

An inverse kinematic solution algorithm for dexterous redundant manipulators

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ABSTRACT

Kinematically redundant manipulators are characterized by an increased degree of motion flexibility over conventional nonredundant structures. An index of this ability is given by the manipulability (or dexterity) measure, and the most dexterous configuration of the manipulator joints can be sought. Manipulability can be also characterized by the compatibility to the task the manipulator is required to execute. In this paper a solution algorithm to the inverse kinematic problem for dexterous redundant manipulators is proposed. The proper augmentation of the direct kinematics of the manipulator to include the constraints provided by the manipulability measures allows the derivation of the solution algorithm in the same formal way as for the unconstrained manipulator. A simple case study with a planar three-bar mechanism is finally discussed, and three different manipulability measures are respectively considered as the additional constraint to the problem.

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Throughout the text underlined letters denote vectors, capital letters denote matrices.

INTRODUCTION

A kinematically redundant manipulator can be defined as a manipulator that contains more than the minimum number of degrees of freedom (DOF's) needed to execute a certain task. Since the usual task of arbitrarily positioning and orienting the end effector in space requires six DOF's, a manipulator with seven or more DOF's will be termed kinematically redundant. The extra (redundant) DOF's result in greater flexibility and dexterity in the manipulator motion. In this case these redundant DOF's can be conveniently exploited to meet additional constraints on the manipulator motion [1].

A natural constraint is to require that the manipulator assumes configurations which are as "dexterous" as possible. To this purpose the human arm provides an excellent model of this ability; it has evenly distributed joint angles and a "natural" appearance. The manipulability measure has been proposed by Yoshikawa [2] as one quantitative measure of the easiness (dexterity) of arbitrarily changing the position and orientation of the end effector of the manipulator. The condition number of the Jacobian matrix has also been recommended as a dexterity measure for selecting the best working point for the manipulator [3].

Another attracting use of redundancy is to make the motion and strength characteristic of the manipulator compatible to the task requirements. An index for measuring the compatibility of the arm configuration with respect to manipulation tasks has been recently proposed by Chiu [4].

In this paper, the concept of the above manipulability measures is used to derive a solution algorithm to the inverse kinematic problem for constrained redundant manipulators. The algorithm is an extension of a general solution algorithm [5,6] which is based only on the computation of the direct kinematics of the manipulator. It is shown how the constraints given by the manipulability measures can be systematically incorporated into the solution, on the condition that a properly augmented direct kinematics is defined. The same approach has already been pursued to meet other constraints such as obstacle avoidance and limited joint range [7]. Finally, a simple planar three-bar mechanism is chosen to work out a case study, where the three different manipulability measures are respectively selected as providing the constraint to the inverse kinematic problem.

MANIPULABILITY MEASURES AND AUGMENTED KINEMATICS

Consider an n -DOF manipulator and assume that the position and orientation of its end effector is described by a set of m variables. Its direct kinematic equation can be established in a unique straightforward manner [8] as the relationship between the $(n \times 1)$ joint vector \underline{q} and the $(m \times 1)$ task vector \underline{x} ,

$$\underline{x} = \underline{f}(\underline{q}) \quad (1)$$

where \underline{f} is a continuous nonlinear function, whose structure and parameters are known. The kinematic equation (1) can be differentiated with respect to time, yielding the relationship between the joint velocity vector $\dot{\underline{q}}$ and the task velocity vector $\dot{\underline{x}}$, through the $(m \times n)$ Jacobian matrix $J(\underline{q}) = \partial \underline{f} / \partial \underline{q}$,

$$\dot{\underline{x}} = J(\underline{q}) \dot{\underline{q}}. \quad (2)$$

For a kinematically redundant manipulator it is $m < n$. It can be assumed that the following condition holds

$$\max_{\underline{q}} \text{rank } J(\underline{q}) = m \quad (3)$$

and $(n - m)$ redundant DOF's will be available. If for some $\bar{\underline{q}}$,

$$\text{rank } J(\bar{\underline{q}}) < m \quad (4)$$

the manipulator is said to be at a singular configuration. In this state the manipulator loses its ability to move along or rotate about some direction of the space, meaning that its manipulability is reduced.

