

OPTIMAL OUTPUT FEEDBACK IN TWO TIME SCALE SYSTEMS

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ABSTRACT

A formulation is presented for designing optimal output feedback regulators of fixed order. The formulation exploits an observer canonical form to represent the compensator dynamics. The formulation precludes the use of direct feedback of the plant output. The major contribution of the paper is to present how this approach to compensator design may be extended to output feedback of singularly perturbed systems. The result is that two compensators are designed that operate on separate time scales, and which may be implemented with different sampling rates.

I. INTRODUCTION

It has been shown [1] that for a multivariable system described by

$$\dot{x} = Ax + Bu \quad x \in R^n \quad (1)$$

$$y = Cx + Du \quad y \in R^p \quad (2)$$

a fixed order compensator without direct feedthrough of the output can be formulated in observer canonical form as:

$$u = -H^\circ z \quad u \in R^m \quad (3)$$

$$\dot{z} = P^\circ z + u_c \quad z \in R^{n_c} \quad (4)$$

$$u_c = Pu - Ny \quad u_c \in R^{n_c} \quad (5)$$

where

$$H^\circ = \text{block diag}\{[0 \dots 0 \ 1]_{1 \times v_i} \quad i=1, \dots, m\} \quad (6)$$

$$P^\circ = \text{block diag} [P_1^\circ, \dots, P_m^\circ] \quad (7)$$

$$P_i^\circ = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix}_{v_i \times v_i} \quad (8)$$

In (3-5) N and P are free parameter matrices with dimensions $(n_c \times p)$ and $(n_c \times m)$, respectively. The dimensions of H° and P° are defined by the observability indices of the compensator, which are chosen to satisfy:

$$i) \sum_{i=1}^m v_i = n_c \quad ii) v_i \leq v_{i+1}$$

The augmented system matrices:

$$\tilde{A} = \begin{bmatrix} A & -BH^\circ \\ 0 & P^\circ \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix} \quad (9)$$

$$\tilde{C} = \begin{bmatrix} C & -DH^\circ \\ 0 & H^\circ \end{bmatrix} \quad G = [N \ P] \quad (10)$$

define an optimal output feedback problem, with the quadratic performance index:

$$J = E_{x_0} \left\{ \int_0^\infty [x^t Q x + u_c^t R u_c] dt \right\} \quad (11)$$

where the augmented state vector is

$$\tilde{x}^t = [x^t \ z^t] \quad (12)$$

and the control is defined as

$$u_c = -G \tilde{C} \tilde{x} \quad (13)$$

Some of the advantages to the above compensator formulation are that the compensator is represented by a minimum number of parameters, and these are compactly placed in the equivalent constant gain matrix in (10). A convergent numerical method for calculating G is given in [2]. Also, because we have precluded the use of direct feedback of the output, the design carries the same advantage of a full order observer in reducing the effect of sensor noise, and improving robustness to high frequency unmodeled dynamics by guaranteeing an additional 20 db/decade roll-off (over that of the open loop plant) at high frequencies.

This paper considers the extension of the above formulation to two time scale systems. We have found that this extension becomes very transparent when one makes use of recent frequency domain results for two time scale systems [2,3].

II. TWO TIME SCALE FORMULATION

A linear time-invariant singularly perturbed system is described by:

$$\dot{x}_1 = A_{11} x_1 + A_{12} x_2 + B_1 u \quad x_1 \in R^{n_1} \quad (14a)$$

$$\epsilon \dot{x}_2 = A_{21} x_1 + A_{22} x_2 + B_2 u \quad x_2 \in R^{n_2} \quad (14b)$$

$$y = C_1 x_1 + C_2 x_2 \quad (14c)$$

where $0 < \epsilon \ll 1$, and $\det \{A_{22}\} \neq 0$. To control systems of this form it is desirable to separately design two compensators, one for the slow subsystem that results from formally setting $\epsilon = 0$, and one for the fast subsystem obtained from the time stretching transformation $\tau = t/\epsilon$. In [3] it is shown that a two-frequency scale transfer function matrix can be decomposed in the form:

$$C(s, \epsilon) = C_1(s, \epsilon) + C_2(\epsilon s, \epsilon) + D(\epsilon) \quad (15)$$

