

ON THE SOLUTION OF INVERSE KINEMATICS
OF REDUNDANT MANIPULATORS

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Abstract

Kinematically redundant manipulators have been recognized by the robotics research community as offering greater flexibility and dexterity in robot design, planning and control. One common way of solving for redundancy in an inverse kinematics setting is to require that the robot meets a set of functional constraints which characterize an increased dexterity. This work addresses this issue and proposes two closed-loop inverse kinematic schemes obtained by a suitable "dynamic" reformulation of the constrained problem. Similarities and differences with other proposed approaches are also discussed.

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Throughout the paper, underlines denote vectors while capitals indicate matrices.

Introduction

In recent years a great deal of research work has been devoted to the adoption of redundancy in robot design as a successful strategy towards a more dexterous manipulator structure.¹⁻⁹ The human arm constitutes a tangible model of the ability of a redundant arm to execute more dexterous motions. Despite of this, there appears to be a great reluctance in industry to produce redundant geometries. The reason perhaps is that redundancy involves mechanical and control complexity and then increased costs. When a manipulator is redundant with respect to a given task, the space of redundancy can be conveniently exploited to meet a number of constraints on the solution of the kinematic control problem. Typical goals are the avoidance of obstacles in the workspace,^{1,2} mechanical joint limits,³ kinematic singularities,⁴ or the minimization of actuator joint forces,⁵ kinematic and dynamic manipulability measures,^{6,7} dexterity measures,⁸ position/force task compatibility indices,⁹ and so forth.

Previous approaches in the literature were aimed at solving redundancy in terms of optimizing quadratic type criteria along with the use of generalized inverses of the manipulator Jacobian. The joint velocity solution vector is formed by two terms, a locally minimum norm term plus a term in the space of redundancy (the null space of the Jacobian matrix) which is used for local optimization purposes.^{3,10,11}

An alternative approach applicable to any robot geometry was proposed by Balestrino et al.¹² and Wolovich and Elliott¹³ which is based on a closed-loop system formulation. A simple "dynamic" system, when driven by a specified end-effector trajectory, yields a joint displacement and velocity trajectory solution of the inverse kinematic problem. In particular, the tracking error is bounded while the positional error is null. The scheme is computationally efficient since it only requires the on-line computation of the end-effector direct kinematic function and the transpose of the Jacobian. Nonetheless, it was shown that the adoption of the (pseudo)inverse of the Jacobian in lieu of its transpose, though more computationally demanding, is an attractive alternative in order to obtain a null tracking error.¹²

This technique has lately been extended by Sciavicco and Siciliano¹⁴⁻¹⁶ to the solution of the inverse kinematic problem for redundant manipulators by imposing a set of differentiable functional constraints on the joint displacements. This approach results in defining an augmented task space vector which can be used as input to the above

dynamic system. Similar task augmentation approaches have been proposed by Baillieul¹⁷ and Egeland.¹⁸

In general, however, it is not possible to arbitrarily choose such constraints so that the joint displacements satisfy the constraints and depend continuously on the trajectory assigned at the end-effector.¹⁹ A task-priority based strategy has been suggested by Maciejewski and Klein¹ and Nakamura et al.²⁰ along with the use of pseudoinverses, in the sense that priority is given to the primary task (typically the end-effector trajectory) and the secondary task (typically the constraints) is satisfied only on condition that it does not disturb the primary task. It is shown here that, in an augmented task space setting, this approach leads to adopting a modified augmented Jacobian in which the constraint Jacobian is projected onto the null space of the end-effector Jacobian so that priority to the end-effector task is ensured. A similar scheme, though only for the simple positioning task, has been lately given by Das et al.²¹

The difficulty of imposing a specified set of functional constraints is overcome by the approach presented by Baillieul,¹⁷ and lately generalized by Chang,²² which is aimed at seeking the constrained local minimum for a scalar criteria function whose gradient with respect to the joint displacements is projected onto a basis of the null space of the end-effector Jacobian. It is shown here how the same inverse kinematic scheme with task space augmentation can be formally applied also in this case. Finally, a comparison between the two approaches is discussed.

