

STABILIZATION OF UNCERTAIN SYSTEMS BY FIXED ORDER COMPENSATION

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ABSTRACT

The problem of optimal output feedback is addressed in the context of controlling uncertain systems. The objective is to control a possibly high order uncertain system with a compensator of low order. The compensator formulation precludes the use of direct feedback of the plant output, and uses an observer canonical form to represent the compensator dynamics. Similar to the case of observer design, a two step design procedure is proposed. First a robust design assuming full state feedback is performed, followed by the compensator design using an approximate loop transfer recovery method.

1. INTRODUCTION

Numerous methods have been proposed for designing a full state controller for stabilizing an uncertain system [1-2]. Approaches have also been outlined for the case of output feedback with minimal order observers [3]. One disadvantage to the use of a minimal order observer is that the controller structure makes use of direct feedback of the measurements. It is generally good practice to avoid having direct feedthrough of sensor outputs to improve robustness to unmodelled (high frequency) dynamics, and to reduce the effect of sensor noise. Aside from these issues, the order of the compensator when designed for large order systems may prove unwarranted. In this note we outline an approach for designing fixed order compensators for uncertain systems, without direct feedthrough of the measurements. A two step design procedure is proposed. First a robust design assuming full state feedback is performed, followed by the compensator design using an approximate loop transfer recovery method for fixed order compensators. The intent is to demonstrate that by recovering the loop transfer function of the full state design, we recover its robustness property as well.

II. PROBLEM FORMULATION

For simplicity, we confine our attention to uncertain systems having the form:

$$\dot{x} = [A + \delta A(r)]x + [B + \delta B(s)]u \quad (1,a)$$

$$y = Cx + Du \quad (1,b)$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$. The model uncertainty $r(t) \in R^k$ and input uncertainty $s(t) \in S^l$ lie in prescribed sets \bar{R} and \bar{S} . In addition, the following standard assumptions are made:

- A1. $\delta A(-)$ and $\delta B(-)$ are continuous.
- A2. \bar{R} and \bar{S} are compact.
- A3. $r(t)$ and $s(t)$ are Lebesgue measurable.
- A4. The nominal system (A,B,C) is controllable and observable.
- A5. There exists a constant matrix $K \in R^{m \times n}$ such that the system (1) is stabilizable with the control $u = -Kx$.

Of interest here are controllers having the following structure:

$$\dot{z} = Pz - Ny \quad (2,a)$$

$$u = -Hz \quad (2,b)$$

Thus, the nominal system dynamics are given by

$$\dot{x}_n = \begin{bmatrix} A & -BH \\ -NC & P+NDH \end{bmatrix} x_n = A_n x_n \quad (3)$$

Assuming that the choice of $\{P,N,H\}$ results in an asymptotically stable nominal system, then associated with (3) we have the Lyapunov equation

$$A_n^T M + M A_n + W = 0 \quad (4)$$

where any $W > 0$ results in a unique $M > 0$ satisfying (4). The closed loop uncertain system dynamics are

$$\dot{x}_n = [A_n + \delta A_n] x_n, \quad \delta A_n = \begin{bmatrix} \delta A & -\delta BH \\ 0 & 0 \end{bmatrix} \quad (5)$$

We can now state the following result:

Theorem: Let the system (1) satisfy A1-A5, and let $\{P,N,H\}$ render the nominal system (3) asymptotically stable. If

$$\eta(r,s) = -W + M \delta A_n(r,s) + \delta A_n^T(r,s) M < 0 \quad (6)$$

for all $r \in \bar{R}$, $s \in \bar{S}$, then $V = x_n^T M x_n$ is a Lyapunov function for the closed loop uncertain system (5).

Proof The time derivative of V along any trajectory of (5) is given by

$$\begin{aligned} \dot{V}(x_n, t) &= 2x_n^T M [A_n + \delta A_n] x_n \\ &= x_n^T (M A_n + A_n^T M) x_n + x_n^T (M \delta A_n + \delta A_n^T M) \\ &= x_n^T (-W + M \delta A_n + \delta A_n^T M) x_n = x_n^T \eta(r,s) x_n < 0 \quad \square \end{aligned}$$

III. DESIGN PROCEDURE

For the compensator design we adopt an observer canonical form, which yields a minimal parameterization for $\{P,N,H\}$. The design philosophy consists of designing a full state feedback gain matrix K which is robust to the specified uncertainties, followed by a compensator design which attempts to recover the loop transfer properties of the full state design.

