

CONTROLLO DEL MOTO

Controllo nello spazio dei giunti

Controllo decentralizzato

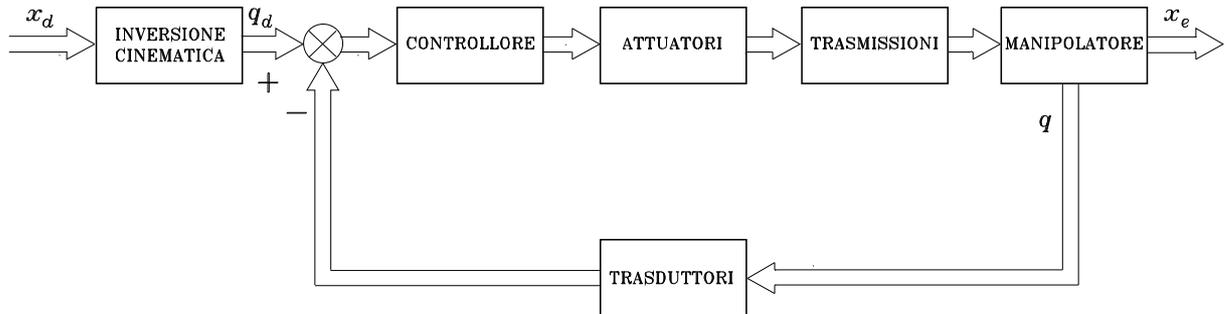
Compensazione in avanti a coppia precalcolata

Controllo centralizzato

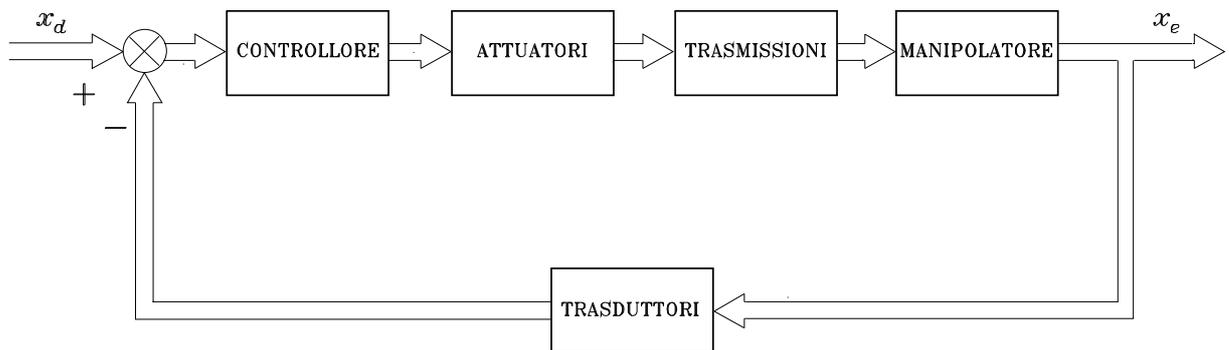
Controllo nello spazio operativo

IL PROBLEMA DEL CONTROLLO

- Controllo nello spazio dei giunti



- Controllo nello spazio operativo



CONTROLLO NELLO SPAZIO DEI GIUNTI

- Modello dinamico

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F_v\dot{q} + g(q) = \tau$$

- Controllo \equiv determinare τ :

$$q(t) = q_d(t)$$

- ★ trasmissioni

$$K_r q = q_m \quad \tau_m = K_r^{-1} \tau$$

- ★ azionamenti

$$\begin{aligned} K_r^{-1} \tau &= K_t i_a \\ v_a &= R_a i_a + K_v \dot{q}_m \\ v_a &= G_v v_c \end{aligned}$$

- Manipolatore controllato in velocità

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = u$$

$$F = F_v + K_r K_t R_a^{-1} K_v K_r$$

$$u = K_r K_t R_a^{-1} G_v v_c$$

$$K_r K_t R_a^{-1} G_v v_c = \tau + K_r K_t R_a^{-1} K_v K_r \dot{q}$$

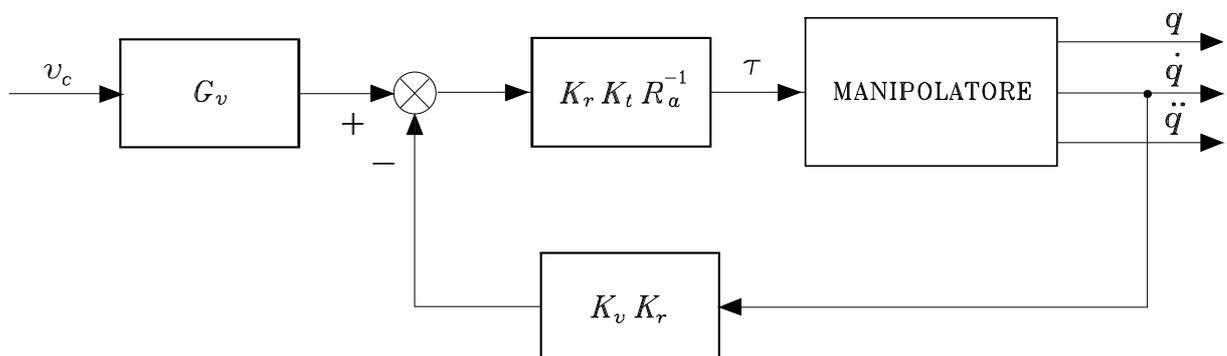
$$\tau = K_r K_t R_a^{-1} (G_v v_c - K_v K_r \dot{q})$$

- ★ K_r con elementi $\gg 1$
- ★ R_a con elementi piccoli
- ★ τ non elevate

⇓

- ★ controllo decentralizzato

$$G_v v_c \approx K_v K_r \dot{q}$$



- Manipolatore controllato in coppia

- ★ riduzione della sensibilità alle variazioni parametriche

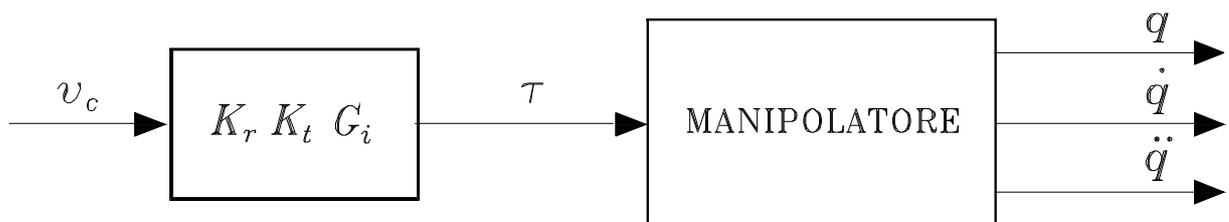
$$\mathbf{K}_t, \mathbf{K}_v, \mathbf{R}_a$$

$$\mathbf{i}_a = \mathbf{G}_i \mathbf{v}_c$$

⇓

- ★ controllo centralizzato

$$\boldsymbol{\tau} = \mathbf{u} = \mathbf{K}_r \mathbf{K}_t \mathbf{G}_i \mathbf{v}_c$$



CONTROLLO DECENTRALIZZATO

- Modello dinamico al lato motore

$$\mathbf{K}_r^{-1} \mathbf{B} \mathbf{K}_r^{-1} \ddot{\mathbf{q}}_m + \mathbf{K}_r^{-1} \mathbf{C} \mathbf{K}_r^{-1} \dot{\mathbf{q}}_m + \mathbf{K}_r^{-1} \mathbf{F}_v \mathbf{K}_r^{-1} \dot{\mathbf{q}}_m + \mathbf{K}_r^{-1} \mathbf{g} = \boldsymbol{\tau}_m$$

- ★ inerzie medie

$$\mathbf{B}(\mathbf{q}) = \bar{\mathbf{B}} + \Delta \mathbf{B}(\mathbf{q})$$

$$\mathbf{K}_r^{-1} \bar{\mathbf{B}} \mathbf{K}_r^{-1} \ddot{\mathbf{q}}_m + \mathbf{F}_m \dot{\mathbf{q}}_m + \mathbf{d} = \boldsymbol{\tau}_m$$

- ★ attrito viscoso

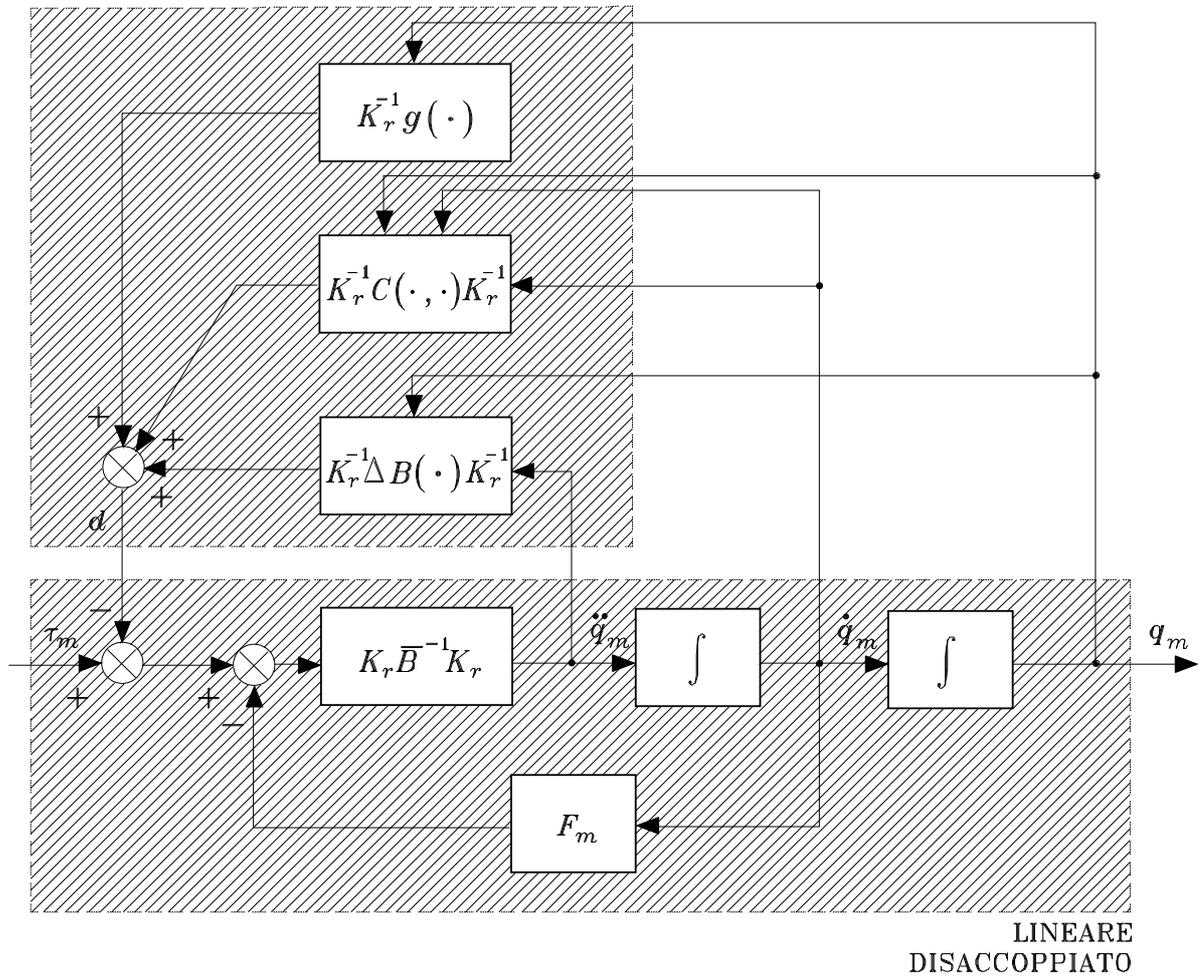
$$\mathbf{F}_m = \mathbf{K}_r^{-1} \mathbf{F}_v \mathbf{K}_r^{-1}$$

- ★ disturbo

$$\mathbf{d} = \mathbf{K}_r^{-1} \Delta \mathbf{B}(\mathbf{q}) \mathbf{K}_r^{-1} \ddot{\mathbf{q}}_m + \mathbf{K}_r^{-1} \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \mathbf{K}_r^{-1} \dot{\mathbf{q}}_m + \mathbf{K}_r^{-1} \mathbf{g}(\mathbf{q})$$