Following Chiu's effective formulation [4] which views the manipulator as a mechanical transformer with joint velocity and force as input and Cartesian velocity and force as output, the velocity and force transmission characteristics of a manipulator at a given configuration can be geometrically represented as ellipsoids. From (1), it can be seen that the unit sphere in R^n defined by

$$\underline{\dot{\theta}}^T \underline{\dot{\theta}} \leq 1 \quad (5)$$

is mapped into the so-called velocity ellipsoid in R^m defined by

$$\underline{\dot{x}}^T (JJ^T)^{-1} \underline{\dot{x}} \leq 1. \quad (6)$$

Analogous to the velocity ellipsoid, an ellipsoid for describing the force transmission characteristics of a manipulator at a given configuration can be defined. Forces in joint space and task space are mapped via the same Jacobian defined in (2) through the relation

$$\underline{\tau} = J^T(\underline{q}) \underline{f} \quad (7)$$

where \underline{f} is the force vector in the task space and $\underline{\tau}$ is the joint torque vector. Thus the unit sphere in R^n

$$\underline{\tau}^T \underline{\tau} \leq 1 \quad (8)$$

maps into the so-called force ellipsoid in R^m

$$\underline{f}^T (JJ^T) \underline{f} \leq 1. \quad (9)$$

It can be shown that the principal axes (eigenvectors) of the velocity and force ellipsoids coincide, whereas the lengths of the axes (eigenvalues) are in inverse proportions [4].

The volume of the velocity ellipsoid (6) has been used as an effective means for singularity avoidance, see eq. (4). Yoshikawa [2] first introduced the concept of manipulability measure at configuration \underline{q} as the scalar quantity

$$w(\underline{q}) = [\det J(\underline{q})J^T(\underline{q})]^{\frac{1}{2}} \quad (10)$$

which is proportional to the volume of the ellipsoid defined by (6). If $m = n$, that is the manipulator is nonredundant, the measure w reduces to

$$w(\underline{q}) = |\det J(\underline{q})| \quad (11)$$

which is the kind of measure used in [9] for analysis of robot wrists. At a singular configuration $\underline{\bar{q}}$ it is obviously

$$w(\underline{\bar{q}}) = 0. \quad (12)$$

The force ellipsoid is at the basis of the condition number reported by Klein [1]

$$\kappa(\underline{q}) = \frac{\Lambda[J(\underline{q})J^T(\underline{q})]}{\lambda[J(\underline{q})J^T(\underline{q})]} \quad (13)$$

where Λ and λ respectively denote the largest and the smallest eigenvalues of the matrix JJ^T . While a determinant going to zero marks the presence of a singularity as in (12), the actual value of the determinant is not a practical measure of the degree of ill-conditioning. Instead the condition number gives a measure of closeness of the force

ellipsoid to a sphere. If $m = n$, the condition number κ reduces to

$$\kappa(\underline{q}) = \frac{\Lambda[J(\underline{q})]}{\lambda[J(\underline{q})]} \quad (14)$$

which is the kind of measure adopted by Salisbury and Craig [3] to select the best working point for a nonredundant manipulator. At a configuration $\bar{\underline{q}}$ where

$$\kappa(\bar{\underline{q}}) = 1 \quad (15)$$

the manipulator will exert task space forces of equal magnitude in all directions (isotropic configuration). Nevertheless, on the basis of the above duality between the force and the velocity ellipsoids, an isotropic configuration will also present task space velocities of equal magnitude in all directions.

The previous two measures serve as quantitative tools to characterize quite the "natural" (dexterous) configuration of a manipulator independently of the specific task it is required to perform in terms of motion and force. Rather different indices have been only recently proposed by Chiu [4], which measure the compatibility of a manipulator configuration with respect to "fine" manipulation tasks, where accurate control of small velocity and force is required, and "coarse" manipulation tasks, where exertion of large velocity and force is required [10].

Based upon the velocity and force ellipsoids respectively defined by eqs. (6) and (9), the force (velocity) transmission ratio along a particular direction can be defined as the distance from the center to the surface of the force (velocity) ellipsoid along the directional vector. Let \underline{u} (\underline{v}) denote the unit vector in the direction of interest, and let α (β) be the distance along the vector \underline{u} (\underline{v}) from the origin to the surface of the force (velocity) ellipsoid. The scalar α (β) is the force (velocity) transmission ratio in the direction of \underline{u} (\underline{v}). Hence, it is

$$\alpha(\underline{q}) = [\underline{u}^T (J(\underline{q}) J^T(\underline{q})) \underline{u}]^{-\frac{1}{2}} \quad (16)$$

and

$$\beta(\underline{q}) = [\underline{v}^T (J(\underline{q}) J^T(\underline{q}))^{-1} \underline{v}]^{-\frac{1}{2}}. \quad (17)$$

In the light of the duality discussed above, it can be stated that the best direction for effecting velocity (maximum β) is also the best direction for controlling force (minimum α). Similarly, the best direction for effecting force (maximum α) is also the best direction for controlling velocity (minimum β). Chiu [4] suggested to combine the ratios defined in (16) and (17) into one compatibility index $c = (1/\alpha\beta)$ as a quantitative measure of good control (exertion) compatibility in the respective directions. This places equal importance on all task directions, whereas the requirement may be more demanding in one direction than in another.

