ.
- Let $c = 2b$ be the chord of the airfoil. The conformal transformation is

$$x + i\,z = X + i\,Z + \frac{b^2}{4(X + i\,Z)} \tag{7}$$

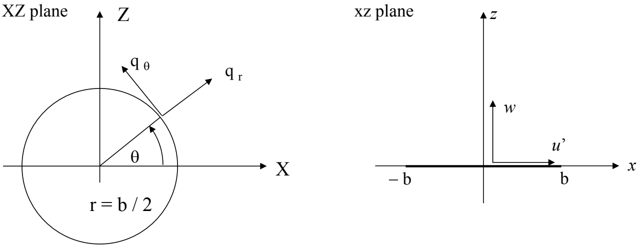
where $i = \sqrt{-1}$ is the imaginary unit.

- The above relation transforms the plate of length c to a circle of radius $r = b/2$.
- In fact, $X = r\cos\theta$ and $Z = r\sin\theta$, we have:

$$x + i\,z = r\,e^{i\theta} + \frac{b^2}{4re^{i\theta}} = 2r\cos\theta$$

Theodorsen Solution

Part 2. Conformal transformation.



Conformal transformation of a circle in the plane XZ to a flat plate in the plane xz.

Theodorsen Solution

Part 2. Conformal transformation. Velocity.

- Now, it is necessary to establish the transformation for the velocity components.
- In the plane XZ, the velocity components are indicated with q_X and q_Z.
- The complex velocity is obtained as

$$u' - iw = (q_x - iq_z)\frac{d(X + i\,Z)}{d(x + i\,z)}$$

where:

$$\frac{d(x + i\,z)}{d(X + i\,Z)} = 1 - \frac{b^2}{4(X + i\,Z)^2} \tag{8}$$

For $r = b/2$,

$$\left[\frac{d(x + i\,z)}{d(X + i\,Z)}\right]_{r=b/2} = 2\sin\theta e^{i\theta} \tag{9}$$

Theodorsen Solution

Part 2. Conformal transformation. Velocity.

- By calculating q_X and q_Z on the circle $r = b/2$, we have:

$$u' - i\,w \quad = \quad \frac{q_X - i\,q_Z}{2\sin\theta}\,e^{\,i\,(\theta-\pi/2)} = \frac{1}{2\sin\theta}\left[q_X\,e^{\,i\,(\theta-\pi/2)} + q_Z\,e^{\,i\,(\theta-\pi)}\right]$$

- Since, $e^{\,i\,(\theta-\pi/2)} = \sin\theta - i\,\cos\theta$, and $e^{\,i\,(\theta-\pi)} = -\cos\theta - i\,\sin\theta$, we have:

$$u' - i\,w \quad = \quad \frac{1}{2\sin\theta}\left[q_X\sin\theta - q_Z\cos\theta - i\,(q_X\cos\theta + q_Z\sin\theta)\right]$$

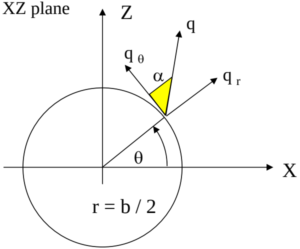
- The modules are:

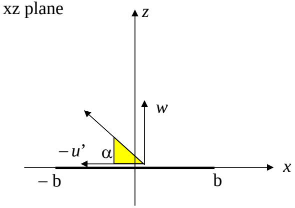
$$|u' - iw| = \sqrt{u'^2 + w^2} = \frac{\sqrt{q_X^2 + q_Z^2}}{|2\sin\theta|} = \frac{\sqrt{q_\theta^2 + q_r^2}}{|2\sin\theta|} \qquad (10)$$

where q_θ and q_r are the radial and tangential components in the XZ plane.

Theodorsen Solution

Part 2. Conformal transformation. Velocity





Conformal transformations preserve the angle at which two lines intersect. Then,

$$\frac{|q_\theta|}{|q|} = \frac{|u'|}{|u|} \Rightarrow |u'| = \frac{|q_\theta|}{|2\sin\theta|} \qquad \text{and} \qquad |w| = \frac{|q_r|}{|2\sin\theta|}$$

According with the sign of θ and x axis, we deduct also that :

$$\boxed{u' = -\frac{q_\theta}{2\sin\theta} \qquad \text{and} \qquad w = \frac{q_r}{2\sin\theta}}$$

Theodorsen Solution

Part 2. Conformal transformation. Potential.

- It is necessary to connect the velocity potentials between the xz and XZ planes in such a way that

$$d\phi(x,z) = d\phi(X,Z)$$

- For a path along the slit in the xz plane (the circle in the XZ plane)

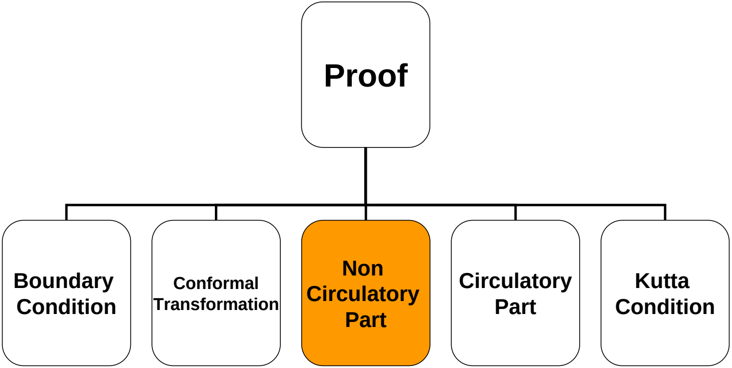
$$d\phi(x,z) = u'dx \qquad d\phi(X,Z) = \frac{b}{2}q_\theta\, d\theta$$

