

OPTIMAL OUTPUT FAST FEEDBACK IN TWO-TIME SCALE CONTROL OF FLEXIBLE ARMS

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ABSTRACT

Control of lightweight flexible arms moving along predefined paths can be successfully synthesized on the basis of a two-time scale approach. A model following control can be designed for the reduced order slow subsystem. The fast subsystem is a linear system in which the slow variables act as parameters. The flexible fast variables which model the deflections of the arm along the trajectory can be sensed through strain gage measurements. For full state feedback design the derivatives of the deflections need to be estimated. The main contribution of this work is the design of an output feedback controller which includes a fixed order dynamic compensator, based on a recent convergent numerical algorithm for calculating LQ optimal gains. The design procedure is tested by means of simulation results for the one link flexible arm prototype in the laboratory.

INTRODUCTION

Control is one of the crucial points to an effective use of lightweight flexible arms. The control problem, however, is more complicated than in case of rigid arms, due to the flexibility distributed along a lightweight mechanical structure. The dynamic model for a flexible arm can be derived via a Lagrangian-assumed modes method [1]. The result is an extended number of generalized coordinates, and then state variables, to handle for control purposes.

An efficient control strategy based on a singular perturbation approach has been proposed in [2]. A two-time scale analysis of the system is performed: a slow subsystem which is of the same order as that of a rigid arm, and a fast linear subsystem in which the slow state variables play the role of parameters. A composite control [3] is then adopted: a slow model following control can be first designed to track a desired joint trajectory, and a linear fast state feedback control provides to stabilize the deflections along the trajectory. The flexible fast variables which model the deflections of the arm along the trajectory can be sensed through strain gage measurements [4]. For full state feedback design, however, the derivatives of the deflections need to be estimated.

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The goal of this paper is the design of an output fast feedback controller which includes a fixed order dynamic compensator [5], based on a recently proposed convergent numerical algorithm for calculating LQ optimal gains [6]. The design procedure is applied to the one link flexible arm prototype in the laboratory [2] and simulation results are presented.

THE TWO-TIME SCALE CONTROL APPROACH

The dynamic model for a flexible link arm can be derived via a Lagrangian-assumed modes method [1]. For the purpose of this work the state space formulation derived in [2] will be adopted. Let $\underline{x}^T = (\underline{q}^T \ \dot{\underline{q}}^T)$ and $\underline{z}^T = (\underline{\delta}^T \ \dot{\underline{\delta}}^T)$ be the state variables, where $\underline{q} \in \mathbb{R}^n$ is the vector of joint variables and $\underline{\delta} \in \mathbb{R}^m$ is the vector of deflection variables obtained via the assumed modes expansion. The model results in the following form:

$$\dot{\underline{x}} = F\underline{x} + \underline{g}_1(\underline{x}) + A_1(\underline{x})\underline{z} + B_1(\underline{x})\underline{u} \quad (1a)$$

$$F = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \quad \underline{g}_1 = \begin{bmatrix} 0 \\ \underline{g}_{10} \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 0 \\ A_{10} & A_{11} \end{bmatrix} \quad B_1 = \begin{bmatrix} 0 \\ B_{10} \end{bmatrix}$$

$$\dot{\underline{z}} = A_2(\underline{x})\underline{z} + \underline{g}_2(\underline{x}) + B_2(\underline{x})\underline{u} \quad (1b)$$

$$A_2 = \begin{bmatrix} 0 & I \\ A_{20} & A_{21} \end{bmatrix} \quad \underline{g}_2 = \begin{bmatrix} 0 \\ \underline{g}_{20} \end{bmatrix}$$

$$B_2 = \begin{bmatrix} 0 \\ B_{20} \end{bmatrix}$$

where $\underline{u} \in \mathbb{R}^n$ is the control vector.

Under the assumption that the spectrum of rigid body motion is well separated from the spectrum of flexible link deflections, the system can be considered a singularly perturbed one. As proposed in [7], the

system of equations (1) can be artificially scaled in the following way:

$$\dot{\underline{x}} = F\underline{x} + \underline{g}_1 + A_1\underline{z} + B_1\underline{u} \quad (2a)$$

$$\mu\dot{\underline{z}} = A_2\underline{z} + \underline{g}_2 + B_2\underline{u} \quad (2b)$$

where the parameter μ is identified as a time scaling parameter so that the variables \underline{z} are scaled on a separate time scale in accordance with the relative speed with which these variables change their magnitudes. The development of singular perturbation theory is on the basis that μ represents a small parameter. Since the system (2) is nonlinear, the procedure for identifying μ is not straightforward and may involve considerable effort, even if flexible dynamics are known to be faster than rigid dynamics. The viewpoint taken in [2] and here is that μ is given by the ratio of the highest frequency of the slow dynamics vs the smallest frequency of the fast dynamics.