Kinematics

For any manipulator with known geometrical parameters, the kinematic equation specifies the relation between the $(n \times 1)$ vector \underline{q} of joint displacements and the $(m \times 1)$ vector \underline{x}_e of end-effector location as²³

$$\underline{x}_e = \underline{f}_e(\underline{q}) \quad (1)$$

where \underline{f}_e is a continuous nonlinear function which associates to each \underline{q} a unique \underline{x}_e .

Differentiating eq. (1) with respect to time yields the relation between the vector $\dot{\underline{q}}$ of joint velocities and the vector $\dot{\underline{x}}_e$ of end-effector velocities, i.e.

$$\dot{\underline{x}}_e = J_e(\underline{q})\dot{\underline{q}} \quad (2)$$

where $J_e(\underline{q}) = \partial \underline{f}_e / \partial \underline{q}$ is the $(m \times n)$ end-effector Jacobian matrix.²³

If the manipulator is kinematically redundant with respect to a certain task, it is $m < n$. Assuming that the Jacobian matrix $J_e(\underline{q})$ has full rank for almost all \underline{q} 's, $(n - m)$ DOF's are available for solving redundancy. If for some $\bar{\underline{q}}$, $J_e(\bar{\underline{q}})$ has rank less than m , the manipulator is said to be at a singular configuration. In this configuration the manipulator loses its ability to move along or rotate about some direction of the space, meaning that its manipulability is reduced.⁷

The Inverse Kinematic Scheme

It is uniformly recognized that the solution of the inverse kinematic problem, i.e. given \underline{x}_e solve eq. (1) for \underline{q} , is of fundamental importance for robot manipulator control.²³ In case of redundant manipulators, the most common approach that was proposed in the robotics literature is based on the instantaneous inversion of the mapping (2). It can be shown that a solution to (2) is given by

$$\dot{\underline{q}} = J_e^+(\underline{q})\dot{\underline{x}}_e \quad (3)$$

where J_e^+ is usually the $(n \times m)$ pseudoinverse matrix of matrix J_e defined as $J_e^+ = J_e^T (J_e J_e^T)^{-1}$.²⁴

A conceptually different approach to the inverse kinematic problem for redundant manipulators is given by a recently proposed general solution scheme which is obtained via a "dynamic" reformulation of the problem.^{12,13} The method is summarized in the following.

Let \underline{q}_d be a solution to (1) relative to a given end-effector location \underline{x}_{ed} . A vector \underline{e}_e of end-effector location errors can be defined between the reference vector \underline{x}_{ed} and the actual vector \underline{x}_e computed from the current joint vector \underline{q} via eq. (1),

$$\underline{e}_e = \underline{x}_{ed} - \underline{x}_e \quad (4)$$

Here the explicit dependence on time is not evidenced. However, differentiating with respect to time yields

$$\dot{\underline{e}}_e = \dot{\underline{x}}_{ed} - J_e(\underline{q})\dot{\underline{q}}. \quad (5)$$

It can be formally proved, by choosing as candidate Lyapunov function $v_e = \frac{1}{2}\underline{e}_e^T K_e \underline{e}_e$, where K_e is a positive definite matrix (usually diagonal), that the choice

$$\dot{\underline{q}} = J_e^T(\underline{q})K_e \underline{e}_e \quad (6)$$

ensures that:

- a) If \underline{e}_e at time $t = 0$ is null (i.e. the initial configuration $\underline{q}(0)$ of the manipulator is known), the tracking error is confined to a region of the error space containing the origin which is attractive for all trajectories $\dot{\underline{x}}_{ed} \in R(J_e)$, where $R(J_e)$ denotes the range space of matrix J_e ; the larger the norm of K_e and the inverse of the norm of $\dot{\underline{x}}_{ed}$, the smaller the region.¹²
- b) The positional error, i.e. when $\dot{\underline{x}}_{ed} = \underline{0}$, is null.^{12,13,21}

The resulting closed-loop scheme is illustrated in Fig. 1. In view of the preceding, the above scheme can be used either on-line for continuously solving an end-effector trajectory into a joint trajectory while guaranteeing an upper-bounded tracking error and a null positional error,¹² or off-line to find a joint solution corresponding to a given constant end-effector location with guaranteed null positional error.^{13,21}