- The potential difference between two points on the slit is

$$\boxed{\phi_2 - \phi_1 = \int_{\theta_1}^{\theta_2} \frac{b}{2}q_\theta\, d\theta = -\int_{x_1}^{x_2} u'\, dx} \qquad (11)$$

Theodorsen Solution

Road Map.



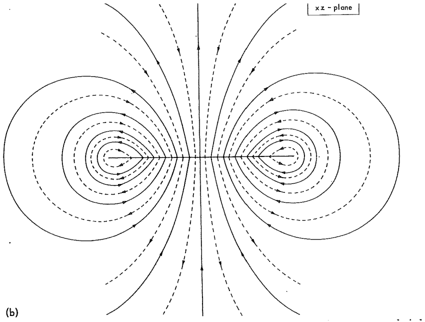
Theodorsen Solution

Part 3. Non circulatory contribution

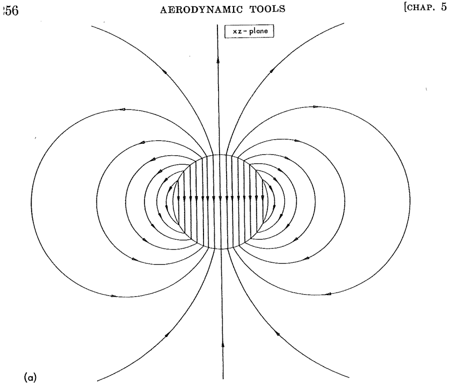
- Theodorsen achieved the solution by distributing sources (upper side) and sinks (lower side) of equal strength.
- In this way, the boundary condition $w_a(x, t)$ was fulfilled.
- The sources and sinks do not cancel each other on the plate surface.
- The points outside the circle in the plane XZ are mapped in the external field around the plate.
- But, the points inside the circle are associated in the external field of the plate as well.
- The whole plane XZ creates two overlapped sheets in the plane xz (Riemann surfaces).
- We pass from a sheet to another only when we cross the plate from upper side to the lower side and viceversa.

Theodorsen Solution

Part 3. Non circulatory contribution



(a) *xz* plane



(b) *XZ* plane

Stream lines of the flow due to the distribution of sources and sinks. From Bisplinghoff et al., pg 256

Theodorsen Solution

Part 3. Non circulatory contribution

- Let H^+ be an infinitesimal source sheet distributed on the upper side of the circle.
- The potential function ϕ' in (x, z) induced by H^+ is given by:

$$\phi'(x, z, t) = \frac{1}{4\pi} \int_{-b}^{b} H^+(\xi, t) \ln\left[(x-\xi)^2 + z^2\right] d\xi \tag{12}$$

- The result is related to the wall normal velocity as:

$$H^+(x, t) = 2\, w_a(x, t) \tag{13}$$

$$H^+(\theta, t) = 4 w_a(x, t) \sin\theta \tag{14}$$

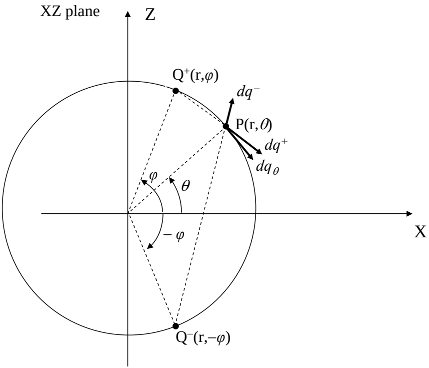
Similarly

$$H^-(x, t) = -2 w_a(x, t) \tag{15}$$

$$H^-(\theta, t) = -4 w_a(x, t) \sin\theta \tag{16}$$

Theodorsen Solution

Part 3. Non circulatory contribution



Scheme of the velocity components on the circumference in the complex plane

- The velocity resulting from the distribution of sources and sinks has to be derived.
- Consider two points, $Q^{+}(r,\psi)$ and $Q^{-}(r,-\psi)$ symmetrically located on the circumference
- The velocity in a point $P(r,\theta)$ induced by $H^{+}\ r\ d\psi$ and $H^{-}\ r\ d\psi$ is built on the basis of geometrical considerations.
- Note that the induced velocity of the source-sink sheet is such that $q_r = 0$, otherwise the circle is a stream line.