On the basis of the definition of the manipulability measures given in (10), (13), (16) and (17), it is possible to define an augmented direct kinematics for a redundant manipulator as

$$\underline{y}(\underline{q}) = \begin{bmatrix} \underline{x}(\underline{q}) \\ \underline{z}(\underline{q}) \end{bmatrix} \quad (18)$$

where $\underline{y}(\underline{q})$ is the augmented $((m+r) \times 1)$ task space vector, formed by \underline{x} as given in (1) and by an $(r \times 1)$ vector \underline{z} whose components are any of the scalar manipulability measures introduced above. Obviously the problem is well posed if

$$r \leq n - m \quad (19)$$

so as to cover at most all the redundant DOF's. Correspondingly, the relation (2) allows

the definition of an augmented Jacobian matrix as

$$J_y(\underline{q}) = \begin{bmatrix} J(\underline{q}) \\ J_z(\underline{q}) \end{bmatrix} \quad (20)$$

where $J_z(\underline{q})$ is the $(v \times n)$ matrix obtained as $\partial z / \partial \underline{q}$. It is worth reporting here that a similar approach based on kinematics augmentation has also been followed in [11,12].

THE INVERSE KINEMATIC SOLUTION ALGORITHM

The crucial point for robot manipulator analysis and control is the capability of mapping the task space vector \underline{x} into the joint space vector \underline{q} , that is solving the kinematic equation (1). In case of redundant manipulators, the redundant DOF's can be adequately used to meet additional constraints on the manipulator motion, such as obstacle avoidance, limited joint range and dexterity.

The most common approach followed in the literature is based on the use of the pseudoinverse of the Jacobian, in connection with the mapping (2). It can be shown that the general solution to (2) is given by

$$\dot{\underline{q}} = J^\dagger \dot{\underline{x}} + (I - J^\dagger J) \underline{h} \quad (21)$$

where J^\dagger is the $(n \times m)$ Moore-Penrose pseudoinverse matrix defined as $J^\dagger = J^T (JJ^T)^{-1}$, I is the $(n \times n)$ identity matrix and \underline{h} is an $(n \times 1)$ arbitrary vector. It can be noted that the solution (21) composes of the least-square solution term of minimum norm [13] plus a homogeneous solution term created by the projection operator $(I - J^\dagger J)$ which selects the components of \underline{h} in the null space of the mapping J . Therefore the vector \underline{h} can be used to optimize some additional criterion, such as limited joint range [14], obstacle avoidance [15,16,17] to reference only a few. An interesting solution with singularity robustness has been more recently proposed in [18].

A rather different approach to the inverse kinematic problem is based on a recently proposed solution algorithm which only requires the computation of direct kinematic functions [5,6]. The technique is briefly summarized in the following.

Let $\hat{\underline{q}}$ be a solution to (1) relative to a given end effector location $\hat{\underline{x}}$ specified in the task space. A task space error vector \underline{e} can be defined between the reference task vector $\hat{\underline{x}}$ and the actual task vector \underline{x} obtained from the current joint vector \underline{q} ,

$$\underline{e} = \hat{\underline{x}} - \underline{x}. \quad (22)$$

Differentiating with respect to time yields

$$\dot{\underline{e}} = \dot{\hat{\underline{x}}} - J(\underline{q}) \dot{\underline{q}}. \quad (23)$$

It can be proved [19] that the choice

$$\dot{\underline{q}} = KJ^T \underline{e} \quad (24)$$

assures that the tracking error \underline{e} is bounded if $\dot{\hat{\underline{x}}} \neq 0$ and is zero if $\dot{\hat{\underline{x}}} = 0$, by means of a proper selection of the positive definite feedback gain matrix K . The resulting closed loop dynamic scheme is illustrated in Fig. 1. As anticipated above, the key feature of this technique is the sole computation of direct kinematic functions (\underline{f} and J). It can be remarked also that the joint velocity vector $\dot{\underline{q}}$ is inherently generated by the solution algorithm; this may turn advantageous for control purposes. A more detailed discussion on the characteristics of the solution algorithm of Fig. 1 and several application examples are referred to [7,19,20].