Formally setting $\mu = 0$ accomplishes a model order reduction from $n + m$ to n , as the differential equations (2b) degenerate into the algebraic transcendental equations

$$\underline{0} = A_2(\bar{\underline{x}})\bar{\underline{z}} + \underline{g}_2(\bar{\underline{x}}) + B_2(\bar{\underline{x}})\underline{u} \quad (3)$$

where the bar is used to indicate that the variables are in the slow time scale with $\mu = 0$. Since the system (3) is linear in the fast variables \underline{z} , it is possible to find the quasi-steady state solution to (2b) [8]

$$\bar{\underline{z}} = \begin{bmatrix} -A_{20}^{-1}(\underline{g}_{20} + B_{20}\bar{\underline{u}}) \\ \underline{0} \end{bmatrix} \quad (4)$$

Substituting (4) in (2a) yields the slow time scale subsystem (of order n)

$$\dot{\bar{\underline{x}}} = F\bar{\underline{x}} + \underline{a}(\bar{\underline{x}}) + B(\bar{\underline{x}})\bar{\underline{u}} \quad (5)$$

$$\underline{a} = \underline{g}_1 + \begin{bmatrix} \underline{0} \\ -A_{10}A_{20}^{-1}\underline{g}_{20} \end{bmatrix}$$

$$B = B_1 + \begin{bmatrix} \underline{0} \\ -A_{10}A_{20}^{-1}B_{20} \end{bmatrix}$$

Defining the fast state variable change around the equilibrium trajectory $\underline{z}_f = \underline{z} - \bar{\underline{z}}$, and correspondingly $\underline{u}_f = \underline{u} - \bar{\underline{u}}$, the fast time scale subsystem (of order m) results

$$\frac{d\underline{z}_f}{d\tau} = A_2^1(\bar{\underline{x}})\underline{z}_f + B_2^1(\bar{\underline{x}})\underline{u}_f \quad (6)$$

where $\tau = t/\mu$ is the fast time scale. It must be emphasized that the system (6) is a linear system parameterized in the slow variables $\bar{\underline{x}}$.

Under these results, the design of a feedback control \underline{u} for the full order system (2) can be performed on the basis of a composite control strategy [3] as

$$\underline{u} = \bar{\underline{u}}(\bar{\underline{x}}) + \underline{u}_f(\bar{\underline{x}}, \underline{z}_f) \quad (7)$$

with the constraint that $\underline{u}_f(\bar{\underline{x}}, \underline{0}) = \underline{0}$ such that \underline{u}_f is inactive along the solution (4).

Since the slow time scale subsystem (5) is of order n , it is quite straightforward to design the slow control as

$$\bar{\underline{u}}(\bar{\underline{x}}) = B^+(\bar{\underline{x}})(-\underline{a}(\bar{\underline{x}}) + \underline{v}(\bar{\underline{x}}, \bar{\underline{x}})) \quad (8)$$

where \underline{v} is a new control input which allows the system (5) to track a reference model specified by $\bar{\underline{x}}$ [2].

At this point the singular perturbation theory requires that the fast time scale subsystem (6) be uniformly stable along the equilibrium trajectory $\bar{\underline{z}}$ given in (4). Assuming that the couple (A_2^1, B_2^1) is uniformly stabilizable for any slow trajectory $\bar{\underline{x}}$, a fast state feedback control of the type

$$\underline{u}_f(\bar{\underline{x}}, \underline{z}_f) = K_f(\bar{\underline{x}})\underline{z}_f \quad (9)$$

would stabilize the system (6) to $\underline{z}_f = \underline{0}$. The synthesis of the control (9), however, requires full fast state feedback. In reality the deflection variables $\underline{\delta}$ can be sensed through strain gage measurements [4], whereas their derivatives $\dot{\underline{\delta}}$ need to be reconstructed, e.g. via an observer. To overcome this drawback, in the following the design of an output feedback controller in lieu of (9) is presented.

FEEDBACK CONTROLLER DESIGN

In [5] it is shown that for a multivariable system described by

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \quad \underline{x} \in R^n \quad (10a)$$

$$\underline{y} = C\underline{x} \quad \underline{y} \in R^p \quad (10b)$$

a fixed order compensator without direct feedthrough of the output can be formulated in observer canonical form as

$$\underline{u} = -H^o \underline{z} \quad \underline{u} \in R^m \quad (11a)$$

$$\dot{\underline{z}} = P^o \underline{z} + I_{nc} \underline{u}_c \quad \underline{z} \in R^{nc} \quad (11b)$$

$$\underline{u}_c = P_z \underline{u} - N\underline{y} \quad \underline{u}_c \in R^{nc} \quad (11c)$$

where

$$H^\circ = \text{block diag} \{ [0 \dots 0 \ 1]_{\ell_i \times 1}, i=1, \dots, m \} \quad (12)$$

and

$$P^\circ = \text{block diag} [P_1^\circ, \dots, P_m^\circ] \quad (13)$$

with

$$P_i^\circ = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}_{\ell_i \times \ell_i} \quad (14)$$

In (11) N and P_z are free parameter matrices with dimensions $(n_c \times p)$ and $(n_c \times m)$, respectively.