Theodorsen Solution

Part 3. Non circulatory contribution

- The final result is:

$$q_\theta(\theta, t) \quad = \quad \frac{2}{\pi} \int_0^\pi \frac{w_a(x,t)\sin^2\psi}{\cos\psi - \cos\theta}\, d\psi \tag{17}$$

- The potential function ϕ' is obtained:

$$\phi'_U(\theta, t) - \phi'(\pi, t) = -\frac{b}{\pi} \int_\theta^\pi \int_0^\pi \frac{w_a(x,t)\sin^2\psi}{\cos\psi - \cos\theta}\, d\psi d\theta \tag{18}$$

- Because of the arbitrary time function in the definition of the potential ϕ', it is possible to put $\phi'(\pi, t) = 0$.
- Besides, for the symmetry, we observe that the following relation subsists between the lower and upper side potential:

$$\phi'_L(-\theta, t) = -\phi'_U(\theta, t)$$

Theodorsen Solution

Part 3. Non circulatory contribution

- By taking into account that $x = b\cos\theta$, the following integral gives the velocity potential:

$$\phi'_U(\theta,t) = \frac{b}{\pi}(\dot{h} + U_\infty \alpha) \int_\theta^\pi \int_0^\pi \frac{\sin^2\psi}{\cos\psi - \cos\theta}\, d\psi d\theta + \frac{b^2\dot{\alpha}}{\pi} \int_\theta^\pi \int_0^\pi \frac{\sin^2\psi(\cos\theta - a)}{\cos\psi - \cos\theta}\, d\psi d\theta \qquad (19)$$

- After some algebra, the final result is achieved in this form:

$$\phi'_U(\theta,t) = b(\dot{h} + U_\infty\,\alpha)\sin\theta + b^2\dot{\alpha}\left(\frac{1}{2}\cos\theta - a\right) \qquad (20)$$

Theodorsen Solution

Part 3. Non circulatory contribution

- By means of Bernoulli, the pressure relates to the potential function. Then

$$p_U - p_L = \left[U_\infty \left(\frac{\partial \phi'_U}{\partial x} - \frac{\partial \phi'_L}{\partial x} \right) + \left(\frac{\partial \phi'_U}{\partial t} - \frac{\partial \phi'_L}{\partial t} \right) \right] \tag{21}$$

- By exploiting the symmetry properties of ϕ', we have:

$$p_U - p_L = -2U_\infty \frac{\partial \phi'_U}{\partial x} - 2\frac{\partial \phi'}{\partial t} = \frac{2U_\infty}{b \sin\theta} \frac{\partial \phi'_U}{\partial \theta} - 2\frac{\partial \phi'}{\partial t} \tag{22}$$

- The *non circulatory* contribution is:

$$L_{NC} \quad = \quad -\int_{-b}^{b} (p_U - p_L) dx = U_\infty \left[\phi'_U \right]_{-b}^{b} - \int_{-b}^{b} 2\rho \frac{\partial \phi'_U}{\partial t}\, dx \tag{23}$$

- Since there is no circulation, $\phi'(\theta, t)$ is single valued. Because of $\phi'_L(-\theta, t) = -\phi'_U(\theta, t)$, we have that $\phi_U(0, t) = \phi_L(0, t) = 0$. Then

$$L_{NC} \quad = \quad 2\frac{\partial}{\partial t} \int_0^\pi \phi'_U \sin\theta \, d\theta \tag{24}$$

Theodorsen Solution

Part 3. Non circulatory contribution

- By means of Bernoulli, the pressure relates to the potential function. Then

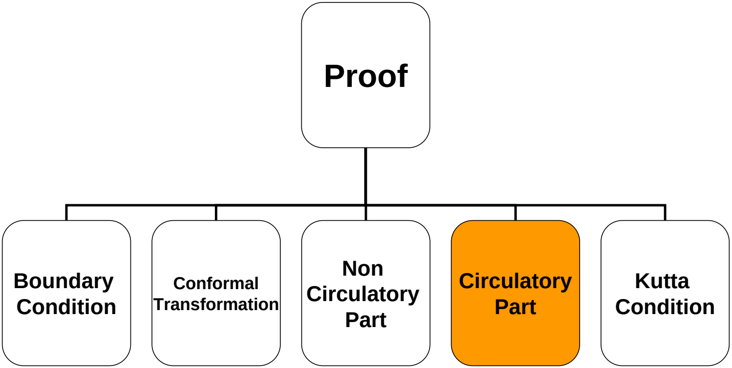
$$\boxed{L_{NC} = \pi b^2 \left(\ddot{h} + U_\infty \dot{\alpha} - a\, b\, \ddot{\alpha}\right)} \tag{25}$$

- We report also the *non circulatory* part of the aerodynamic moment:

$$\boxed{M_{NC} = \pi b^2 \left[U_\infty \dot{h} + ba\ddot{h} + U_\infty^2 \alpha - b^2 \left(\frac{1}{8} + a^2\right) \ddot{\alpha}\right]} \tag{26}$$

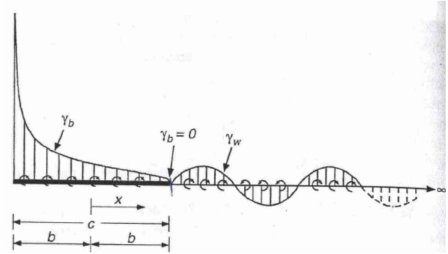
Theodorsen Solution

Road Map.