• Manipolatore + azionamenti

NON LINEARE
ACCOPPIATO



CONTROLLO INDIPENDENTE AI GIUNTI

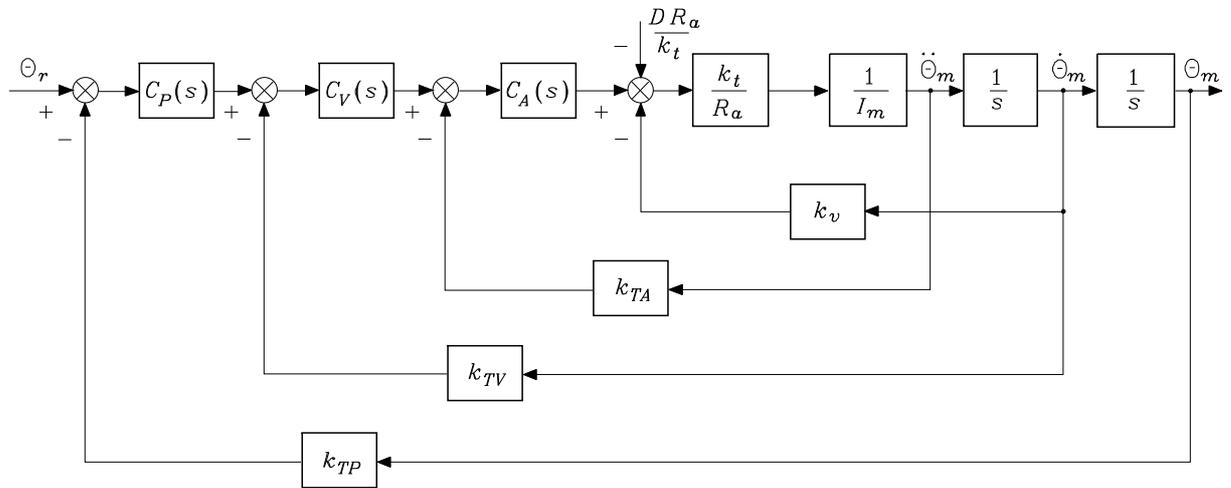
- Manipolatore $\equiv n$ sistemi indipendenti (azionamenti di giunto)
 - ★ controllo di ogni asse di giunto come un *sistema a un ingresso e una uscita*
 - ★ effetti di accoppiamento tra i vari giunti trattati come ingressi di disturbo
- Riduzione degli effetti di d su θ
 - ★ elevato valore del guadagno degli amplificatori a monte del punto di applicazione del disturbo
 - ★ presenza nel controllore di una azione integrale al fine di annullare a regime (θ costante) l'effetto della componente gravitazionale sull'uscita



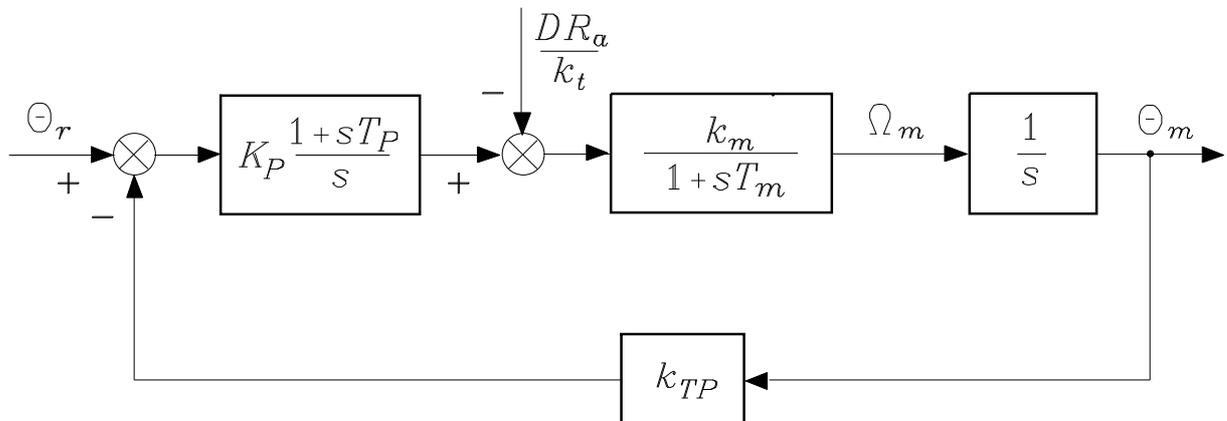
- Controllore *proporzionale-integrale* (PI)

$$C(s) = K_c \frac{1 + sT_c}{s}$$

- Struttura generale



- Retroazione di posizione



$$C_P(s) = K_P \frac{1 + sT_P}{s} \quad C_V(s) = 1 \quad C_A(s) = 1$$

$$k_{TV} = k_{TA} = 0$$

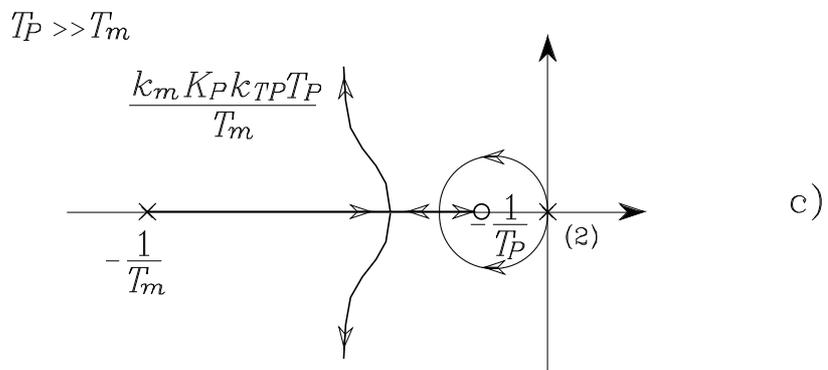
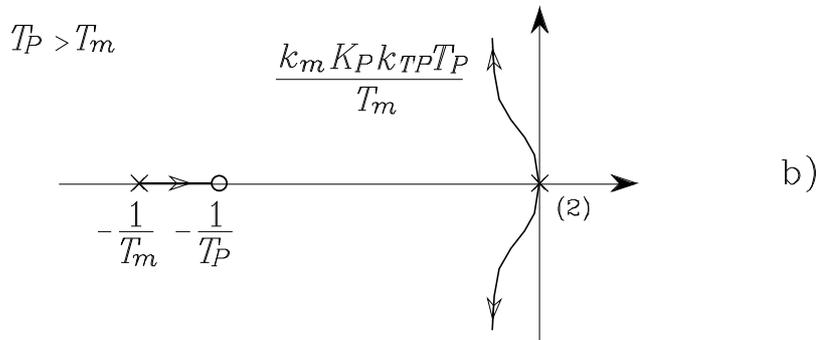
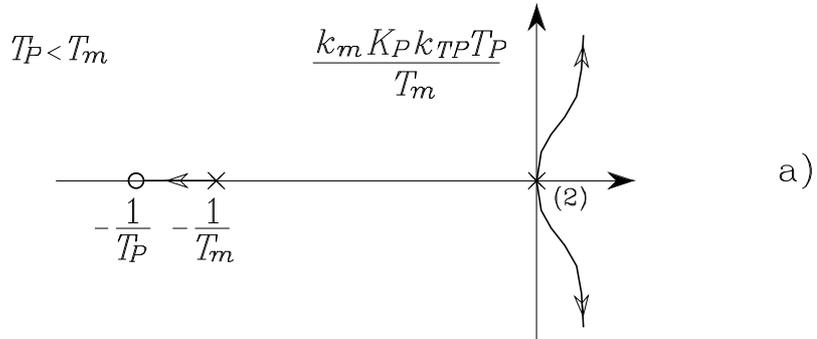
★ f.d.t. del ramo di azione diretta

$$C(s)G(s) = \frac{k_m K_P (1 + sT_P)}{s^2 (1 + sT_m)}$$

★ f.d.t. del ramo in retroazione

$$H(s) = k_{TP}$$

★ analisi con il l.d.r.



★ f.d.t. ingresso–uscita a ciclo chiuso

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\frac{1}{k_{TP}}}{1 + \frac{s^2(1 + sT_m)}{k_m K_P k_{TP}(1 + sT_P)}}$$

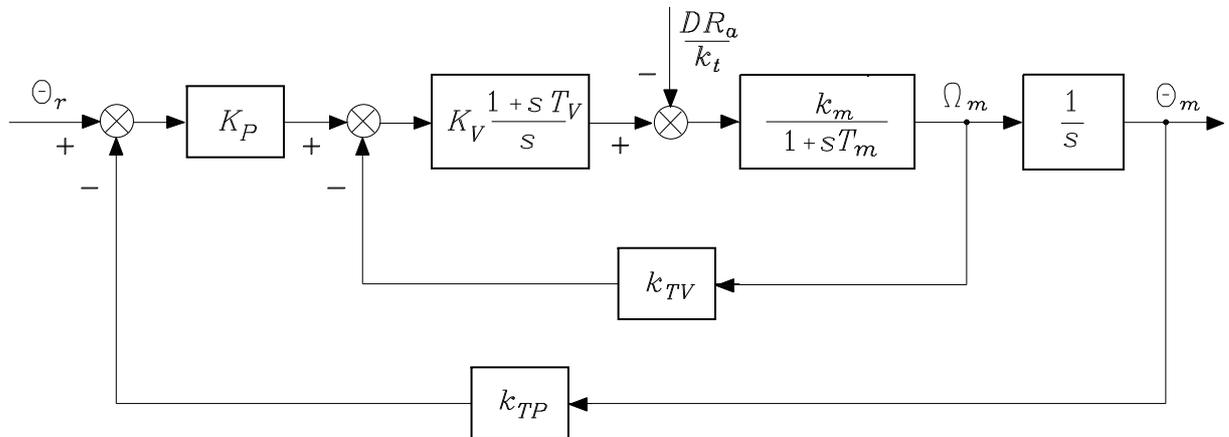
$$W(s) = \frac{\frac{1}{k_{TP}}(1 + sT_P)}{\left(1 + \frac{2\zeta s}{\omega_n} + \frac{s^2}{\omega_n^2}\right)(1 + s\tau)}$$

★ f.d.t. disturbo–uscita

$$\frac{\Theta_m(s)}{D(s)} = -\frac{\frac{sR_a}{k_t K_P k_{TP}(1 + sT_P)}}{1 + \frac{s^2(1 + sT_m)}{k_m K_P k_{TP}(1 + sT_P)}}$$

$$X_R = K_P k_{TP} \quad T_R = \max \left\{ T_P, \frac{1}{\zeta \omega_n} \right\}$$

- Retroazione di posizione e di velocità



$$C_P(s) = K_P \quad C_V(s) = K_V \frac{1 + sT_V}{s} \quad C_A(s) = 1$$

$$k_{TA} = 0$$

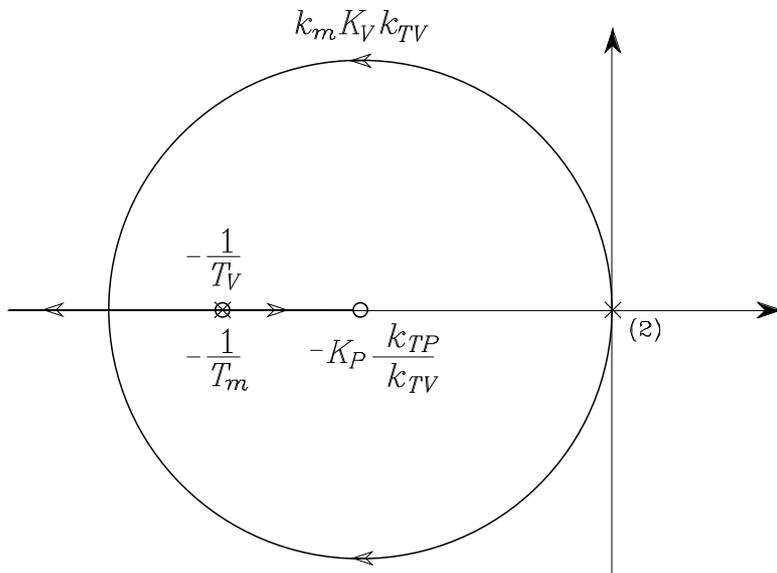
★ f.d.t. del ramo di azione diretta

$$C(s)G(s) = \frac{k_m K_P K_V (1 + sT_V)}{s^2 (1 + sT_m)}$$

★ f.d.t. del ramo in retroazione

$$H(s) = k_{TP} \left(1 + s \frac{k_{TV}}{K_P k_{TP}} \right)$$

★ analisi con il l.d.r.