If a null tracking error is desired, the solution (6) can be modified into the more computational demanding solution¹² (see the scheme in Fig. 2)

$$\dot{\underline{q}} = J_e^+(\underline{q})[\dot{\underline{x}}_{ed} + K_e \underline{e}_e] \quad (7)$$

which resembles the pseudoinverse solution of eq. (3), but it is inherently closed-loop, i.e. it avoids cumulative errors associated with the instantaneous inversion of the mapping J_e . A solution similar to (7) has been recently proposed by Vaccaro and Hill,²⁵ but it was set $\dot{\underline{x}}_{ed} = \underline{0}$ which ensured only a null positional error.

An important remark is in order concerning the occurrence of rank deficiency in the Jacobian matrix $J_e(\underline{q})$ of both schemes of Figs. 1 and 2, as well as in the open-loop solution of eq. (3). For the first scheme, it

can be recognized that when $K_{e-e} \in N(J_e^T)$ with $\underline{e}_e \neq \underline{0}$, where $N(J_e^T)$ indicates the null space of matrix J_e^T , the scheme yields $\dot{\underline{q}} = \underline{0}$. Thus, if $\dot{\underline{x}}_{ed} = \underline{0}$, the solution gets stuck and \underline{e}_e no further decreases. If $\dot{\underline{x}}_{ed} \neq \underline{0}$, instead, the solution gets stuck as long as $\dot{\underline{x}}_{ed}$ does take K_{e-e} out of $N(J_e^T)$.

On the other hand, for the second scheme and the open-loop scheme based on eq. (3), a generalized inverse must be used in lieu of the pseudoinverse, since the matrix $J_e J_e^T$ in the definition of the pseudoinverse is no longer invertible. The expression of the generalized inverse is given by $J_e^I = J_e^T J_{eb} (J_{eb}^T J_e J_e^T J_{eb})^{-1} J_{eb}^T$ where J_{eb} is a matrix of vectors which form a basis of $R(J_e)$.²⁴ For the second closed-loop scheme, it can be recognized that when $[K_{e-e} + \dot{\underline{x}}_{ed}] \in N(J_e^T)$ with $\underline{e}_e \neq \underline{0}$ the scheme yields $\dot{\underline{q}} = \underline{0}$, since $N(J_{eb}^T) = N(J_e^T)$. Thus similarly to the above case, if $\dot{\underline{x}}_{ed} = \underline{0}$, the solution gets stuck and \underline{e}_e no further decreases. If $\dot{\underline{x}}_{ed} \neq \underline{0}$, instead, the solution gets stuck as long as $\dot{\underline{x}}_{ed}$ does take $[K_{e-e} + \dot{\underline{x}}_{ed}]$ out of $N(J_e^T)$.

Inclusion of Constraints

As discussed in the introduction, redundancy can be conveniently exploited to meet additional constraints in order to obtain greater manipulability in terms of manipulator kinematical configuration and interaction with the environment. To this purpose, the human arm constitutes a tangible model of this ability.

If the robot is required to move in a cluttered environment, for instance, obstacle avoidance^{1,2} and limited joint range³ represent two types of constraints to account for in the trajectory planning and inverse kinematics solving. Of interest could also be the minimization of actuator joint forces⁵ along any given trajectory.

The other important point in purposely making a manipulator redundant is the avoidance of singular configurations.^{4,19} The manipulability measure defined by Yoshikawa⁶ and more generally the dexterity measures surveyed by Klein and Baho,⁸ such as the matrix condition number, the minimum singular value, all based on the matrix $J_e(\underline{q})J_e^T(\underline{q})$, constitute indices of the ability of a manipulator to avoid encountering a singularity, and more generally quantitative measures of manipulating ability in positioning and orienting the end-effector. The dynamic manipulability measure introduced by Yoshikawa,⁷ instead, takes the arm

dynamics into consideration. Related to the above points is also the concept of task compatibility,⁹ according to which the matrix $J_e(\underline{q})J_e^T(\underline{q})$ is utilized to determine quantitative indices of the ability to execute a certain exertion/control task along a given direction.