Theodorsen Solution

Part 4. Circulatory contribution



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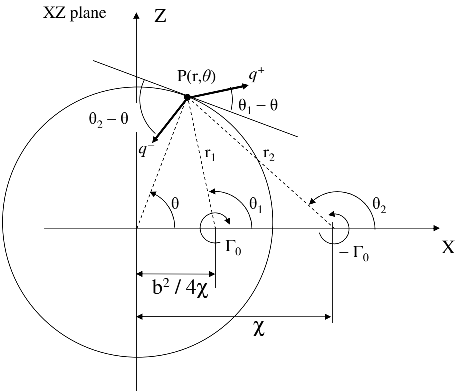
- The *non circolatory* part is not able to fulfill Kutta's condition.
- Theodorsen resolves the problem by superimposing a vorticity distribution on the body surface (*bound* vorticity) and in the wake (*free* vorticity).
- The technique of the image vortices are used.

Theodorsen Solution

Part 4. Circulatory contribution



Bound vortex of intensity Γ_0 and its image of intensity $-\Gamma_0$.

- A vortex located at $(\chi, 0)$ of intensity $-\Gamma_0$ in the plane XZ has its image Γ_0 in $(b^2/4\chi, 0)$.
- The vortex pair respects the condition of a stream line for the circumference.
- It is possible to show $q_r = 0$ on the circle.

Theodorsen Solution

Part 4. Circulatory contribution

- The tangential component q_θ is

$$q_\theta = \frac{\Gamma_0}{2\pi}\left[\frac{r_2\cos(\theta_2-\theta)}{r_2^2} - \frac{r_1\cos(\theta_1-\theta)}{r_1^2}\right]$$

- It can also be observed that

$$r_2^2 = \chi^2 + \left(\frac{b}{2}\right)^2 - \chi b\cos\theta; \qquad r_1^2 = \left(\frac{b^2}{4\chi}\right)^2 + \left(\frac{b}{2}\right)^2 - \frac{b^3}{4\chi}\cos\theta; \tag{27}$$

and

$$r_2\cos(\theta_2-\theta) = \frac{b}{2} - \chi\cos\theta; \qquad r_1\cos(\theta_1-\theta) = \frac{b}{2} - \frac{b^2}{4\chi}\cos\theta; \tag{28}$$

Theodorsen Solution

Part 4. Circulatory contribution

- By substituting the previous relations we have:

$$q_\theta = -\frac{\Gamma_0}{\pi b}\left[\frac{\chi^2 - \frac{b^2}{4}}{\chi^2 + \frac{b^2}{4} - \chi b\cos\theta}\right] \tag{29}$$

- The velocity potential is calculated

$$\begin{aligned} \phi'_U(\theta,t) &= -\int_\theta^\pi q_\theta \, \frac{b}{2}\, d\theta = \\ &= \frac{\Gamma_0}{2\,\pi}\left(\chi^2 - \frac{b^2}{4}\right)\int_\theta^\pi \frac{1}{\chi^2 + \frac{b^2}{4} - \chi b\cos\theta}\, d\theta \end{aligned} \tag{30}$$

Theodorsen Solution

Part 4. Circulatory contribution

- The result is

$$\phi'_U(\theta, t) = \frac{\Gamma_0}{\pi} \tan^{-1} \left(\frac{(\chi - \frac{1}{2} b)}{(\chi + \frac{1}{2} b)} \sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} \right) \tag{31}$$

- By means of equation (31), we are able to compute the pressure distribution by using Bernoulli.
- Note that the time dependency appears through the variable $\chi(t)$ which indicated the instantaneous position of the wake vortex.
- The hypothesis that the vortex is shed at the free stream velocity is adopted.

Theodorsen Solution

Part 4. Circulatory contribution

- This assumption allows for the following transformation:

$$\frac{d\xi}{dt} = U_\infty \tag{32}$$

where ξ is the vortex location in the plane xz which corresponds to

$$\xi = \chi + \frac{b^2}{4\chi} \tag{33}$$

in the plane XZ.

- Equation (33) can be cast as follows:

$$\sqrt{\frac{\xi - b}{\xi + b}} = \frac{\chi - (b/2)}{\chi + (b/2)} \tag{34}$$

In this way, equation (31) can be written as:

$$\phi'_U(\theta, t) \quad = \quad \frac{\Gamma_0}{\pi} \tan^{-1} \sqrt{\frac{(\xi - b)(1 + \cos\theta)}{(\xi + b)(1 - \cos\theta)}} \tag{35}$$

Theodorsen Solution

Part 4. Circulatory contribution

- The lift produced by the pair of vortices of intensity Γ_0 is determined:

$$L_{\Gamma_0} = -\int_0^{\pi} (p_U - p_L)\, b\, \sin\theta\, d\theta = \frac{U_\infty\, \Gamma_0\, \xi}{\sqrt{\xi^2 - b^2}} \tag{36}$$

It can be noted that for $\xi \to \infty$, (i.e., $t \to \infty$) the lift tends to the value produced by a single vortex of intensity Γ_0.