★ scelta dello zero

$$T_V = T_m$$

★ f.d.t. ingresso–uscita a ciclo chiuso

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\frac{1}{k_{TP}}}{1 + \frac{sk_{TV}}{K_P k_{TP}} + \frac{s^2}{k_m K_P k_{TP} K_V}}$$

$$W(s) = \frac{\frac{1}{k_{TP}}}{1 + \frac{2\zeta s}{\omega_n} + \frac{s^2}{\omega_n^2}}$$

★ specifiche di progetto

$$K_V k_{TV} = \frac{2\zeta\omega_n}{k_m}$$

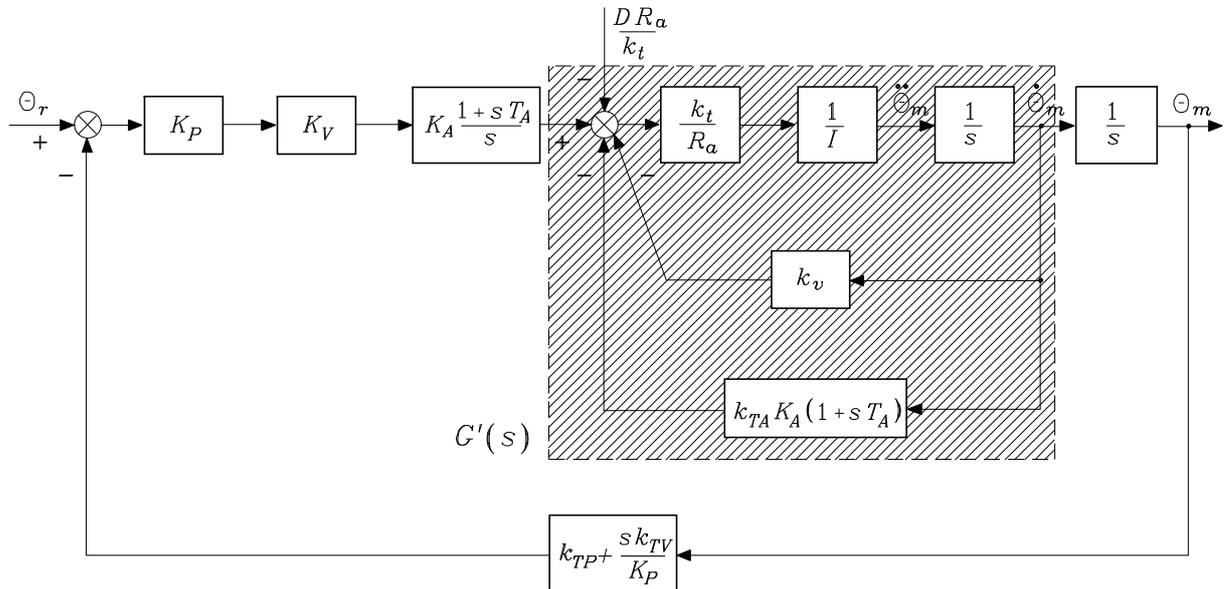
$$K_P k_{TP} K_V = \frac{\omega_n^2}{k_m}$$

★ f.d.t. disturbo–uscita

$$\frac{\Theta_m(s)}{D(s)} = - \frac{\frac{sR_a}{k_t K_P k_{TP} K_V (1 + sT_m)}}{1 + \frac{sk_{TV}}{K_P k_{TP}} + \frac{s^2}{k_m K_P k_{TP} K_V}}$$

$$X_R = K_P k_{TP} K_V \quad T_R = \max \left\{ T_m, \frac{1}{\zeta\omega_n} \right\}$$

- Retroazione di posizione, di velocità e di accelerazione



$$C_P(s) = K_P \quad C_V(s) = K_V \quad C_A(s) = K_A \frac{1 + sT_A}{s}$$

$$G'(s) = \frac{k_m}{(1 + k_m K_A k_{TA}) \left(1 + \frac{sT_m \left(1 + k_m K_A k_{TA} \frac{T_A}{T_m} \right)}{(1 + k_m K_A k_{TA})} \right)}$$

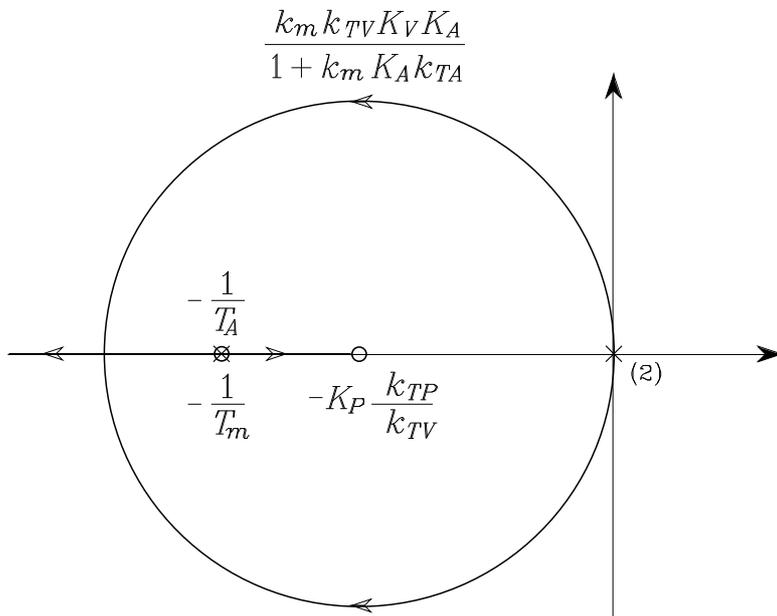
★ f.d.t. del ramo di azione diretta

$$C(s)G(s) = \frac{K_P K_V K_A (1 + sT_A)}{s^2} G'(s)$$

★ f.d.t. del ramo in retroazione

$$H(s) = k_{TP} \left(1 + \frac{s k_{TV}}{K_P k_{TP}} \right)$$

★ analisi con il l.d.r.



★ scelta dello zero

$$T_A = T_m$$

$$k_m K_A k_{TA} T_A \gg T_m \quad k_m K_A k_{TA} \gg 1$$

★ f.d.t. ingresso–uscita a ciclo chiuso

$$\frac{\Theta_m(s)}{\Theta_r(s)} = \frac{\frac{1}{k_{TP}}}{1 + \frac{sk_{TV}}{K_P k_{TP}} + \frac{s^2(1 + k_m K_A k_{TA})}{k_m K_P k_{TP} K_V K_A}}$$

★ f.d.t. disturbo–uscita

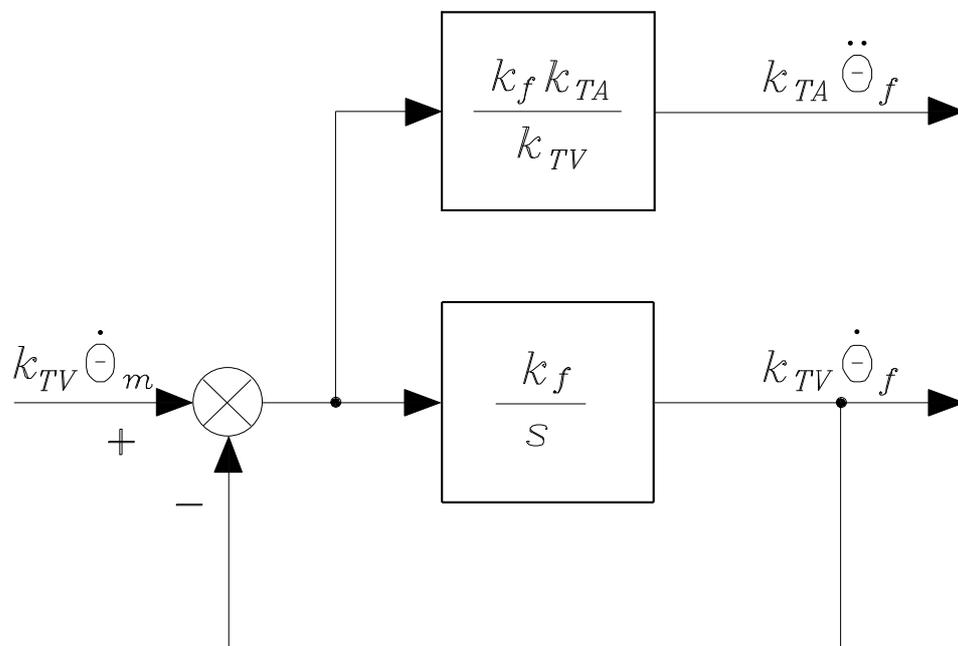
$$\frac{\Theta_m(s)}{D(s)} = - \frac{\frac{sR_a}{k_t K_P k_{TP} K_V K_A (1 + sT_A)}}{1 + \frac{sk_{TV}}{K_P k_{TP}} + \frac{s^2(1 + k_m K_A k_{TA})}{k_m K_P k_{TP} K_V K_A}}$$

$$X_R = K_P k_{TP} K_V K_A \quad T_R = \max \left\{ T_A, \frac{1}{\zeta \omega_n} \right\}$$

★ specifiche di progetto

$$\begin{aligned} \frac{2K_P k_{TP}}{k_{TV}} &= \frac{\omega_n}{\zeta} \\ k_m K_A k_{TA} &= \frac{k_m X_R}{\omega_n^2} - 1 \\ K_P k_{TP} K_V K_A &= X_R \end{aligned}$$

★ ricostruzione dell'accelerazione

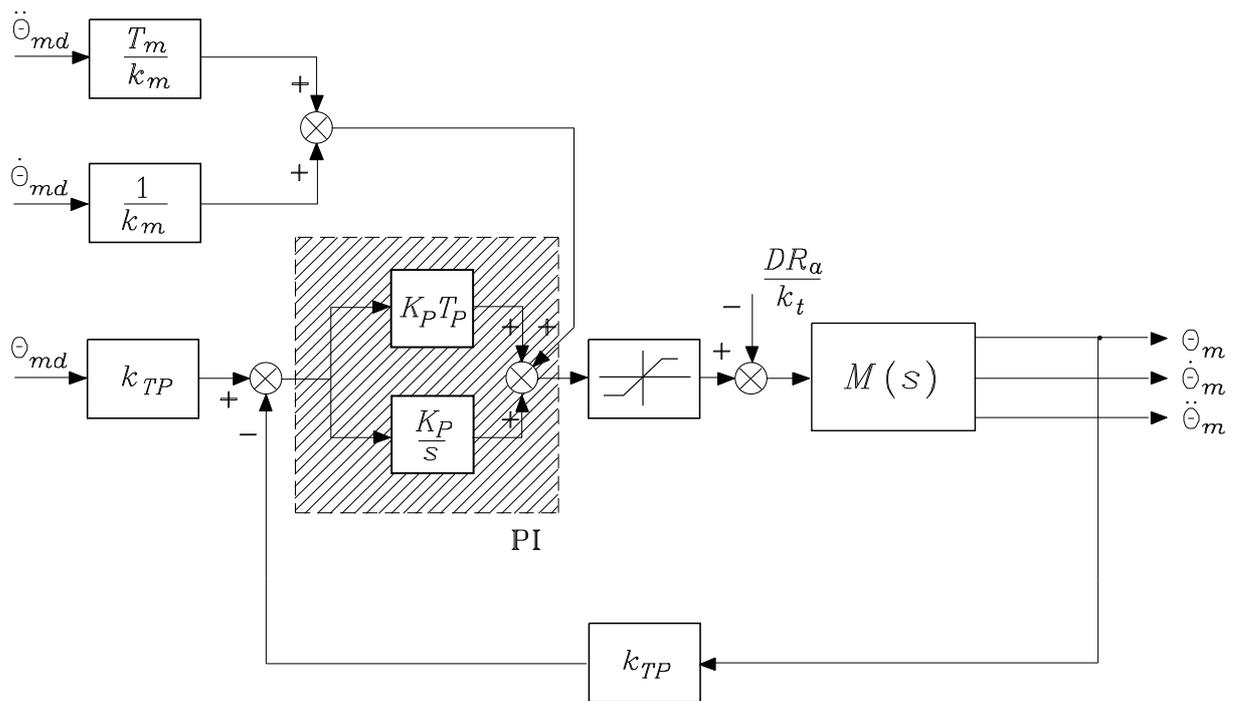
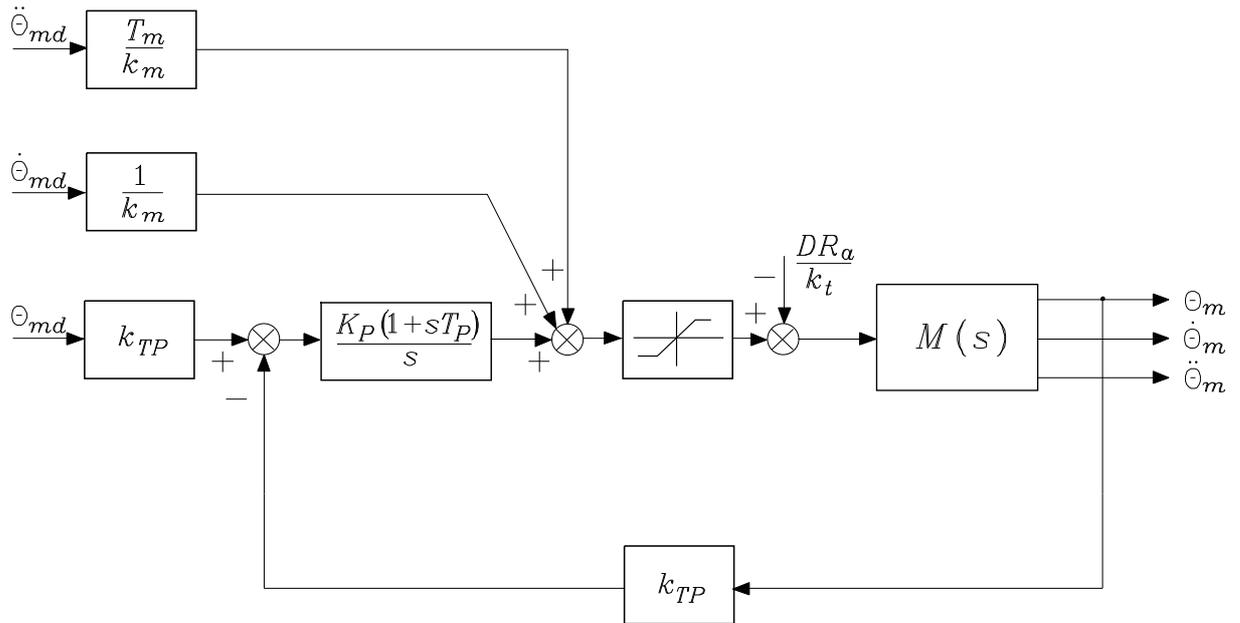