Thus, a strictly proper two time scale compensator in the canonical form of (3-5) would have the following structure:

$$u = -H_1^\circ \zeta_1 - H_2^\circ \zeta_2 \quad (16)$$

$$\dot{\zeta}_1 = P_1^\circ \zeta_1 + u_{c1} \quad \zeta_1 \in R^{n_{c1}} \quad (17a)$$

$$\epsilon \dot{\zeta}_2 = P_2^\circ \zeta_2 + u_{c2} \quad \zeta_2 \in R^{n_{c2}} \quad (17b)$$

$$u_{ci} = P_i u - N_i y \quad i = 1, 2 \quad (18)$$

In [4] it is shown that the closed loop poles of the system (14,16-18), with the exception of "hidden" and "lost" modes, can be approximated for sufficiently small ϵ by the roots associated with the return difference matrix expressions:

$$\det \{I + C_s(s) P_s(s)\} = 0 \quad (19a)$$

$$\det \{I + C_f(p) P_f(p)\} = 0, \quad p = \epsilon s \quad (19b)$$

where

$$P_s(s) = C_o (sI_{n1} - A_o)^{-1} B_o + D_o \quad (20a)$$

$$P_f(p) = C_2 (pI_{n2} - A_{22})^{-1} B_2 \quad (20b)$$

$$C_s(s) = C_1(s, 0) + C_2(0, 0) \quad (20c)$$

$$C_f(p) = C_2(p, 0) \quad (20d)$$

and $A_o = A_{11} - A_{12} A_{22}^{-1} A_{21}$, $B_o = B_1 - A_{12} A_{22}^{-1} B_2$, $C_o = C_1 - C_2 A_{22}^{-1} A_{21}$, $D_o = -C_2 A_{22}^{-1} B_2$. The hidden modes within P and C , and the lost modes arising from setting $\epsilon = 0$, are stable if the triples (A_o, B_o, C_o) and (A_{22}, B_2, C_2) are stabilizable-detectable.

Note that (20c) and (20d) imply

$$C_s(\infty) = C_f(0) \quad (21)$$

The compensator transfer function $C(s, \epsilon)$ associated with (16-18) can then be approximated as

$$C(s, \epsilon) \approx \hat{C}_s(s) + C_f(p) \quad (22)$$

where $\hat{C}_s(s) = C_s(s) - C_s(\infty)$. Comparison of (22) with (16-18) shows that

$$\hat{C}_s(s) = H_1^\circ (sI_{n1} - P_1^\circ + P_1 H_1^\circ) N_1 \quad (23a)$$

$$C_f(p) = H_2^\circ (pI_{n2} - P_2^\circ + P_2 H_2^\circ) N_2 \quad (23b)$$

The difficulty that arises here is that the expression for the slow poles given in (19a) is in terms of $C_s(s)$. This implies that $\hat{C}_s(s)$ should be designed for the reduced plant $P_s(s)$ with $C_f(0)$ as an inner loop feedback, as illustrated in Fig. 1, where from (23b)

$$C_f(0) = H_2^\circ (-P_2^\circ + P_2 H_2^\circ) N_2 \quad (24)$$

Thus the fast subsystem design should be performed first using the augmented system matrices:

$$\begin{aligned} \tilde{A}_2 &= \begin{bmatrix} A_{22} & -B_2 H_2^\circ \\ 0 & P_2^\circ \end{bmatrix} & \tilde{B}_2 &= \begin{bmatrix} 0 \\ I_{n_{c2}} \end{bmatrix} \\ \tilde{C}_2 &= \begin{bmatrix} C_2 & 0 \\ 0 & H_2^\circ \end{bmatrix} & G_2 &= \begin{bmatrix} N_2 & P_2 \end{bmatrix} \end{aligned} \quad (25)$$

which follow directly from (9,10), (20b) and (23b). This is followed by a slow subsystem design where the augmented system matrices, including the inner loop feedback through $C_f(0)$, have the following structures:

$$\begin{aligned} \tilde{A}_1 &= \begin{bmatrix} A_o & -B_o H_1^\circ \\ 0 & P_1^\circ \end{bmatrix} & \tilde{B}_1 &= \begin{bmatrix} 0 \\ I_{n_{c1}} \end{bmatrix} \\ \tilde{C}_1 &= \begin{bmatrix} C_o & -D_o H_1^\circ \\ 0 & H_1^\circ \end{bmatrix} & G_1 &= \begin{bmatrix} N_1 & P_1 \end{bmatrix} \end{aligned} \quad (26)$$

where $A_o' = A_o - B_o C_f(0) C_o$, $B_o' = B_o - C_f(0) D_o$.

SUMMARY

A formulation has been presented for designing output feedback compensators for two time scale systems. The formulation results in a design that avoids direct feedback of outputs to inputs, and minimizes the number of free parameters needed in the compensator representation.

REFERENCES

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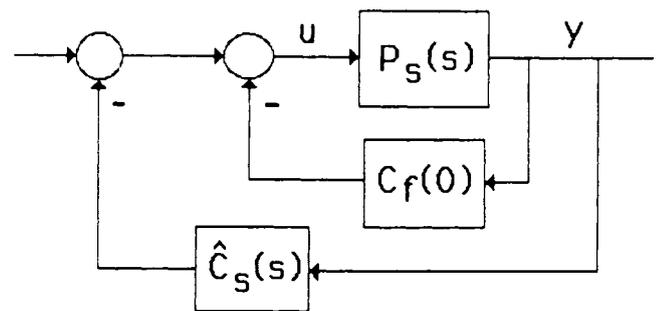


Fig. 1. Equivalent slow subsystem design loop.