The solution (24) apparently serves as inverse kinematic solution for a general "unconstrained" redundant manipulator. The versatility that is inherent in a redundant

manipulator can be naturally exploited by imposing any of the manipulability measures proposed in the previous section as constraint(s) to the solution. Based upon the direct kinematics augmentation introduced above, it is reasonable indeed to define a reference vector \hat{z} for the extra task space variables introduced in (18). Correspondingly, the error vector in the task space is defined by

$$\underline{e}_y = \hat{y} - y \quad (25)$$

and the inverse kinematic solution for the "constrained" redundant manipulator is given by

$$\dot{q} = \kappa_y J_y^T \underline{e}_y \quad (26)$$

which, besides assuring a bounded end effector tracking error, keeps the manipulator in a dexterous configuration, whose manipulability indices are set up by the components of the reference vector \hat{z} .

A CASE STUDY

The planar three-bar mechanism of Fig. 2 is analyzed in the following to develop a case study. The direct kinematics (1) is given by

$$p = \begin{bmatrix} p_x \\ p_y \end{bmatrix} = \begin{bmatrix} l_1 s_1 + l_2 s_{12} + l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} \end{bmatrix} \quad (27)$$

where $s_{ij..} = \sin(\theta_i + \theta_j + \dots)$ and $c_{ij..} = \cos(\theta_i + \theta_j + \dots)$. The Jacobian matrix J in (2) is given by

$$J = \begin{bmatrix} p_y & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ -p_x & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \end{bmatrix} \quad (28)$$

The matrix JJ^T which constitutes the key for any manipulability measure is symbolically given by

$$JJ^T = B = \begin{bmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \quad (29)$$

where the expressions of its elements have been omitted for brevity.

It can be found that the manipulability measure w in (10) is given by

$$w = [l_1^2(l_2 s_2 + l_3 s_{23})^2 + l_3^2(l_1 s_{23} + l_2 s_3)^2 + l_2^2 l_3^2 s_3^2]^{\frac{1}{2}} \quad (30)$$

Then it follows that singularities occur when the three links are aligned, $\theta_2 = 0, 180$ and $\theta_3 = 0, 180$ (four distinct configurations at which $w = 0$).

As far as the condition number κ in (13), it can be observed that a measure of the difference between the two eigenvalues of the matrix B in (29) is directly given by the discriminant of the associated characteristic equation, i.e.

$$\Delta = [(b_{11} - b_{22})^2 + 4b_{12}^2]^{\frac{1}{2}} \quad (31)$$

Lastly, the task compatibility indices in (16) and (17) depend upon the directions \underline{u} and \underline{v} of the manipulation task.

In the following, it is assumed that the arm is in a given configuration and a desired value is assigned to each manipulability measure. Each task consists in maintaining the given end point position while forcing the manipulability measure to the desired value.

The link lengths have been chosen as $l_1 = 1.0$, $l_2 = 1.0$, $l_3 = 0.5$.

For the first measure in (30) the arm is purposely placed in a singular configuration $q^T = (45 \ 0 \ 180)$, at which it is obviously $w = 0$. It is required that $w = 1.0$. The final configuration of the arm is shown in Fig. 3, along with the manipulability measure as a function of time. The final configuration is more dexterous from the point of view of singularity avoidance, since it corresponds to having a larger volume of the force ellipsoid defined in (9).

The arm is then placed in the configuration $q^T = (160 \ 90 \ -90)$, at which the discriminant in (31) results $\Delta = 3.324$. It is required that $\Delta = 3.0$. The final configuration of the arm and the discriminant as a function of time are shown in Fig. 4. The final configuration is characterized by having an associated force ellipsoid which is closer to a sphere than it was the ellipsoid associated with the initial configuration.

Finally, it is assumed that a vertical force ($u^T = (0 \ 1)$) and a horizontal velocity ($v^T = (1 \ 0)$) are wished to control. This implies that the transmission ratios defined in (16) and (17) are respectively $\alpha = b_{22}^{-1}$ and $\beta = b_{22}^{-1}w$, where b_{22} and w are the same as in (29) and (30). The task compatibility index has then been chosen as $c = 1/\alpha\beta = b_{22}/w$ [4]. The initial configuration of the arm is the same as in the previous example, where it results $c = 2.754$. It is required that $c = 3.0$. The final configuration and the compatibility index are shown in Fig. 5. It can be recognized that the arm posture for fine control of vertical force and horizontal velocity resembles that of the human arm during writing. It is interesting also to notice that the final configuration in this case is less "articulated" than the final configuration in the previous example. Requiring fine control in the assigned directions, in fact, corresponds to having unbalanced lengths of the principal axes of the ellipsoids (large condition number), whereas in the previous case the goal is to obtain closer eigenvalues (balanced lengths of the principal axes) and then a best working point for exerting forces of proximal magnitude in all directions, independently, however, of the particular task it is required to perform.