Without going into any further details about the definition of measures characterizing redundant manipulator dexterity, for the purpose of the following discussion, it can be said that the above measures are usually adopted to determine a number of constraints on the solution of the inverse kinematics of redundant manipulators.

Reconsidering the pseudoinverse solution of eq. (3), it can be shown that the general solution to (2) is given by

$$\dot{\underline{q}} = J_e^+(\underline{q})\dot{\underline{x}}_{ed} + [I - J_e^+(\underline{q})J_e(\underline{q})]\dot{\underline{q}}_0 \quad (8)$$

where I is the $(n \times n)$ identity matrix and $\dot{\underline{q}}_0$ is an $(n \times 1)$ arbitrary vector of joint velocities. It is worth emphasizing that the solution (3), which represented the least-square minimum norm solution to (2), has been suitably modified into the solution (8) by the addition of the homogeneous term created by the projection operator $(I - J_e^+J_e)$ which selects the components of $\dot{\underline{q}}_0$ in the null space of the mapping J_e .¹⁰ Usually, the vector $\dot{\underline{q}}_0$ was chosen as the gradient of some scalar quadratic function of the joint displacements obtained via the above discussed constraints, with the purpose of locally minimizing such function.¹⁻¹¹

On the other hand, the inverse kinematic schemes of Figs. 1 and 2 can be easily extended by formally embedding a set of constraints on the joint displacements, characterizing for instance any of the above discussed dexterity measures. More specifically, a functional constraint equation can be considered in the form

$$\underline{x}_c = \underline{f}_c(\underline{q}) \quad (9)$$

where \underline{f}_c is an $(r \times 1)$ vector with continuous first-order partial derivatives with respect to joint displacements; also it is $r \leq (n - m)$, so as to span at most the whole redundant space. As a consequence, the augmented kinematic equation becomes

$$\begin{bmatrix} \underline{x}_e \\ \underline{x}_c \end{bmatrix} = \begin{bmatrix} \underline{f}_e(\underline{q}) \\ \underline{f}_c(\underline{q}) \end{bmatrix} \quad (10)$$

whose joint vector solution \underline{q} not only places the end-effector at the desired location \underline{x}_e , but should also meet the required constraints specified by eq. (9). Hence, it would seem that an augmented task space vector based on eq. (10) can be used as input to the schemes of Figs. 1 and 2, which basically remain the same except for a suitably augmented Jacobian matrix

$$J(\underline{q}) = \begin{bmatrix} J_e(\underline{q}) \\ J_c(\underline{q}) \end{bmatrix} \quad (11)$$

where $J_c(\underline{q}) = \partial f_c / \partial \underline{q}$. This approach has been applied to the case of dexterity measure constraints¹⁴ and to the case of obstacle avoidance and/or limited joint range constraints.^{15,16} Conceptually similar is also the application to a small fast manipulator mounted on a large positioning part.¹⁸

Nonetheless, the question is whether or not it is possible to ensure that the joint displacements satisfy those arbitrarily defined constraints of eq. (9) while the end-effector trajectory is tracked.¹⁹ This corresponds to the fact that the augmented Jacobian matrix J in (11) is not guaranteed at all to have full rank $(m + r)$ for almost all \underline{q} 's. In terms of the above two schemes, when the augmented task space error $\underline{K}_e \in N(J^T)$, with $K = \text{diag}(K_e, K_c)$, $K_c > 0$ and $\underline{e} = (\underline{e}_e^T \ \underline{e}_c^T)^T$, the solution may get stuck while not even exactly performing the end-effector task. In particular, even if J_e has full rank m and J_c has full rank r , the matrix J may have rank less than $(m + r)$ as long as $R(J_e^T) \cap R(J_c^T) \neq 0$. In order to account for such occurrence, a so-called task-priority strategy^{1,20} is advisable, meaning that priority is given to the primary task, the end-effector trajectory \underline{x}_e in this case. The secondary task, the constraints \underline{x}_c in this case, is satisfied only on condition that it does not disturb the primary task. This is obtained by modifying the solution (6) for the scheme of Fig. 1 into