Theodorsen Solution

Part 4. Circulatory contribution

- When we deal with a distribution of wake vorticity, the treatment must be referred to an element of vorticity:

$$\Gamma_0 = -\gamma_w(\xi, t)\, d\xi \tag{37}$$

- Now, the velocity is expressed by:

$$q_\theta = \int_b^\infty \frac{\gamma_w\, d\xi}{\pi b}\left[\frac{\chi^2 - \frac{b^2}{4}}{\chi^2 + \frac{b^2}{4} - \chi b\cos\theta}\right] \tag{38}$$

- The pressure difference due to the complete system of wake vorticity is obtained by

$$p_U - p_L = \frac{U_\infty}{\pi b\, \sin\theta}\int_b^\infty \left(\frac{\xi + b\cos\theta}{\sqrt{\xi^2 - b^2}}\right)\gamma_w(\xi, t)d\xi \tag{39}$$

Theodorsen Solution

Part 4. Circulatory contribution

- By integrating from the trailing edge to infinity we find the complete effect of the wake vorticity on the lift:

$$L_C = -U_\infty \int_b^\infty \frac{\xi}{\sqrt{\xi^2 - b^2}} \gamma_w(\xi, t) d\xi \tag{40}$$

- Theodorsen indicates with Q the following integral:

$$Q = -\frac{1}{2\pi b} \int_b^\infty \sqrt{\frac{\xi + b}{\xi - b}} \gamma_w(\xi, t)\, d\xi$$

- Then, we can write the circulatory part of the lift as:

$$L_C = 2\pi\, b\, U_\infty Q \frac{\displaystyle\int_b^\infty \frac{\xi}{\sqrt{\xi^2 - b^2}} \gamma_w(\xi, t) d\xi}{\displaystyle\int_b^\infty \sqrt{\frac{\xi + b}{\xi - b}} \gamma_w(\xi, t)\, d\xi} \tag{41}$$

Theodorsen Solution

Part 4. Circulatory contribution

- By assuming simple harmonic oscillations in time the wake vorticity $\gamma_w(\xi, t)$ takes the form:

$$\gamma_w(\xi, t) = \bar{\gamma}_w e^{i\omega(t - \frac{\xi}{U_\infty})} \tag{42}$$

- By defining the reduced frequency $k = \omega\, b/U_\infty$ and $\xi^* = \xi/\, b$,

$$\gamma_w(\xi, t) = \bar{\gamma}_w e^{i\omega(t - k\xi^*)} \tag{43}$$

- The ratio of the integrals can be manipulated as:

$$\frac{\displaystyle\int_b^\infty \frac{\xi}{\sqrt{\xi^2 - b^2}} \gamma_w(\xi, t) d\xi}{\displaystyle\int_b^\infty \sqrt{\frac{\xi + b}{\xi - b}} \gamma_w(\xi, t)\, d\xi} = \frac{\displaystyle\int_1^\infty \frac{\xi^*}{\sqrt{\xi^{*2} - 1}} e^{-ik\xi^*}\, d\xi^*}{\displaystyle\int_1^\infty \sqrt{\frac{\xi^* + 1}{\xi^* - 1}}\, e^{-ik\xi^*}\, d\xi^*} = C(k) \tag{44}$$

- $C(k)$ is a complex function of the reduced frequency only.
- $C(k)$ is said Theodorsen's function.

Theodorsen Solution

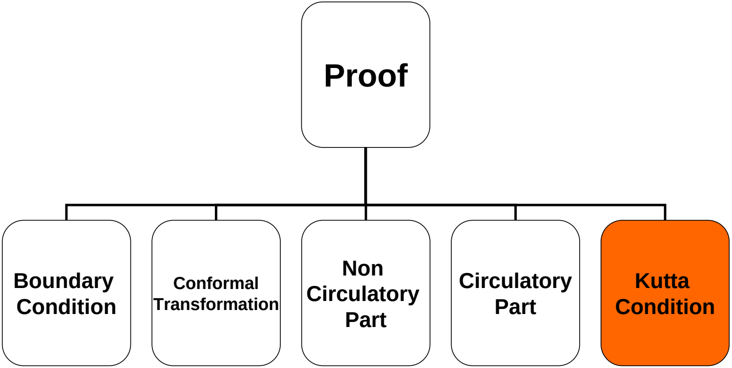
Part 4. Circulatory contribution

- The circulatory lift is

$$\boxed{L_C = 2\pi\, b\, U_\infty Q\, C(k)} \tag{45}$$

Theodorsen Solution

Road Map.



Theodorsen Solution

Part 5. Use of Kutta condition

- Kutta's condition establishes the velocity q_θ at the trailing edge ($\theta = 0$) is zero.
- By means of this further relation, the integral ratio Q can be computed.
- The relations (17) and (29) provide the velocity at T.E.

$$q_\theta(\theta = 0) = \frac{2}{\pi}\int_0^\pi \frac{w_a(x,t)\sin^2\psi}{\cos\psi - 1}\,d\psi + \int_b^\infty \frac{\gamma_w\,d\xi}{\pi b}\left[\frac{\chi^2 - \frac{b^2}{4}}{\chi^2 + \frac{b^2}{4} - \chi b}\right] = 0$$

- By taking into account the relation (34) between χ and ξ, the Kutta condition can be written as:

$$\frac{2}{\pi}\int_0^\pi \frac{w_a(x,t)\sin^2\psi}{\cos\psi - 1}\,d\psi + \frac{1}{\pi b}\int_b^\infty \sqrt{\frac{\xi + b}{\xi - b}}\gamma_w(\xi,t)\,d\xi = 0$$

Theodorsen Solution

Part 5. Use of Kutta condition

- Then,

$$q_\theta = \frac{2}{\pi}\int_0^\pi \frac{w_a(x,t)\sin^2\psi}{\cos\psi - 1}\,d\psi - 2Q = 0$$

By substituting the expression of $w_a(x,t)$ in equation (6), we have:

$$\boxed{Q = \dot{h} + U_\infty \alpha + b\left(\frac{1}{2} - a\right)\dot{\alpha}} \qquad (46)$$

Theodorsen Solution

Final expression.