Compensazione in avanti decentralizzata

- Elevati valori di velocità e accelerazioni
 - ★ capacità di inseguimento degradate
- Impiego di una azione in avanti

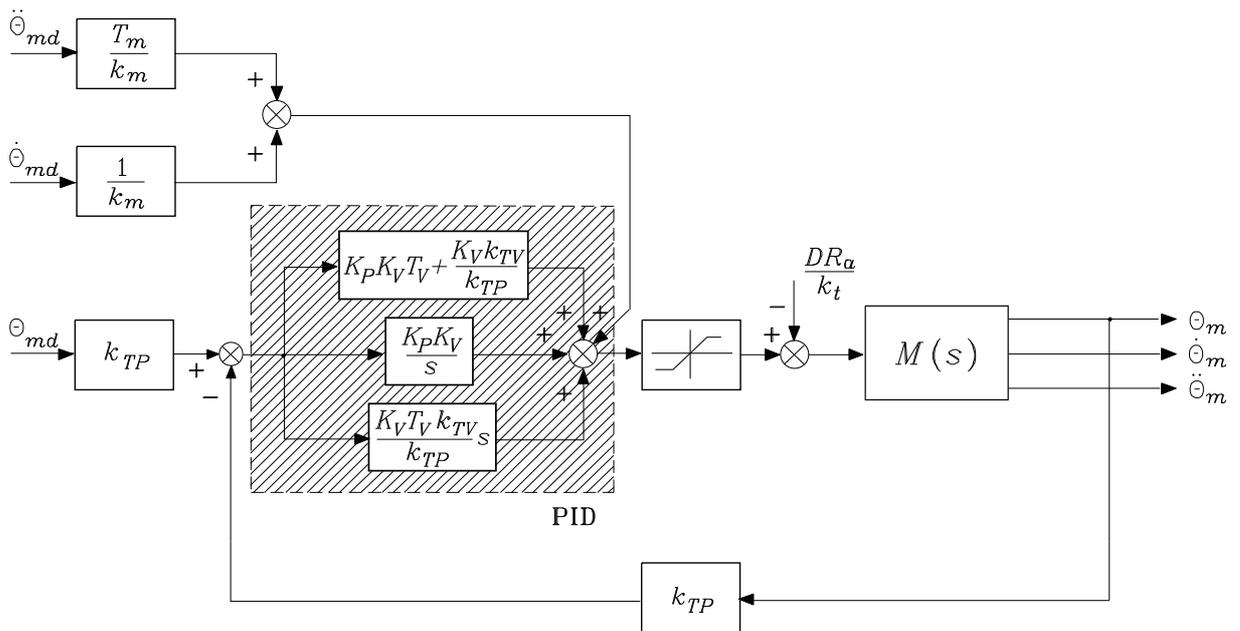
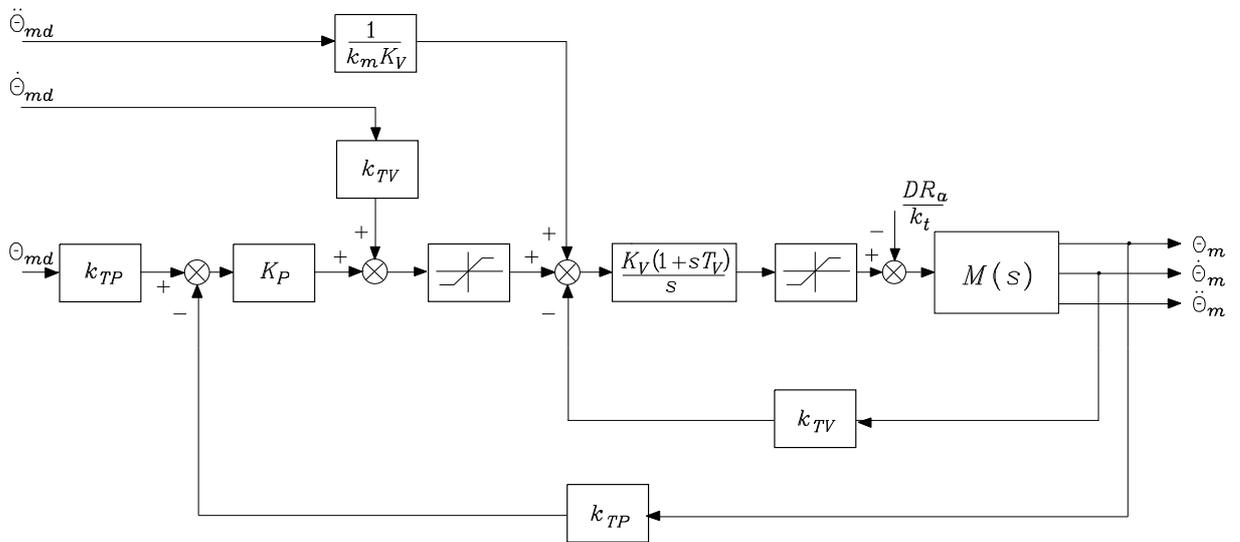
• Retroazione di posizione

$$\Theta'_r(s) = \left(k_{TP} + \frac{s^2(1 + sT_m)}{k_m K_P(1 + sT_P)} \right) \Theta_{md}(s)$$



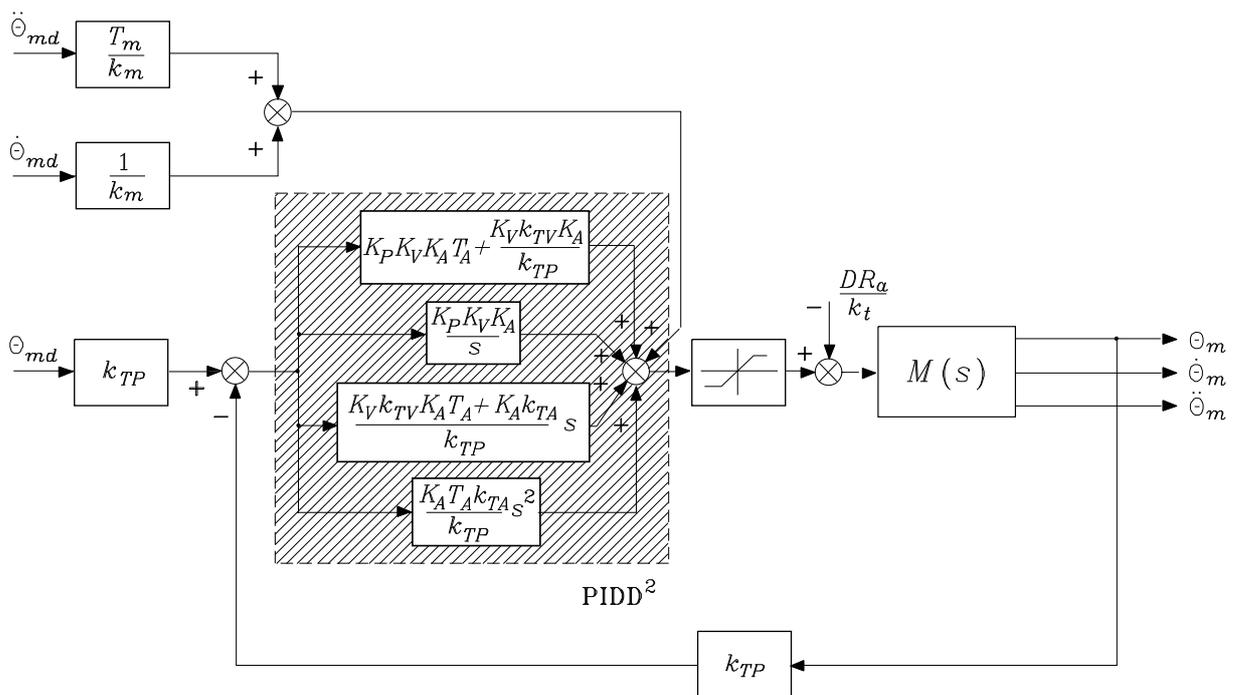
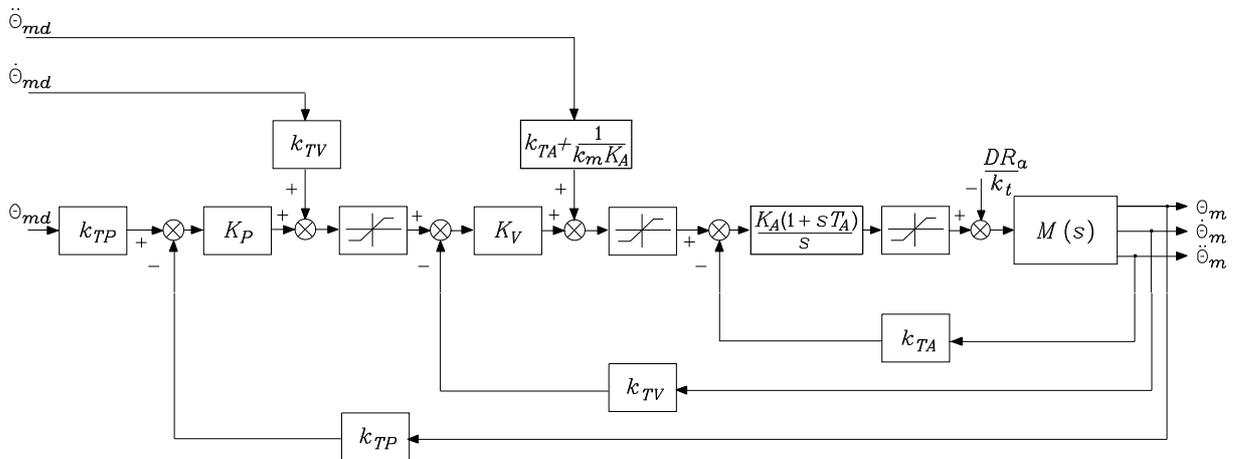
• Retroazione di posizione e di velocità

$$\Theta'_r(s) = \left(k_{TP} + \frac{sk_{TV}}{K_P} + \frac{s^2}{k_m K_P K_V} \right) \Theta_{md}(s)$$