$$\dot{\underline{q}} = J_e^T(\underline{q})K_e\underline{e}_e + [I - J_e^+(\underline{q})J_e(\underline{q})]J_c^T(\underline{q})K_c\underline{e}_c \quad (12)$$

where the operator $(I - J_e^+J_e)$ projects the vector $J_c^TK_c\underline{e}_c$ onto the null space of J_e , similarly to eq. (8). Under the solution (12), the end-effector error dynamics becomes

$$\dot{\underline{e}}_e = \dot{\underline{x}}_{ed} - J_e(\underline{q})J_e^T(\underline{q})K_e\underline{e}_e \quad (13)$$

which is apparently the same as in the case of solution (6). The constraint error dynamics is given by

$$\dot{\underline{e}}_c = \dot{\underline{x}}_{cd} - J_c(\underline{q})J_e^T(\underline{q})K_{e-e} \underline{e}_e - J_c(\underline{q})[I - J_e^+(\underline{q})J_e(\underline{q})]J_c^T(\underline{q})K_{c-c} \underline{e}_c. \quad (14)$$

By choosing as Lyapunov function $v = v_e + v_c$ with $v_c = \frac{1}{2}\underline{e}_c^T K_{c-c} \underline{e}_c$, it can be recognized that the constraint tracking error is norm-bounded and the constraint positional error ($\dot{\underline{x}}_{cd} = \underline{0}$) is null except when $J_c^T K_{c-c} \underline{e}_c \in R(J_e^+)$, in which case the solution gets stuck as long as $\dot{\underline{x}}_{cd}$ does take $J_c^T K_{c-c} \underline{e}_c$ out of $R(J_e^+)$. A similar result has been recently obtained,²¹ although the inverse kinematic scheme was a purely positional scheme, and no explicit discussion on the satisfaction of both tasks was included.

Nevertheless, if the end-effector Jacobian J_e is guaranteed to have full rank m , the solution (12) can be modified into

$$\dot{\underline{q}} = J_e^+(\underline{q})[\dot{\underline{x}}_{ed} + K_{e-e} \underline{e}_e] + [I - J_e^+(\underline{q})J_e(\underline{q})]J_c^T(\underline{q})K_{c-c} \underline{e}_c \quad (15)$$

since the computation of the pseudoinverse of J_e is already performed during the computation of the projector onto the null space $(I - J_e^+ J_e)$. The solution (15) ensures a null end-effector tracking error and a norm-bounded constraint tracking error.

Conversely, if the scheme of Fig. 2 is adopted, the solution derived from (7) can be applied as long as the matrix J has full rank $(m + r)$. A simpler computational solution equivalent to (7) can be directly derived according to the task-priority strategy, i.e.

$$\dot{\underline{q}} = J_e^+(\underline{q})[\dot{\underline{x}}_{ed} + K_{e-e} \underline{e}_e] + \tilde{J}_c^+(\underline{q})\{\dot{\underline{x}}_{cd} - J_c(\underline{q})J_e^+(\underline{q})[\dot{\underline{x}}_{ed} + K_{e-e} \underline{e}_e] + K_{c-c} \underline{e}_c\} \quad (16)$$

where $\tilde{J}_c = J_c(I - J_e^+ J_e)$ and $(I - J_e^+ J_e)\tilde{J}_c = \tilde{J}_c$ since the projection operator $(I - J_e^+ J_e)$ is both hermitian and idempotent.¹ The inverse kinematic scheme based on the solution (16) can be considered as a closed-loop extension of the schemes given by Maciejewski and Klein¹ and Nakamura et al.²⁰ In particular, in analogy with Nakamura et al.,²⁰ it can be shown that, if $(m + r) < n$, another term can be added to the solution (16) of the type $(I - J_e^+ J_e)(I - \tilde{J}_c^+ \tilde{J}_c)\underline{z}$ where \underline{z} is a vector resulting from the addition of another constraint with lower priority.