- By collecting equations (25), (45) and (46) the Theodorsen solution is obtained:

$$\begin{aligned} L &= \pi b^2 \left(\ddot{h} + U_\infty \dot{\alpha} - a\, b\, \ddot{\alpha}\right) + \\ &+ 2\pi\, b\, U_\infty C(k) \left[\dot{h} + U_\infty \alpha + b\left(\frac{1}{2} - a\right)\, \dot{\alpha}\right] \end{aligned} \tag{47}$$

- The expression of the aerodynamic moment is also reported:

$$\begin{aligned} M &= \pi b^2 \left[b a \ddot{h} - U_\infty b \left(\frac{1}{2} - a\right)\, \dot{\alpha} - b^2 \left(\frac{1}{8} + a^2\right) \ddot{\alpha}\right] \\ &+ 2\pi U_\infty b^2 \left(a + \frac{1}{2}\right) C(k) \left[\dot{h} + U_\infty \alpha + b\left(\frac{1}{2} - a\right)\, \dot{\alpha}\right] \end{aligned} \tag{48}$$

Theodorsen Solution

Considerations

- In the circulatory part there is an equivalence between $\dot{h}$ and $U_\infty \alpha$.
- The body motion must produce small velocities according to the hypothesis of small disturbances.
- As a consequence, the reduced frequency is limited.
- The effect of a mean steady angle of attack is taken into account by adding the steady linear contribution.
- Some special cases:

$$\dot{h} = \ddot{h} = 0; \qquad a = -1/2; \qquad \alpha = \bar{\alpha}\, e^{i\omega t};$$

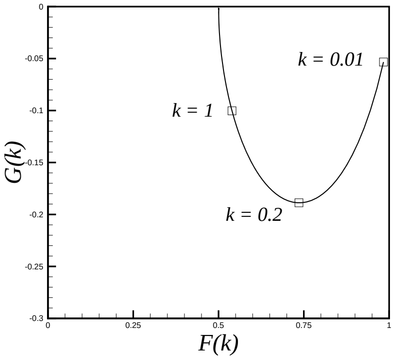
The lift coefficient is

$$C_l = 2\pi \left[F(1 + ik) + G(i - k) \right] \bar{\alpha}\, e^{i\omega t} + \pi \left(i - \frac{k}{2} \right) \bar{\alpha}\, e^{i\omega t}$$

where $F = Re(C)$ and $G = Im(C)$.

Theodorsen Solution

Considerations



- $F \to 1$ as $k \to 0$
- $G \to 0$ as $k \to 0$
- $F \to 0.5$ as $k \to \infty$
- $G \to 0$ as $k \to \infty$

Theodorsen Solution

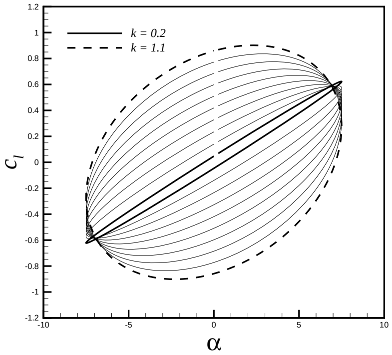
Considerations

Approximated expression of Theodorsen's function.

$$
\begin{aligned}
F &= \frac{0.5005k^3 + 0.51261k^2 + 0.21040k + 0.021573}{k^3 + 1.03538k^2 + 0.25124k + 0.02151} \\
G &= -\frac{0.00015k^3 + 0.12240k^2 + 0.32721k + 0.001990}{k^3 + 2.48148k^2 + 0.93453k + 0.08932}
\end{aligned}
$$

Theodorsen Solution

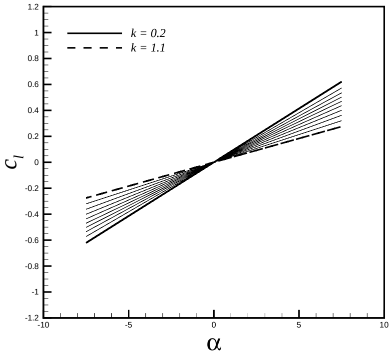
Considerations



- Various Theodorsen solutions. $C_l - \alpha$ curves.

Theodorsen Solution

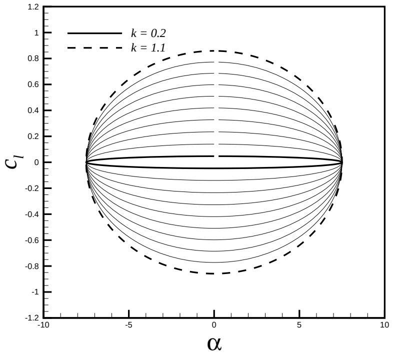
Considerations



- Theodorsen solutions. $C_l - \alpha$ curves.In-phase contribution.