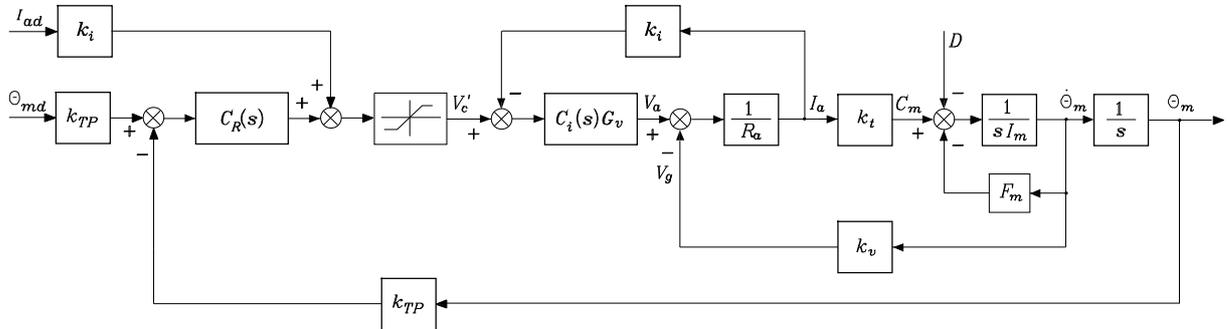


- Retroazione di posizione, di velocità e di accelerazione

$$\Theta'_r(s) = \left(k_{TP} + \frac{sk_{TV}}{K_P} + \frac{(1 + k_m K_A k_{TA})s^2}{k_m K_P K_V K_A} \right) \Theta_{md}(s)$$



- Controllo con azionamento controllato in coppia e azione in avanti in corrente



COMPENSAZIONE IN AVANTI A COPPIA PRECALCOLATA

- All'uscita del PIDD²

$$\begin{aligned}
 a_2\ddot{e} + a_1\dot{e} + a_0e + a_{-1} \int^t e(\varsigma)d\varsigma + \frac{T_m}{k_m}\ddot{\theta}_{md} + \frac{1}{k_m}\dot{\theta}_{md} - \frac{R_a}{k_t}d \\
 = \frac{T_m}{k_m}\ddot{\theta}_m + \frac{1}{k_m}\dot{\theta}_m
 \end{aligned}$$

⇓

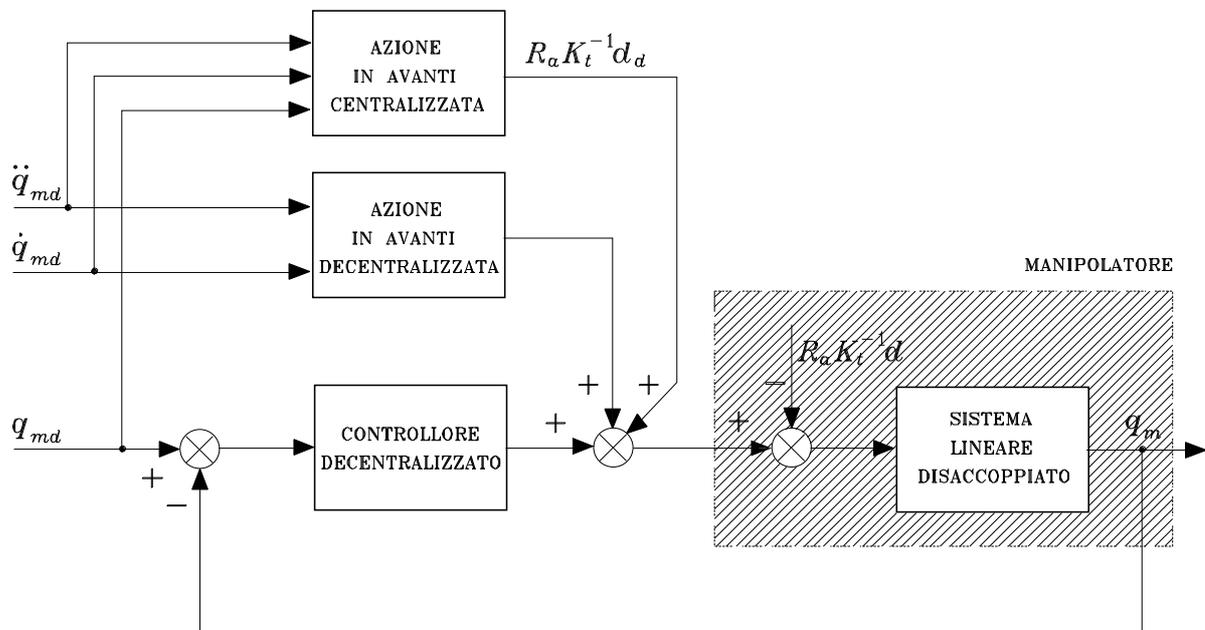
$$a'_2\ddot{e} + a'_1\dot{e} + a'_0e + a'_{-1} \int^t e(\varsigma)d\varsigma = \frac{R_a}{k_t}d$$

$$\frac{E(s)}{D(s)} = \frac{\frac{R_a}{k_t}s}{a'_2s^3 + a'_1s^2 + a'_0s + a'_{-1}}$$

★ adozione di guadagni di anello troppo elevati

- Coppia precalcolata

- ★ azione *in avanti* (inversione di modello)



$$d_d = K_r^{-1} \Delta B(q_d) K_r^{-1} \ddot{q}_{md} + K_r^{-1} C(q_d, \dot{q}_d) K_r^{-1} \dot{q}_{md} + K_r^{-1} g(q_d)$$

- ★ riduzione degli sforzi di reiezione del disturbo (guadagni contenuti)
- ★ calcolo fuori linea/in linea
- ★ compensazione parziale

CONTROLLO CENTRALIZZATO

- Manipolatore \equiv Sistema multivariabile accoppiato e non lineare

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = u$$

Controllo PD con compensazione di gravità

- Regolazione a una postura di equilibrio *costante* q_d
- Metodo diretto di Lyapunov

★ stato $[\tilde{q}^T \quad \dot{q}^T]^T$ $\tilde{q} = q_d - q$

- ★ funzione candidata di Lyapunov

$$V(\dot{q}, \tilde{q}) = \frac{1}{2}\dot{q}^T B(q)\dot{q} + \frac{1}{2}\tilde{q}^T K_P \tilde{q} > 0 \quad \forall \dot{q}, \tilde{q} \neq 0$$

$$\dot{V} = \dot{q}^T B(q)\ddot{q} + \frac{1}{2}\dot{q}^T \dot{B}(q)\dot{q} - \dot{q}^T K_P \tilde{q}$$

$$= \frac{1}{2}\dot{q}^T (\dot{B}(q) - 2C(q, \dot{q}))\dot{q} - \dot{q}^T F\dot{q} + \dot{q}^T (u - g(q) - K_P \tilde{q})$$

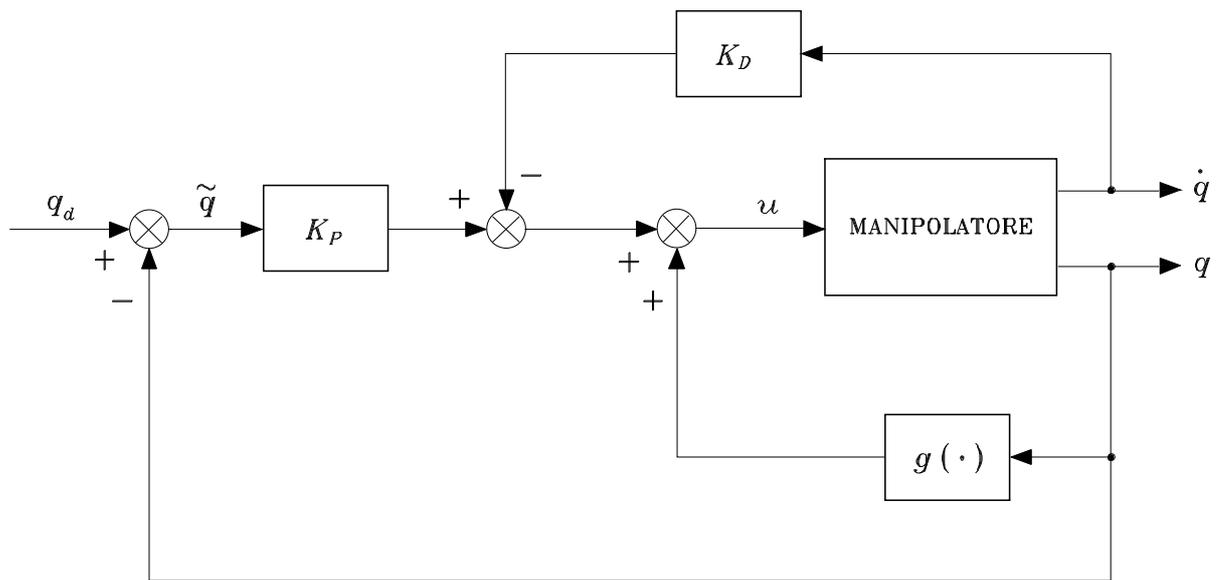
★ scelta del controllo

$$\mathbf{u} = \mathbf{g}(\mathbf{q}) + \mathbf{K}_P \tilde{\mathbf{q}} - \mathbf{K}_D \dot{\mathbf{q}}$$

⇓

$$\dot{V} = -\dot{\mathbf{q}}^T (\mathbf{F} + \mathbf{K}_D) \dot{\mathbf{q}}$$

$$\dot{V} = 0 \quad \dot{\mathbf{q}} = \mathbf{0}, \forall \tilde{\mathbf{q}}$$



★ dinamica del sistema controllato

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = g(q) + K_P\tilde{q} - K_D\dot{q}$$

★ all'equilibrio ($\dot{q} \equiv \ddot{q} \equiv 0$)

$$K_P\tilde{q} = 0 \quad \implies \quad \tilde{q} = q_d - q \equiv 0$$

Controllo a dinamica inversa

- Modello dinamico

$$B(q)\ddot{q} + n(q, \dot{q}) = u$$

$$n(q, \dot{q}) = C(q, \dot{q})\dot{q} + F\dot{q} + g(q)$$

- Retroazione non lineare dello stato (linearizzazione *globale*)