The dimensions of H° and P° are defined by the observability indices of the compensator, which are chosen to satisfy:

$$i) \sum_{i=1}^m \ell_i = n_c \quad ii) \ell_i < \ell_{i+1}$$

The augmented system matrices

$$\bar{A} = \begin{bmatrix} A & -BH^\circ \\ 0 & P^\circ \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix} \quad (15a)$$

$$\bar{C} = \begin{bmatrix} C & 0 \\ 0 & H^\circ \end{bmatrix} \quad \bar{G} = [N \ P_z] \quad (15b)$$

define an optimal output feedback problem, with the quadratic performance index

$$J = E_{x_0} \int_0^\infty [x^t Q x + u_c^t R u_c] dt \quad (16)$$

where the augmented state vector is

$$\bar{x}^t = [x^t \ z^t]$$

FREQUENCY SHAPING

It is well known that frequency shaped cost functions are a more direct approach to damping

structural modes. However, past papers on this subject have required that the frequency shaping be realized as part of the compensator design [5,9,10]. We present here an approach to frequency shaping that does not increase the order of the compensator. The idea is to adopt the following performance index

$$J = E_{x_0} \int_0^\infty [x^t Q x + y_2^t y_2 + u_c^t R u_c] dt \quad (17)$$

where y_2 is defined by

$$\dot{w} = Fw + My_1, \quad y_1 = C_1 x \quad (18a)$$

$$y_2 = Ew + Jy_1 \quad (18b)$$

That is, y_2 is the output of a filter driven by a suitably chosen linear combination of the plant states. The augmented system matrices become:

$$\bar{A} = \begin{bmatrix} A & 0 & -H^\circ B \\ C_1 M & F & 0 \\ 0 & 0 & P^\circ \end{bmatrix} \quad (19a)$$

$$\bar{B} = \begin{bmatrix} 0 \\ 0 \\ I_{nc} \end{bmatrix} \quad \bar{C} = \begin{bmatrix} C & 0 & 0 \\ 0 & 0 & H^\circ \end{bmatrix} \quad (19b)$$

where now $\bar{x}^t = [x^t \ w^t \ z^t]$. The resulting weighting matrix, when the performance index is reformulated in the form (16) becomes

$$\bar{Q} = \begin{bmatrix} Q + C_1^t J^t J C_1 & C_1^t J^t E & 0 \\ E y^t J C_1 & E^t E & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (20)$$

NUMERICAL RESULTS

We present in this section the numerical results based on the flexible arm model developed in [2]. The fast subsystem as defined by (6) is parameterized by x . To simplify the design of an output feedback compensator, x was chosen as the final joint configuration. This was also done in [2], where the fast subsystem design uses full state feedback. The resulting fast subsystem matrices are:

$$A_2' = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -205.4 & -1900 & 0 & 0 \\ -53.01 & -8051 & 0 & 0 \end{bmatrix} \quad B_2' = \begin{bmatrix} 0 \\ 0 \\ -2.33 \\ -0.75 \end{bmatrix} \quad (21)$$

The flexible fast state variables are $z_f^t = [\delta_1, \delta_2, \dot{\delta}_1, \dot{\delta}_2]$ where strain gauge measurements of δ_1 and δ_2 represent deflections of the arm at the endpoint and at the midpoint, respectively. Thus, C in (10 b) is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (22)$$

The open loop modes are at $\pm \omega_i j$, where $\omega_1 = 13.88$ and $\omega_2 = 89.8$. The frequency shaping dynamics in (18) were defined as the realizations of the transfer functions:

$$y_{1_i} / \dot{\delta}_2 = k_i \frac{s^2/\omega_i + 2\zeta_d s/\omega_i + 1}{s^2/\omega_i + 2\zeta s/\omega_i + 1} \quad i = 1, 2 \quad (23)$$

where $\zeta_d = 0.7$, $\zeta = .01$. This in effect amplifies the weightings on the plant state $\dot{\delta}_2$ in the vicinity of ω_i . The weightings on each mode are independently controlled by selecting the weighting parameters, k_i . A second order compensator ($n_c = 2$) was designed using the numerical algorithm in [6] for calculating LQ optimal output feedback gains, with $Q = 0$, $R = 0.1$ in (20). The parameters k_i were adjusted to achieve the desired damping in the structural modes.