Theodorsen Solution

Considerations



- Theodorsen solutions. $C_l - \alpha$ curves. Out-phase contribution.

CFD of the oscillating airfoils.

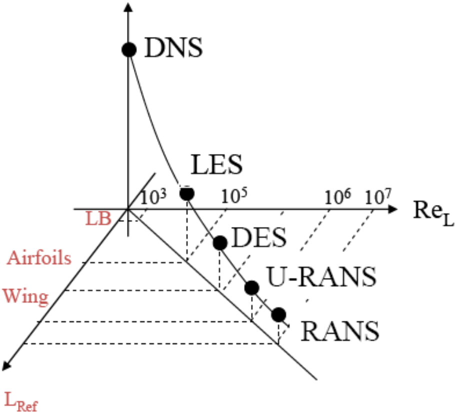
CFD of the oscillating airfoils

Real flow around oscillating airfoils

- Unsteadiness
- Turbulence
- Three-dimensional effects (even for airfoils)
- Compressibility effects (even for low free stream Mach numbers)
- Transition from laminar to turbulence
- Vibration and structure deformation

CFD of the oscillating airfoils

Numerical methods.



- The numerical methods can be listed as the accuracy increases.
- The numerical simulation around an airfoil at flight Reynolds number:
- (2080) Direct Navier Stokes
- (2045) Large Eddy Simulation
- (2000) Detached Eddy Simulation
- (1995) Unsteady RANS
- (1985) RANS

From Spalart 1999, 2000.

CFD of the oscillating airfoils

Numerical methods.

The progress in the prediction of the dynamic stall features is limited by the following factors:

- The numerical simulations are very time-consuming. Difficulties in tuning the parameters.
- The experimental data often are not very accurate and detailed in order to make close comparisons.
- The experimental measures of the dynamic stall are made complex by the presence of the mechanical devices for the oscillatory motion.
- The pressure taps give reliable values only for the lift and not for the drag and moment
- Other techniques, PIV, LDV, provide more information but far from the solid wall
- There no information about the presence of laminar separation bubble, transition location ...

CFD of the oscillating airfoils

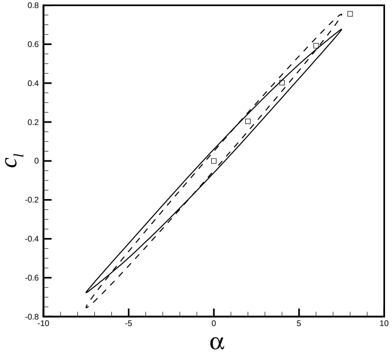
Numerical methods.

Some examples of dynamic stall with the CIRA CFD code, ZEN.

- Geometry NACA0012
- $Re = 1.35 \times 10^5$
- 2D Grid, 153600 cells.
- $\kappa - \omega$ SST Menter.
- No model for the transition.
- Moving grid technique.

CFD of the oscillating airfoils

Applications. Pre stall



- $Re = 1.35 \times 10^5$, $M = 0.1$.
- $\alpha = 7.5^\circ \sin(2kt)$, $k = 0.05$
- $C_l - \alpha$ curve
- Solid Line, unsteady RANS
- Dashed Line, Theodorsen

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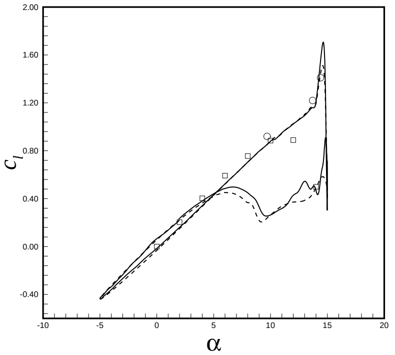
CFD of the oscillating airfoils

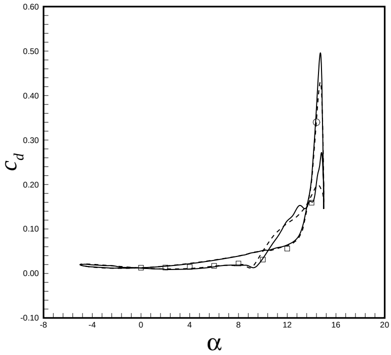
Applications. Light stall



- $Re = 1.35 \times 10^5$, $M = 0.1$.
- $\alpha = 5^\circ + 10^\circ \sin(2kt)$, $k = 0.05$
- $C_l - \alpha$ curve
- Solid Line, unsteady RANS

CFD of the oscillating airfoils

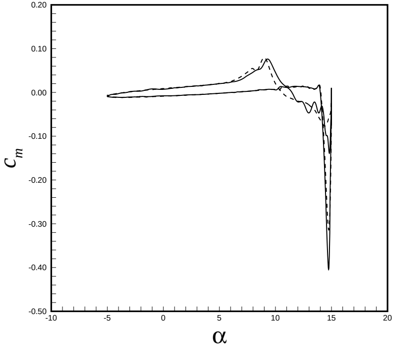
Applications. Light stall



- $Re = 1.35 \times 10^5$, $M = 0.1$.
- $\alpha = 5^\circ + 10^\circ \sin(2kt)$, $k = 0.05$
- $C_d - \alpha$ curve
- Solid Line, unsteady RANS