On the other hand, if J has rank less than $(m + r)$ the solution (7) can no longer be adopted since no task-priority is accomplished. The solution (16) can be utilized on condition that \tilde{J}_c^+ be a generalized

inverse of \tilde{J}_c , but it is apparently more computationally demanding. Therefore, the solutions (12) and (15) seem to be preferred when J has not full rank.

A nice representation of the manipulable spaces and the redundant spaces associated to the task variables defined in eq. (10) is given by Nakamura et al.²⁰ It is worth noticing, here, that not only does such representation apply to the solution (16) but also to the solutions (12) and (15) since $R(J^+) = R(J^T)$.

In the above formulation of constraints, it is implicitly assumed that they are set in a completely arbitrary manner. This may not be appropriate in certain cases. It is for this reason that the approach first presented by Baillieu¹⁷ and lately formalized for the most general case by Chang²² represents a valid alternative to the problem of solving redundancy at inverse kinematic level. The main difference with the previous approach is that a scalar quadratic criteria function $c(\underline{q})$ with continuous first-order partial derivatives with respect to joint displacements is chosen which describes the desired performance. Focusing directly on the formulation given by Chang,²² the Lagrangian multiplier method is used to derive a set of $(n - m)$ scalar equations which are combined with the m scalar equations (1). The result is a full task space augmentation, i.e.

$$\begin{bmatrix} \underline{x}_e \\ \underline{0} \end{bmatrix} = \begin{bmatrix} \underline{f}_e(\underline{q}) \\ Z_e(\underline{q})\underline{f}_c(\underline{q}) \end{bmatrix} \quad (17)$$

where $\underline{f}_c(\underline{q}) = \partial c / \partial \underline{q}$ and $Z_e(\underline{q})$ is a vector basis of $N(J_e)$ which can be computed in symbolic form, once the end-effector Jacobian J_e is expressed in symbolic form. It must be outlined, here, that in order that a solution \underline{q} to (17) exists, it must be $\underline{f}_c \in R(J_e^T)$. If this is the case, the solution found minimizes $c(\underline{q})$ subjected to the kinematic constraint (1). Chang²² has proposed to solve eq. (17) with a numerical package based on Powell's hybrid method. Baillieu¹⁷ has derived an extended Jacobian $J(\underline{q})$ through differentiation of the right half side of eq. (17) with respect to the joint displacements; the solution is then obtained in the form $\dot{\underline{q}} = J^{-1}(\underline{q})[\underline{x}_e^T \ \underline{0}^T]^T$. The occurrence of so-called algorithmic singularities¹⁷ (when J has rank less than n , despite J_e has rank m), however, is a drawback for such solution.

Nonetheless, based on this different approach, the scheme of Fig. 1 or the scheme of Fig. 2 with a pure inverse in lieu of the pseudoinverse, can be formally applied to solve eq. (17), with the inherent advantage of

obtaining a closed-loop formulation to the problem.

Concluding Remarks

A closed-loop formulation to the inverse kinematic problem for redundant manipulators has been given. Two computational schemes have been obtained; one is based on the use of the transpose of the Jacobian, the other on the pseudoinverse of the Jacobian. It has been shown how those schemes can be suitably extended to deal with the inclusion of constraints on the solution which describe the dexterity of the redundant arm.

If a set of independent constraints is specified, a task-priority strategy serves as an effective means to guarantee that the primary end-effector task is correctly executed whereas the secondary constraint task is executed in the best compatible way with the end-effector task. It is to be remarked that the scheme based on the transpose of the constraint Jacobian should be preferred to the scheme using the pseudoinverse of the constraint Jacobian, since it is less computationally demanding.

If the specification of constraints is not desired, but the minimization of a scalar quadratic function incorporating one or more constraints is of primary concern, a different approach can be pursued which leads to establish a structural condition of orthogonality between the gradient of the above function and the null space of the end-effector Jacobian. A closed-loop scheme can be obtained also in this case, although it is apparently more computational demanding. This indicates that the choice between the two approaches has to be made on the basis of a trade-off between the necessity of specifying the constraint task and the burden of computing power required.