Experience with solution procedure showed that the damping on the closed loop structural modes could be individually adjusted by the choice of k_i . Moreover, the damping can be introduced with only minor change in natural frequency (less than 5%). The compensator introduces two additional closed loop, low frequency poles that are almost unobservable in the plant states. For $k_1 = 350$, $k_2 = 345$, the closed loop structural mode dampings were $\zeta_1 = 0.52$ and $\zeta_2 = 0.70$ respectively. Attempts to further increase the damping resulted in convergence difficulties with the numerical algorithm used to find the optimal \bar{G} . Normally, the algorithm converged in fewer than 10 iterations.

The final solution for \bar{G} was

$$\bar{G} = \begin{bmatrix} -54.3 & -628.5 & 0.687 \\ -922.0 & -1.19 \times 10^6 & 126.8 \end{bmatrix} \quad (24)$$

Figures 1 and 2 illustrate the closed loop fast subsystem response for an initial condition, $\delta_1(0) = 1.0$.

CONCLUSIONS

An output feedback controller with a fixed order dynamic compensator is successfully designed to damp the fast structural modes of a flexible arm. It is found that the use of frequency shaped cost functionals in an output feedback setting leads to a straightforward design procedure, without the need to realize the frequency shaping dynamics as part of the compensator. This permits the design of low order compensators for structural mode damping.

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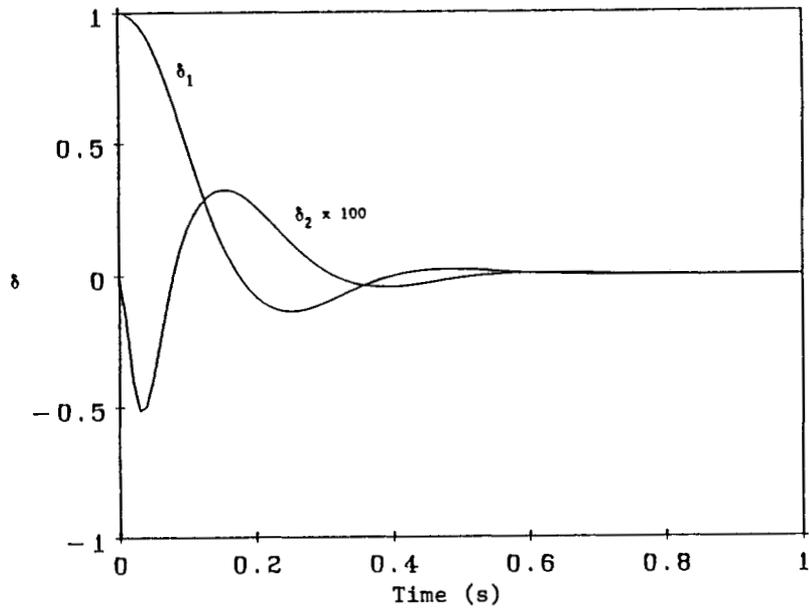


Figure 1. Deflection Response For An Initial Condition $\delta_1(0) = 1.0$.

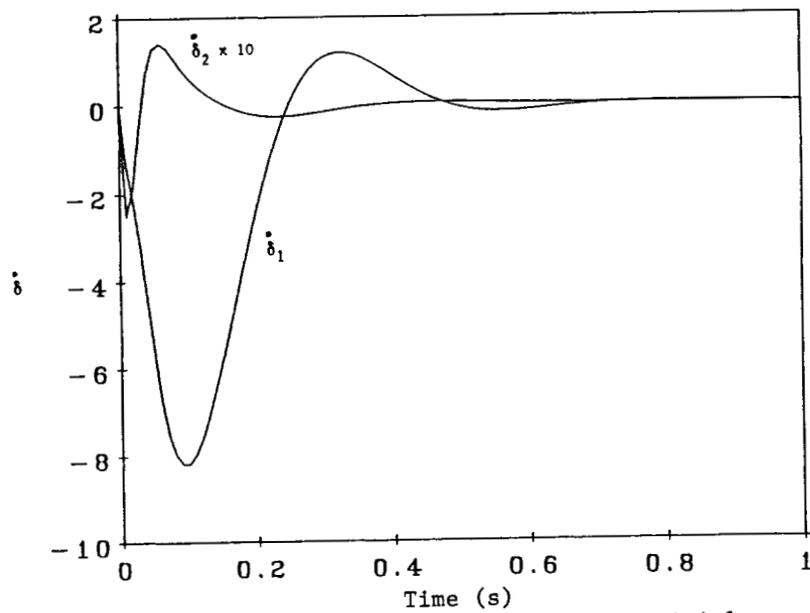


Figure 2. Deflection Rates For An Initial Condition $\delta_1(0) = 1.0$.