CFD of the oscillating airfoils

Applications. Light stall



- $Re = 1.35 \times 10^5$, $M = 0.1$.
- $\alpha = 5^\circ + 10^\circ \sin(2kt)$, $k = 0.05$
- $C_m - \alpha$ curve
- Solid Line, unsteady RANS

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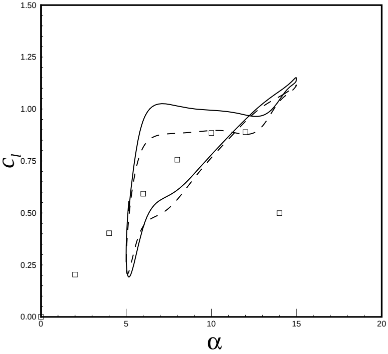
CFD of the oscillating airfoils

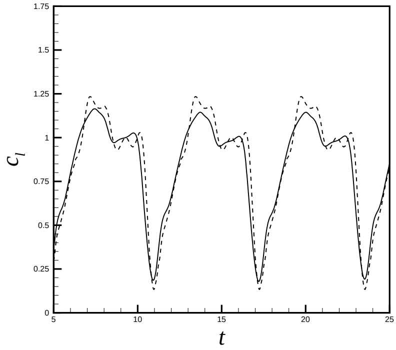
Applications. Deep stall



- $Re = 1.35 \times 10^5,\ M = 0.3.$
- $\alpha = 10^\circ + 5^\circ \sin(2kt),\ k = 0.5$
- $C_l - \alpha$ curve
- Solid Line, unsteady RANS

CFD of the oscillating airfoils

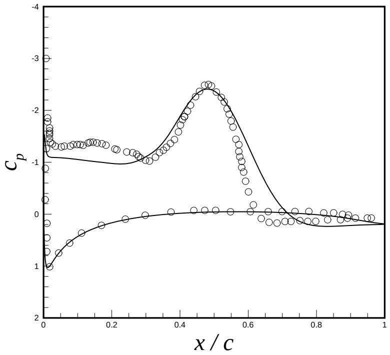
Applications. Deep stall



- $Re = 1.35 \times 10^5$, $M = 0.3$.
- $\alpha = 10^\circ + 5^\circ \sin(2kt)$, $k = 0.5$
- $C_l -$ time curve
- Solid Line, unsteady RANS
- dashed, LES (from Nagarayan et al. 2006)

CFD of the oscillating airfoils

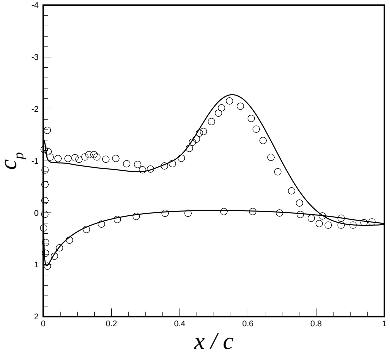
Applications. Deep stall



- $Re = 1.35 \times 10^5$, $M = 0.3$.
- $\alpha = 10^\circ + 5^\circ \sin(2kt)$, $k = 0.5$
- C_P distribution on the airfoil.
- Solid Line, unsteady RANS
- ○, LES (from Nagarayan et al. 2006)

CFD of the oscillating airfoils

Applications. Deep stall



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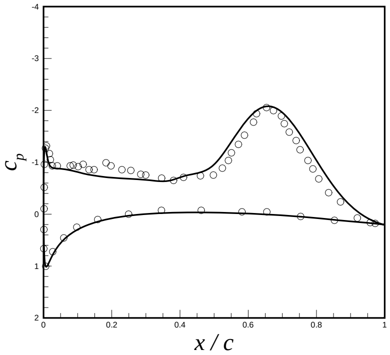
- Solid Line, unsteady RANS
- $\circ$, LES (from Nagarayan et al. 2006)

CFD of the oscillating airfoils

Applications. Deep stall



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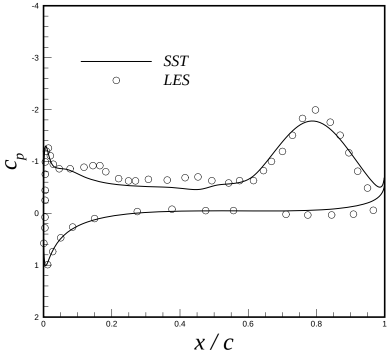
CFD of the oscillating airfoils

Applications. Deep stall



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- $\alpha = 10^\circ + 5^\circ \sin(2kt)$, $k = 0.5$
- C_P distribution on the airfoil.
- Solid Line, unsteady RANS
- $\circ$, LES (from Nagarayan et al. 2006)

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Conclusions

- In case of ideal flow, the aerodynamic theories provide useful explanations of the airfoil behavior.
- When the angular amplitudes are wide enough (dynamic stall) there is no exact theory able to predict the aerodynamics.
- The analysis of the dynamic stall can be made only by CFD and by experimental measurements.
- Currently the dynamic stall is object of many industrial researches for its strong technological impact.