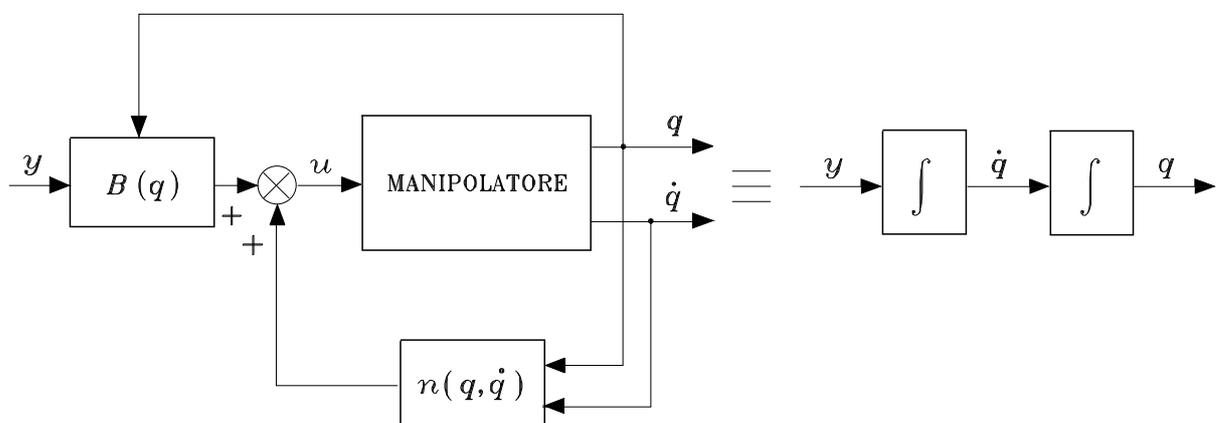
★ struttura lineare in u

★ $\rho(B(q)) = n \quad \forall q$

$$u = B(q)y + n(q, \dot{q})$$

⇓

$$\ddot{q} = y$$



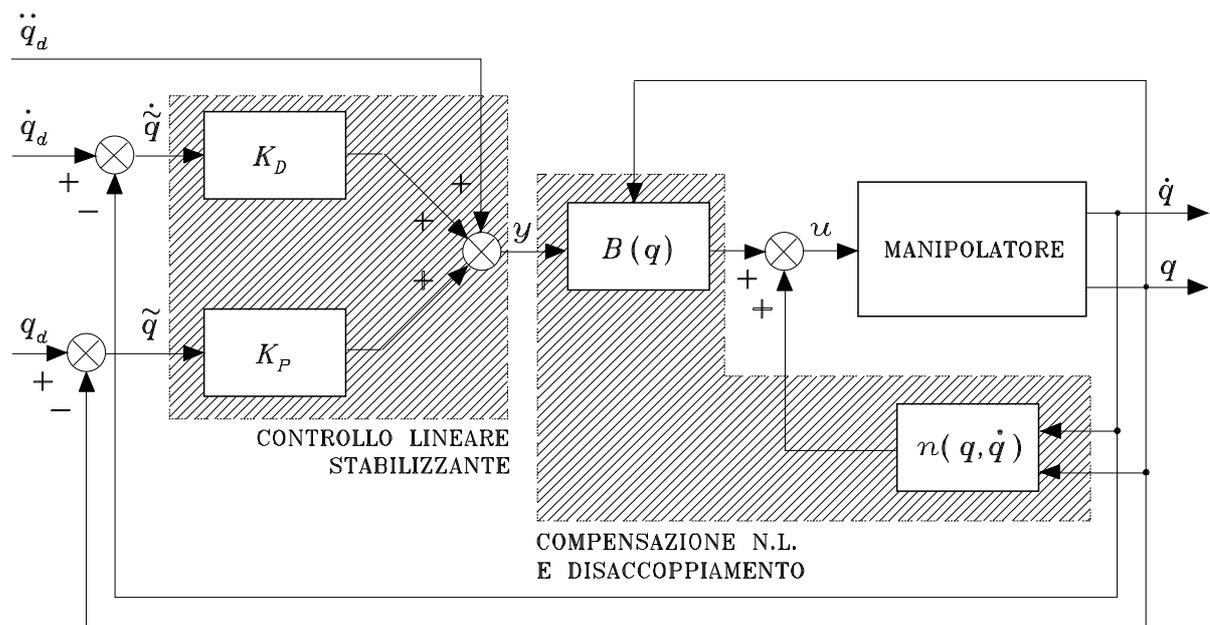
- Scelta di un controllo y stabilizzante

$$y = -K_P q - K_D \dot{q} + r$$

$$r = \ddot{q}_d + K_D \dot{q}_d + K_P q_d$$

⇓

$$\ddot{\tilde{q}} + K_D \dot{\tilde{q}} + K_P \tilde{q} = 0$$



- ★ cancellazione perfetta
- ★ vincoli sull'architettura hardware/software dell'unità di governo

Controllo robusto

- Compensazione *imperfetta*

$$\mathbf{u} = \widehat{\mathbf{B}}(\mathbf{q})\mathbf{y} + \widehat{\mathbf{n}}(\mathbf{q}, \dot{\mathbf{q}})$$

★ incertezza

$$\widetilde{\mathbf{B}} = \widehat{\mathbf{B}} - \mathbf{B} \quad \widetilde{\mathbf{n}} = \widehat{\mathbf{n}} - \mathbf{n}$$

⇓

$$\mathbf{B}\ddot{\mathbf{q}} + \mathbf{n} = \widehat{\mathbf{B}}\mathbf{y} + \widehat{\mathbf{n}}$$

$$\ddot{\mathbf{q}} = \mathbf{y} + (\mathbf{B}^{-1}\widehat{\mathbf{B}} - \mathbf{I})\mathbf{y} + \mathbf{B}^{-1}\widetilde{\mathbf{n}} = \mathbf{y} - \boldsymbol{\eta}$$

$$\boldsymbol{\eta} = (\mathbf{I} - \mathbf{B}^{-1}\widehat{\mathbf{B}})\mathbf{y} - \mathbf{B}^{-1}\widetilde{\mathbf{n}}$$

- Scelta del controllo

$$\mathbf{y} = \ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{q}}_d - \dot{\mathbf{q}}) + \mathbf{K}_P(\mathbf{q}_d - \mathbf{q})$$

⇓

$$\ddot{\widetilde{\mathbf{q}}} + \mathbf{K}_D\dot{\widetilde{\mathbf{q}}} + \mathbf{K}_P\widetilde{\mathbf{q}} = \boldsymbol{\eta}$$

$$\ddot{\tilde{\mathbf{q}}} = \ddot{\mathbf{q}}_d - \mathbf{y} + \boldsymbol{\eta}$$

★ equazione differenziale del primo ordine

$$\dot{\boldsymbol{\xi}} = \mathbf{H}\boldsymbol{\xi} + \mathbf{D}(\ddot{\mathbf{q}}_d - \mathbf{y} + \boldsymbol{\eta})$$

- Stima del campo di variabilità dell'incertezza

$$\sup_{t \geq 0} \|\ddot{\mathbf{q}}_d\| < Q_M < \infty \quad \forall \ddot{\mathbf{q}}_d$$

$$\|\mathbf{I} - \mathbf{B}^{-1}(\mathbf{q})\hat{\mathbf{B}}(\mathbf{q})\| \leq \alpha \leq 1 \quad \forall \mathbf{q}$$

$$\|\tilde{\mathbf{n}}\| \leq \Phi < \infty \quad \forall \mathbf{q}, \dot{\mathbf{q}}$$

- Scelta del controllo

$$\mathbf{y} = \ddot{\mathbf{q}}_d + \mathbf{K}_D \dot{\tilde{\mathbf{q}}} + \mathbf{K}_P \tilde{\mathbf{q}} + \mathbf{w}$$

↓

$$\dot{\boldsymbol{\xi}} = \widetilde{\mathbf{H}} \boldsymbol{\xi} + \mathbf{D}(\boldsymbol{\eta} - \mathbf{w})$$

$$\widetilde{\mathbf{H}} = (\mathbf{H} - \mathbf{DK}) = \begin{bmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{K}_P & -\mathbf{K}_D \end{bmatrix}$$

- Metodo di Lyapunov

$$V(\boldsymbol{\xi}) = \boldsymbol{\xi}^T \mathbf{Q} \boldsymbol{\xi} > 0 \quad \forall \boldsymbol{\xi} \neq \mathbf{0}$$

$$\begin{aligned} \dot{V} &= \dot{\boldsymbol{\xi}}^T \mathbf{Q} \boldsymbol{\xi} + \boldsymbol{\xi}^T \mathbf{Q} \dot{\boldsymbol{\xi}} \\ &= \boldsymbol{\xi}^T (\widetilde{\mathbf{H}}^T \mathbf{Q} + \mathbf{Q} \widetilde{\mathbf{H}}) \boldsymbol{\xi} + 2\boldsymbol{\xi}^T \mathbf{Q} \mathbf{D}(\boldsymbol{\eta} - \mathbf{w}) \\ &= -\boldsymbol{\xi}^T \mathbf{P} \boldsymbol{\xi} + 2\boldsymbol{\xi}^T \mathbf{Q} \mathbf{D}(\boldsymbol{\eta} - \mathbf{w}) \\ &= -\boldsymbol{\xi}^T \mathbf{P} \boldsymbol{\xi} + 2z^T(\boldsymbol{\eta} - \mathbf{w}) \end{aligned}$$

- Legge di controllo

$$\mathbf{w} = \frac{\rho}{\|\mathbf{z}\|} \mathbf{z} \quad \rho > 0$$

⇓

$$\begin{aligned} \mathbf{z}^T (\boldsymbol{\eta} - \mathbf{w}) &= \mathbf{z}^T \boldsymbol{\eta} - \frac{\rho}{\|\mathbf{z}\|} \mathbf{z}^T \mathbf{z} \\ &\leq \|\mathbf{z}\| \|\boldsymbol{\eta}\| - \rho \|\mathbf{z}\| \\ &= \|\mathbf{z}\| (\|\boldsymbol{\eta}\| - \rho) \end{aligned}$$

- Scelta di ρ

$$\rho \geq \|\boldsymbol{\eta}\| \quad \forall \mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}_d$$

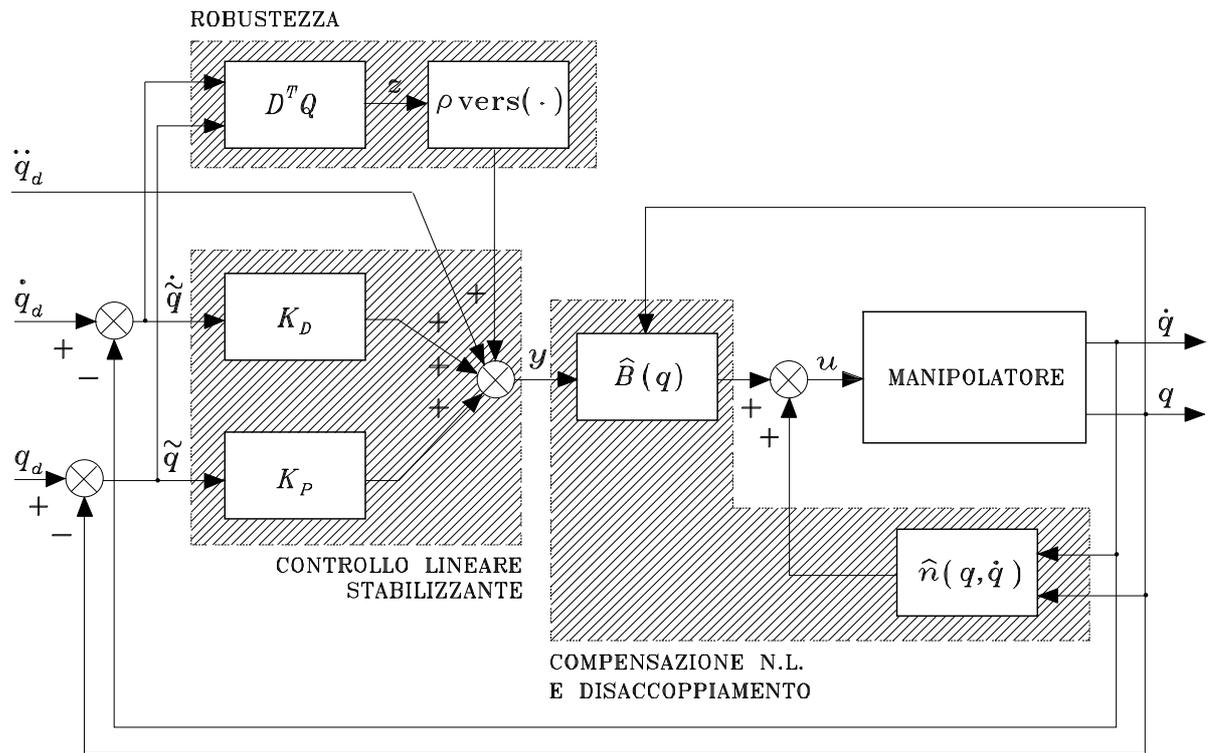
★ stima dell'incertezza

$$\begin{aligned} \|\boldsymbol{\eta}\| &\leq \|\mathbf{I} - \mathbf{B}^{-1} \widehat{\mathbf{B}}\| (\|\ddot{\mathbf{q}}_d\| + \|\mathbf{K}\| \|\boldsymbol{\xi}\| + \|\mathbf{w}\|) + \|\mathbf{B}^{-1}\| \|\tilde{\mathbf{n}}\| \\ &\leq \alpha Q_M + \alpha \|\mathbf{K}\| \|\boldsymbol{\xi}\| + \alpha \rho + B_M \Phi \end{aligned}$$

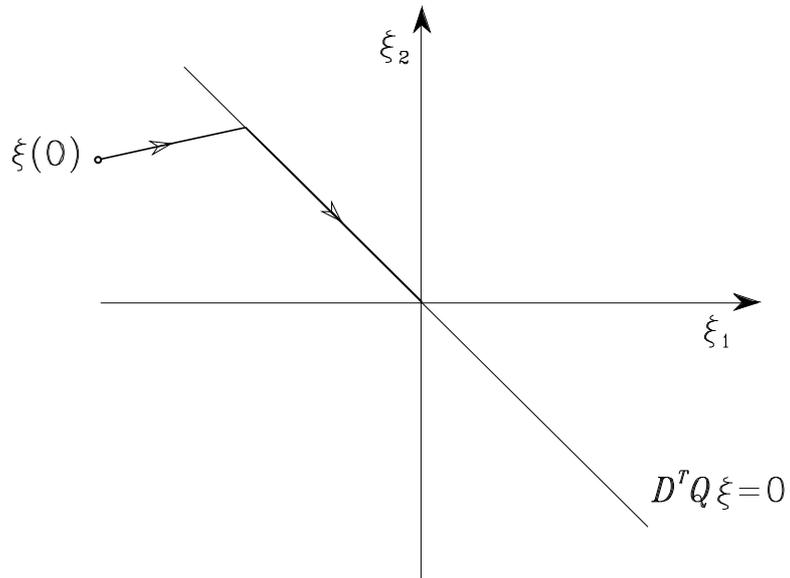
$$\rho \geq \frac{1}{1 - \alpha} (\alpha Q_M + \alpha \|\mathbf{K}\| \|\boldsymbol{\xi}\| + B_M \Phi)$$