Observer Canonical Form [4]

The compensator parameterization is given as follows:

$$u = -H^0 z \quad u \in R^m \quad (7)$$

$$\dot{z} = P^0 z + u_c \quad z \in R^n \quad (8)$$

$$u_c = Pu - Ny \quad u_c \in R^n \quad (9)$$

where

$$H^0 = \text{block diag}\{[0 \dots 0 \ 1]_{1 \times v_i} \quad i=1, \dots, m\} \quad (10)$$

$$P^0 = \text{block diag} [P_1^0, \dots, P_m^0] \quad (11)$$

$$P_i^0 = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} v_i x v_i \quad (12)$$

In (7-9) N and P are free parameter matrices with dimensions $(n_c \times p)$ and $(n_c \times m)$, respectively. The dimensions of H^0 and P^0 are defined by the observability indices of the compensator, which are chosen to satisfy:

$$i) \sum_{i=1}^m v_i = n_c \quad ii) v_i \leq v_{i+1}$$

The augmented system matrices:

$$\tilde{A} = \begin{bmatrix} A & -BH^0 \\ 0 & P^0 \end{bmatrix} \quad \tilde{B} = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix} \quad (13)$$

$$\tilde{C} = \begin{bmatrix} C & -DH^0 \\ 0 & H^0 \end{bmatrix} \quad G = [N \ P] \quad (14)$$

define an optimal output feedback problem, with the quadratic performance index:

$$J = E_{x_0} \left\{ \int_0^\infty [x^T Q x + u_c^T R u_c] dt \right\} \quad (15)$$

where the augmented state vector is

$$\tilde{x}^T = [x^T, z^T] \quad (16)$$

and the fictitious control is defined as

$$u_c = -G \tilde{C} \tilde{x} \quad (17)$$

Some of the advantages to the above compensator formulation are that are the compensator is represented by a minimum number of parameters, and these are compactly placed in the equivalent constant gain matrix in (14). Note that for this form, the matrix H needed to define δA_n in (6) is completely specified in (10).

The necessary conditions for optimality are:

$$A_n^T M + M A_n + W = 0, \quad W = Q + C^T G^T R G C \quad (18)$$

$$A_n L + L A_n^T + X_0 = 0, \quad X_0 = E\{x_0 x_0^T\} \quad (19)$$

$$R G C L^T - B M L C^T = 0 \quad (20)$$

Loop Transfer Recovery [5]

Breaking the loop at the plant input, the return signal in the case of full state feedback is $-Kx$. Referring to (7), the return signal in the case of fixed order compensation is $-H^0 z$. Thus, the objective in designing the compensator should be to minimize

$$J = E_{x_0} \left\{ \int_0^\infty [y_1^T y_1 + u_c^T u_c] dt \right\}, \quad y_1 = Kx - H^0 z \quad (21)$$

for a suitably chosen open loop input and for zero initial conditions. This leads to the following choice for the weighing matrices in (15):

$$Q = \begin{bmatrix} K^T K & -K^T H^0 \\ -H^0 K & H^0 T H^0 \end{bmatrix}, \quad R = \rho I_{nc} \quad (22)$$

Selecting the open loop input waveforms as impulses with magnitudes uniformly distributed on the unit sphere, results in the following expression for the equivalent distribution of initial conditions:

$$x_0 = \begin{bmatrix} I \\ BB^T & 0 \\ 0 & 0 \end{bmatrix} \quad (23)$$

needed in (19). A convergent numerical method for calculating G is given in Ref. 6.

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REFERENCES

1. Barmish, B.R., M. Corless, G. Leitmann, "A New Class of Stabilizing Controllers for Uncertain Dynamic Systems," *SIAM J. Cont. and Optim.*, Vol. 21, pp. 246-255, 1983.
2. Schmitendorf, W.E., "Design Methodology for Robust Stabilizing Controllers," *AIAA J. Guid., Cont. & Dynam.*, Vol. 10, pp. 250-254, 1987.
3. Schmitendorf, W.E., "Design of Observer-Based Robust Stabilizing Controllers," Northwestern U., Dept. of Engr. Sciences & Appl. Math., TR No. 8714, 1987.
4. Kramer, F., A.J. Calise, "Fixed Order Dynamic Compensation for Multivariable Linear Systems," *AIAA J. Guid., Cont., and Dynam.*, Vol. 11, pp. 80-85, 1988.
5. Calise, A.J., J. Prasad, "An Approximate Loop Transfer Recovery Method for Designing Fixed-Order Compensators," *AIAA Guid., Nav. and Cont. Conf.*, Minneapolis, MN, 1988.
6. Moerder, D.D., A.J. Calise, "Convergence of a Numerical Algorithm for Calculating Optimal Output Feedback Gains," *IEEE Trans. A.C.*, Vol. 30, pp. 900-903, 1985.