A final remark seems in order regarding the issues discussed in this work. All the techniques presented as well as most of the proposed approaches in the robotics literature¹⁻²² are inherently aimed at solving redundancy instantaneously, which has the advantage of real-time computation, but it lacks the guarantee of global optimality. The simple reason is that the Jacobian matrix appearing in all above formulations is configuration-dependent. Nakamura and Hanfusa²⁶ have recently shown how to obtain, if it exists, the global optimal joint trajectory for a redundant manipulator with specified constraints, by using the Pontryagin's maximum principle. This approach has also been revisited by Baillieul et al.¹⁹

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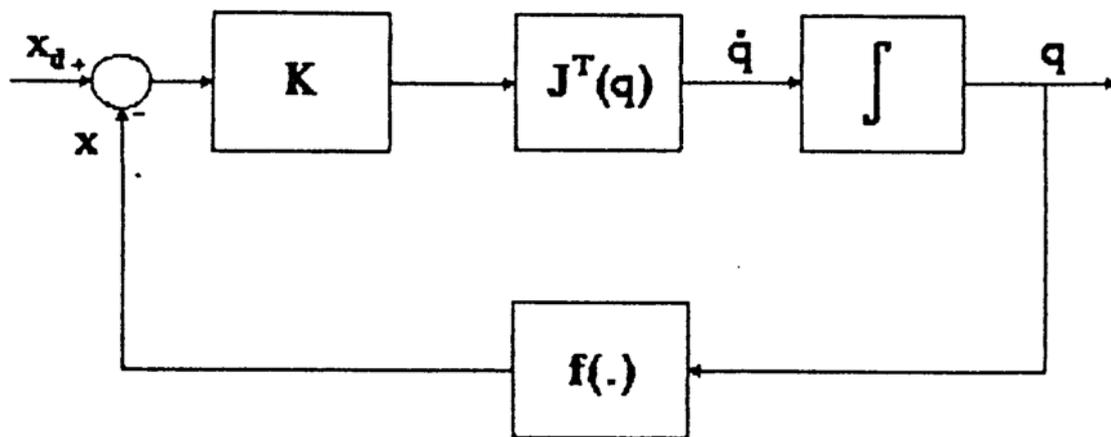


Fig. 1 - The closed-loop inverse kinematic scheme based on the transpose of the Jacobian.

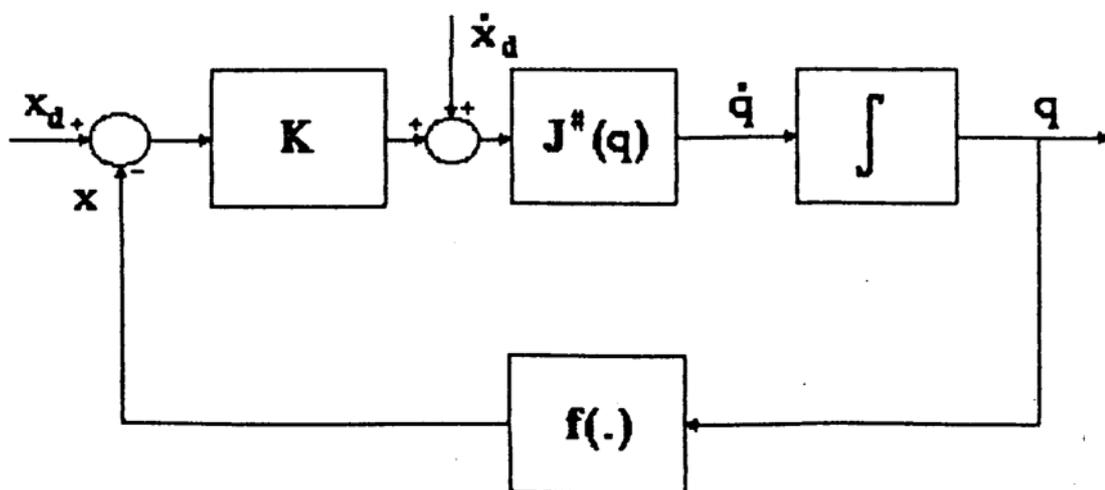


Fig. 2 - The closed-loop inverse kinematic scheme based on the pseudoinverse of the Jacobian.