⇓

$$\dot{V} = -\boldsymbol{\xi}^T \mathbf{P} \boldsymbol{\xi} + 2\mathbf{z}^T \left(\boldsymbol{\eta} - \frac{\rho}{\|\mathbf{z}\|} \mathbf{z} \right) < 0 \quad \forall \boldsymbol{\xi} \neq \mathbf{0}$$

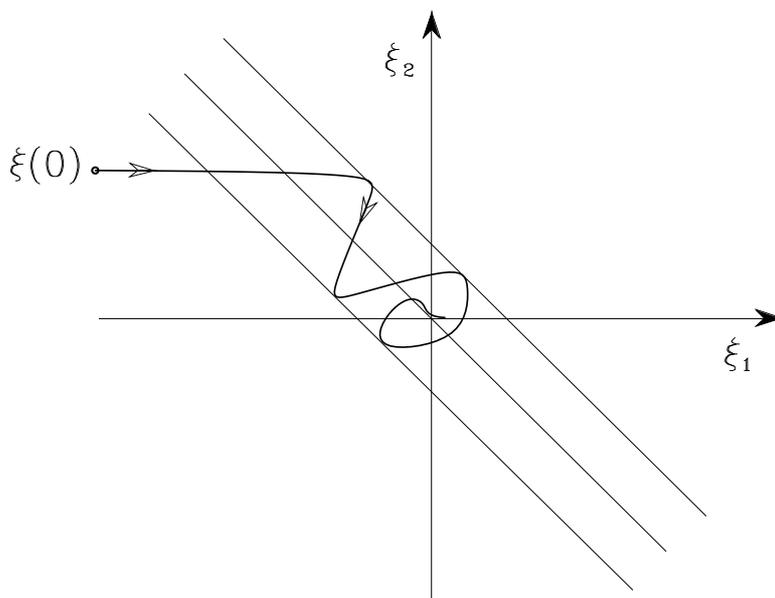


★ sottospazio di *scivolamento* (attrattivo)



- Eliminazione delle componenti in alta frequenza

$$w = \begin{cases} \frac{\rho}{\|z\|} z & \text{per } \|z\| \geq \epsilon \\ \frac{\rho}{\epsilon} z & \text{per } \|z\| < \epsilon \end{cases}$$



Controllo adattativo

- Modello dinamico (lineare nei parametri)

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + F\dot{q} + g(q) = Y(q, \dot{q}, \ddot{q})\pi = u$$

- Controllo

$$u = B(q)\ddot{q}_r + C(q, \dot{q})\dot{q}_r + F\dot{q}_r + g(q) + K_D\sigma$$

$$\dot{q}_r = \dot{q}_d + \Lambda\tilde{q}$$

$$\ddot{q}_r = \ddot{q}_d + \Lambda\dot{\tilde{q}}$$

$$\sigma = \dot{q}_r - \dot{q} = \dot{\tilde{q}} + \Lambda\tilde{q}$$

↓

$$B(q)\dot{\sigma} + C(q, \dot{q})\sigma + F\sigma + K_D\sigma = 0$$

- Metodo di Lyapunov

$$V(\boldsymbol{\sigma}, \tilde{\mathbf{q}}) = \frac{1}{2} \boldsymbol{\sigma}^T \mathbf{B}(\mathbf{q}) \boldsymbol{\sigma} + \frac{1}{2} \tilde{\mathbf{q}}^T \mathbf{M} \tilde{\mathbf{q}} > 0 \quad \forall \boldsymbol{\sigma}, \tilde{\mathbf{q}} \neq \mathbf{0}$$

$$\begin{aligned} \dot{V} &= \boldsymbol{\sigma}^T \mathbf{B}(\mathbf{q}) \dot{\boldsymbol{\sigma}} + \frac{1}{2} \boldsymbol{\sigma}^T \dot{\mathbf{B}}(\mathbf{q}) \boldsymbol{\sigma} + \tilde{\mathbf{q}}^T \mathbf{M} \dot{\tilde{\mathbf{q}}} \\ &= -\boldsymbol{\sigma}^T (\mathbf{F} + \mathbf{K}_D) \boldsymbol{\sigma} + \tilde{\mathbf{q}}^T \mathbf{M} \dot{\tilde{\mathbf{q}}} \\ &= -\boldsymbol{\sigma}^T \mathbf{F} \boldsymbol{\sigma} - \dot{\tilde{\mathbf{q}}}^T \mathbf{K}_D \dot{\tilde{\mathbf{q}}} - \tilde{\mathbf{q}}^T \boldsymbol{\Lambda} \mathbf{K}_D \boldsymbol{\Lambda} \tilde{\mathbf{q}} \end{aligned}$$

★ $\dot{V} = 0$ solo per $\tilde{\mathbf{q}} = \dot{\tilde{\mathbf{q}}} \equiv \mathbf{0} \implies [\tilde{\mathbf{q}}^T \quad \boldsymbol{\sigma}^T]^T = \mathbf{0}$
globalmente asintoticamente stabile

- Controllo basato su stime dei parametri

$$\begin{aligned} \mathbf{u} &= \widehat{\mathbf{B}}(\mathbf{q})\ddot{\mathbf{q}}_r + \widehat{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r + \widehat{\mathbf{F}}\dot{\mathbf{q}}_r + \widehat{\mathbf{g}} + \mathbf{K}_D\boldsymbol{\sigma} \\ &= \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\widehat{\boldsymbol{\pi}} + \mathbf{K}_D\boldsymbol{\sigma} \end{aligned}$$

↓

$$\begin{aligned} \mathbf{B}(\mathbf{q})\dot{\boldsymbol{\sigma}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\boldsymbol{\sigma} + \mathbf{F}\boldsymbol{\sigma} + \mathbf{K}_D\boldsymbol{\sigma} \\ &= -\widetilde{\mathbf{B}}(\mathbf{q})\ddot{\mathbf{q}}_r - \widetilde{\mathbf{C}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_r - \widetilde{\mathbf{F}}\dot{\mathbf{q}}_r - \widetilde{\mathbf{g}}(\mathbf{q}) \\ &= -\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\widetilde{\boldsymbol{\pi}} \end{aligned}$$

- Modifica di V

$$V(\boldsymbol{\sigma}, \widetilde{\mathbf{q}}, \widetilde{\boldsymbol{\pi}}) = \frac{1}{2}\boldsymbol{\sigma}^T \mathbf{B}(\mathbf{q})\boldsymbol{\sigma} + \widetilde{\mathbf{q}}^T \boldsymbol{\Lambda} \mathbf{K}_D \widetilde{\mathbf{q}} + \frac{1}{2}\widetilde{\boldsymbol{\pi}}^T \mathbf{K}_\pi \widetilde{\boldsymbol{\pi}} > 0 \quad \forall \boldsymbol{\sigma}, \widetilde{\mathbf{q}}, \widetilde{\boldsymbol{\pi}} \neq 0$$

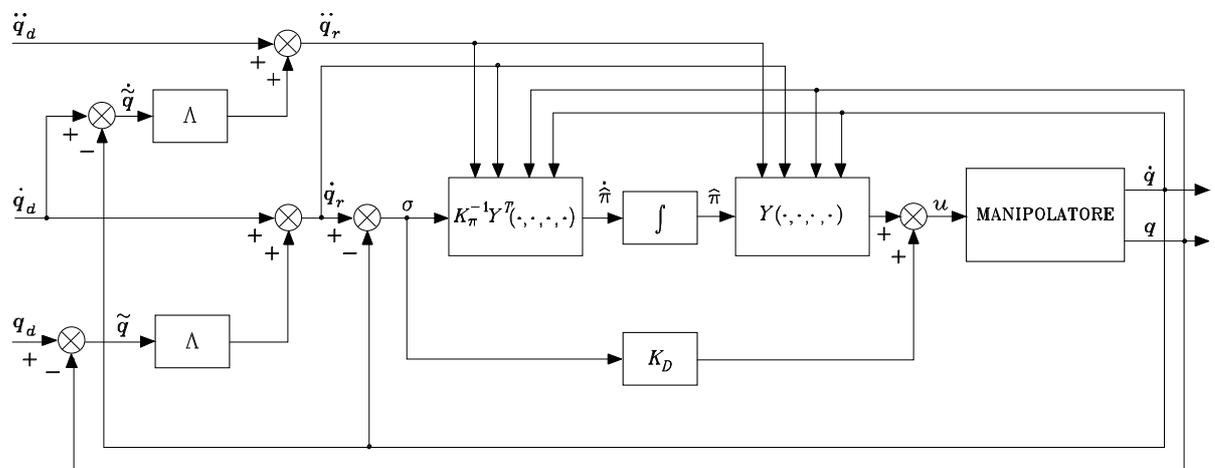
$$\begin{aligned} \dot{V} &= -\boldsymbol{\sigma}^T \mathbf{F}\boldsymbol{\sigma} - \dot{\widetilde{\mathbf{q}}}^T \mathbf{K}_D \dot{\widetilde{\mathbf{q}}} - \widetilde{\mathbf{q}}^T \boldsymbol{\Lambda} \mathbf{K}_D \boldsymbol{\Lambda} \dot{\widetilde{\mathbf{q}}} \\ &\quad + \widetilde{\boldsymbol{\pi}}^T (\mathbf{K}_\pi \dot{\widetilde{\boldsymbol{\pi}}} - \mathbf{Y}^T(\mathbf{q}, \dot{\mathbf{q}}, \dot{\mathbf{q}}_r, \ddot{\mathbf{q}}_r)\boldsymbol{\sigma}) \end{aligned}$$

- Legge di adattamento

$$\dot{\hat{\pi}} = K_{\pi}^{-1} Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r) \sigma$$

⇓

$$\dot{V} = -\sigma^T F \sigma - \tilde{q}^T K_D \dot{\tilde{q}} - \tilde{q}^T \Lambda K_D \Lambda \tilde{q}$$



$$\star \tilde{q} \rightarrow 0$$

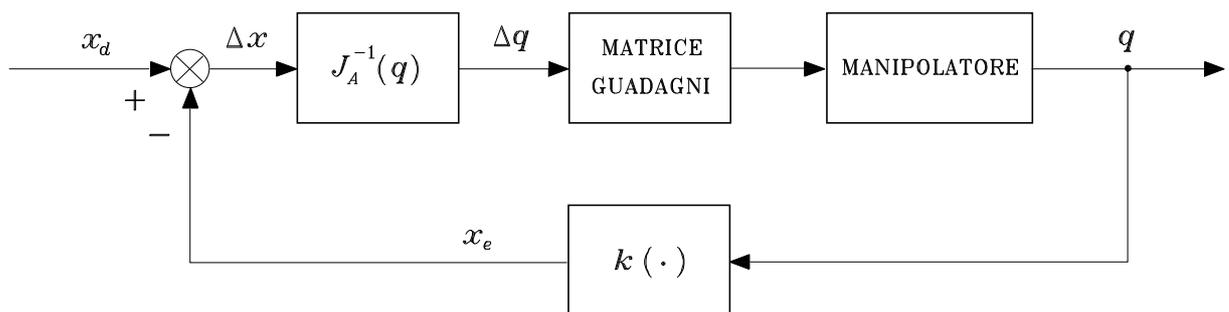
$$\star Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)(\hat{\pi} - \pi) \rightarrow 0$$

CONTROLLO NELLO SPAZIO OPERATIVO

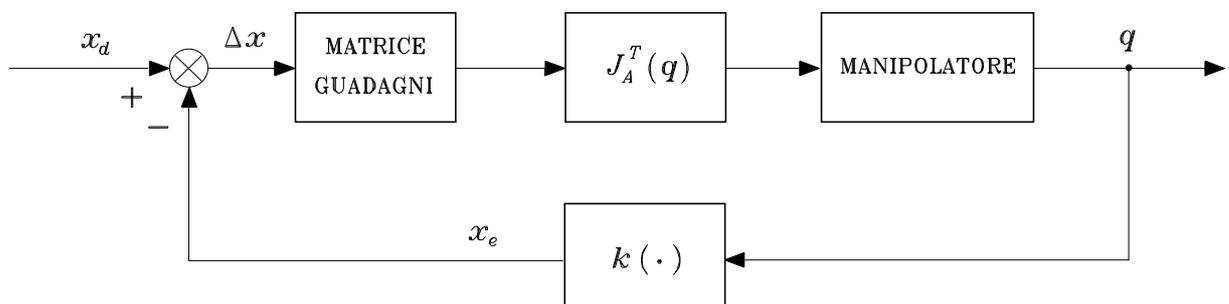
- ★ specifiche di moto
- ★ relazioni cinematiche
- ★ carico computazionale

Schemi di principio

- Controllo con *inversa* dello Jacobiano



- Controllo con *trasposta* dello Jacobiano



Controllo PD con compensazione di gravità

- Regolazione a una posizione di equilibrio *costante* \mathbf{x}_d
- Metodo di Lyapunov

$$V(\dot{\mathbf{q}}, \tilde{\mathbf{x}}) = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{B}(\mathbf{q}) \dot{\mathbf{q}} + \frac{1}{2} \tilde{\mathbf{x}}^T \mathbf{K}_P \tilde{\mathbf{x}} > 0 \quad \forall \dot{\mathbf{q}}, \tilde{\mathbf{x}} \neq \mathbf{0}$$

$$\begin{aligned} \dot{V} &= \dot{\mathbf{q}}^T \mathbf{B}(\mathbf{q}) \ddot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \dot{\mathbf{B}}(\mathbf{q}) \dot{\mathbf{q}} + \dot{\tilde{\mathbf{x}}}^T \mathbf{K}_P \tilde{\mathbf{x}} \\ &= \dot{\mathbf{q}}^T \mathbf{B}(\mathbf{q}) \ddot{\mathbf{q}} + \frac{1}{2} \dot{\mathbf{q}}^T \dot{\mathbf{B}}(\mathbf{q}) \dot{\mathbf{q}} - \dot{\mathbf{q}}^T \mathbf{J}_A^T(\mathbf{q}) \mathbf{K}_P \tilde{\mathbf{x}} \\ &= -\dot{\mathbf{q}}^T \mathbf{F} \dot{\mathbf{q}} + \dot{\mathbf{q}}^T (\mathbf{u} - \mathbf{g}(\mathbf{q}) - \mathbf{J}_A^T(\mathbf{q}) \mathbf{K}_P \tilde{\mathbf{x}}) \end{aligned}$$

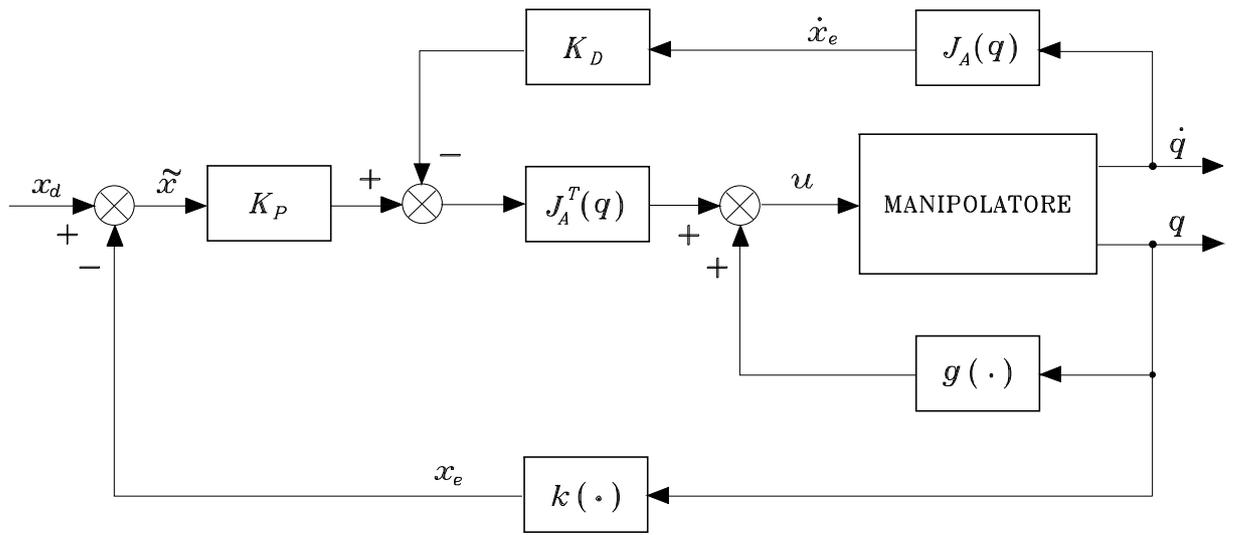
★ scelta del controllo

$$\mathbf{u} = \mathbf{g}(\mathbf{q}) + \mathbf{J}_A^T(\mathbf{q}) \mathbf{K}_P \tilde{\mathbf{x}} - \mathbf{J}_A^T \mathbf{K}_D \mathbf{J}_A(\mathbf{q}) \dot{\mathbf{q}}$$

⇓

$$\dot{V} = -\dot{\mathbf{q}}^T \mathbf{F} \dot{\mathbf{q}} - \dot{\mathbf{q}}^T \mathbf{J}_A^T(\mathbf{q}) \mathbf{K}_D \mathbf{J}_A(\mathbf{q}) \dot{\mathbf{q}}$$

$$\dot{V} = 0 \quad \mathbf{J}_A^T(\mathbf{q}) \mathbf{K}_P \tilde{\mathbf{x}} = \mathbf{0}$$



Controllo a dinamica inversa

- Modello

$$\mathbf{B}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{u}$$

- Controllo linearizzante

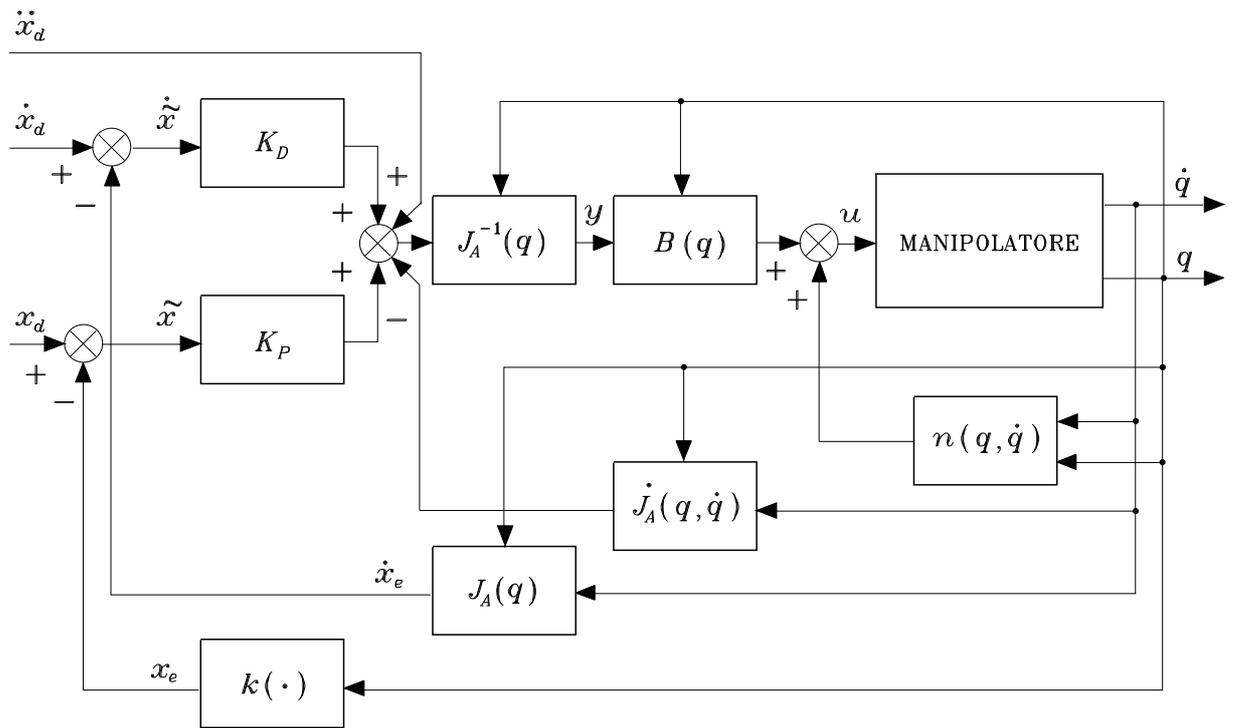
$$\mathbf{u} = \mathbf{B}(\mathbf{q})\mathbf{y} + \mathbf{n}(\mathbf{q}, \dot{\mathbf{q}})$$

↓

$$\ddot{\mathbf{q}} = \mathbf{y}$$

$$= \mathbf{J}_A^{-1}(\mathbf{q}) (\ddot{\mathbf{x}}_d + \mathbf{K}_D \dot{\tilde{\mathbf{x}}} + \mathbf{K}_P \tilde{\mathbf{x}} - \dot{\mathbf{J}}_A(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}})$$

$$\ddot{\tilde{\mathbf{x}}} + \mathbf{K}_D \dot{\tilde{\mathbf{x}}} + \mathbf{K}_P \tilde{\mathbf{x}} = \mathbf{0}$$



CONFRONTO TRA GLI SCHEMI DI CONTROLLO

- Manipolatore planare a due bracci

$$\star a_1 = a_2 = 1 \text{ m} \quad \ell_1 = \ell_2 = 0.5 \text{ m}$$

$$m_{\ell_1} = m_{\ell_2} = 50 \text{ kg} \quad I_{\ell_1} = I_{\ell_2} = 10 \text{ kg} \cdot \text{m}^2$$

$$\star k_{r1} = k_{r2} = 100$$

$$m_{m_1} = m_{m_2} = 5 \text{ kg} \quad I_{m_1} = I_{m_2} = 0.01 \text{ kg} \cdot \text{m}^2$$

$$F_{m_1} = F_{m_2} = 0.01 \text{ N} \cdot \text{m} \cdot \text{s/rad} \quad R_{a1} = R_{a2} = 10 \text{ ohm}$$

$$k_{t1} = k_{t2} = 2 \text{ N} \cdot \text{m/A} \quad k_{v1} = k_{v2} = 2 \text{ V} \cdot \text{s/rad}$$

- Schemi di controllo

A. Indipendente ai giunti pos + vel

$$K_P = 5 \quad K_V = 10 \quad k_{TP} = k_{TV} = 1$$

B. Indipendente ai giunti pos + vel + acc

$$K_P = 5 \quad K_V = 10 \quad K_A = 2 \quad k_{TP} = k_{TV} = k_{TA} = 1$$

C. Come A + compensazione in avanti decentralizzata

D. Come B + compensazione in avanti decentralizzata

E. Coppia precalcolata ai giunti (termini diag. inerzia + gravità) +
Indipendente ai giunti come A

F. PD + compensazione di gravità + $\mathbf{K}_D \dot{\mathbf{q}}_d$

$$\mathbf{K}_P = 3750\mathbf{I} \quad \mathbf{K}_D = 750\mathbf{I}$$

G. Dinamica inversa

$$\mathbf{K}_P = 25\mathbf{I} \quad \mathbf{K}_D = 5\mathbf{I}$$

H. Robusto ($\hat{\mathbf{B}} = \bar{\mathbf{B}}, \hat{\mathbf{n}} = \mathbf{F}_v \dot{\mathbf{q}} + \mathbf{g}$)

$$\mathbf{K}_P = 25\mathbf{I} \quad \mathbf{K}_D = 5\mathbf{I} \quad \rho = 70 \quad \epsilon = 0.004$$

I. Come H $\epsilon = 0.01$

J. Adattativo

$$\Delta m_2 = 10 \text{ kg} \quad \Delta m_2 \ell_{C2} = 11 \text{ kg} \cdot \text{m} \quad \Delta \hat{I}_2 = 12.12 \text{ kg} \cdot \text{m}^2$$

$$\mathbf{\Lambda} = 5\mathbf{I} \quad \mathbf{K}_D = 750\mathbf{I} \quad \mathbf{K}_\pi = 0.01\mathbf{I}$$

K. PD cartesiano + compensazione di gravità + $\mathbf{K}_D \dot{\mathbf{x}}_d$

$$\mathbf{K}_P = 16250\mathbf{I} \quad \mathbf{K}_D = 3250\mathbf{I}$$

L. Dinamica inversa cartesiano

$$\mathbf{K}_P = 25\mathbf{I} \quad \mathbf{K}_D = 5\mathbf{I}$$