CONCLUDING REMARKS

This paper has presented a solution algorithm to the inverse kinematic problem for redundant manipulators under the constraint that the arm assumes a configuration that is as dexterous as possible. Three different manipulability measures have been considered as quantitative indices of the inherent dexterity associated with a redundant manipulator. The first two of them characterize the "natural" appearance of the arm, whereas the third one is purposely thought of as matching the motion and force task direction requirements. The algorithm has been then derived by properly augmenting the direct kinematics of the manipulator and adopting a recently derived technique which is based on the sole computation of the direct kinematics. A case study for a planar three-bar mechanism has been finally developed.

Here it is important to conclude with emphasizing that the augmented kinematics approach has been applied with the intent to satisfy some desired values of the manipulability measures proposed. It is not fully understood, however, how to select those values. Other simulation results not reported here, in fact, have shown that the solution algorithm in the present form does not always guarantee that the manipulability measure of interest is taken to its extremum (i.e. maximum volume, smallest condition number, etc.). This is primarily due to the fact that the augmented Jacobian J_y in (20) will present not only the same singularities as those of the end effector Jacobian J , but also other singularities, due to the addition of the r rows. This issue has been raised also by Baillieul in [11]. In other words, if the augmented error e_y happens to span the null space of J_y , the solution get stuck and no further improvement of the manipulability measure is possible, along with the fact that a position error may occur. This certainly represents a challenging point for future research.

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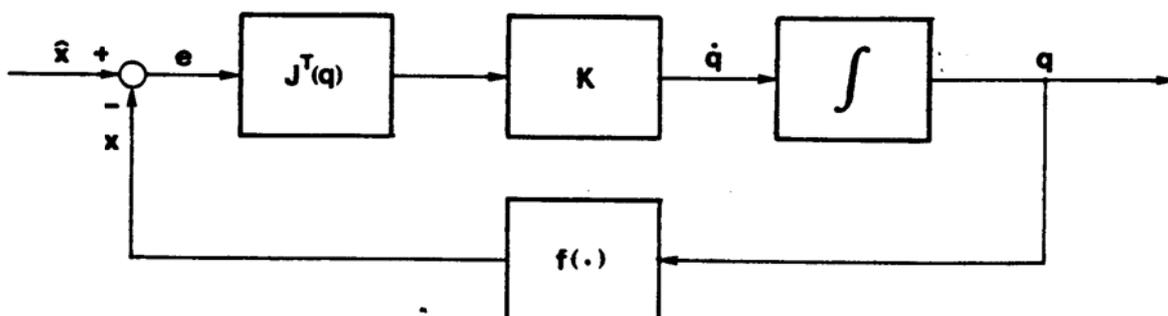


Figure 1. The inverse kinematic solution algorithm.

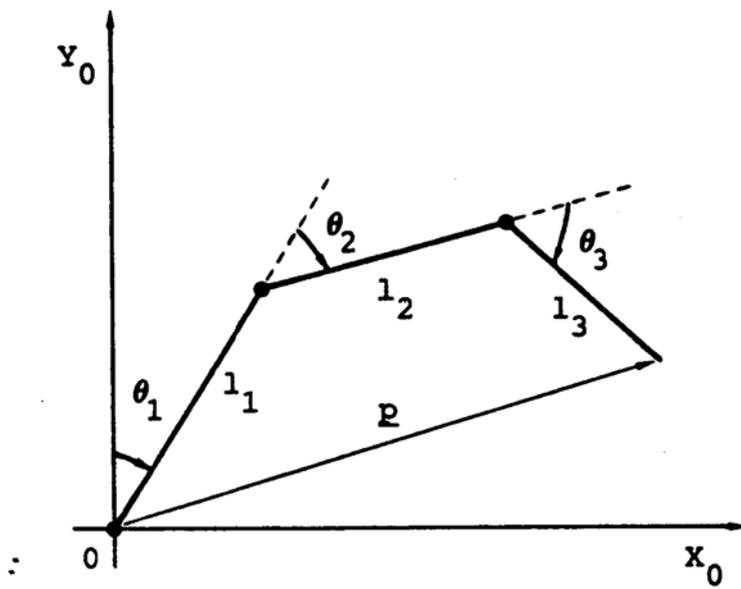


Figure 2. The planar three bar mechanism.

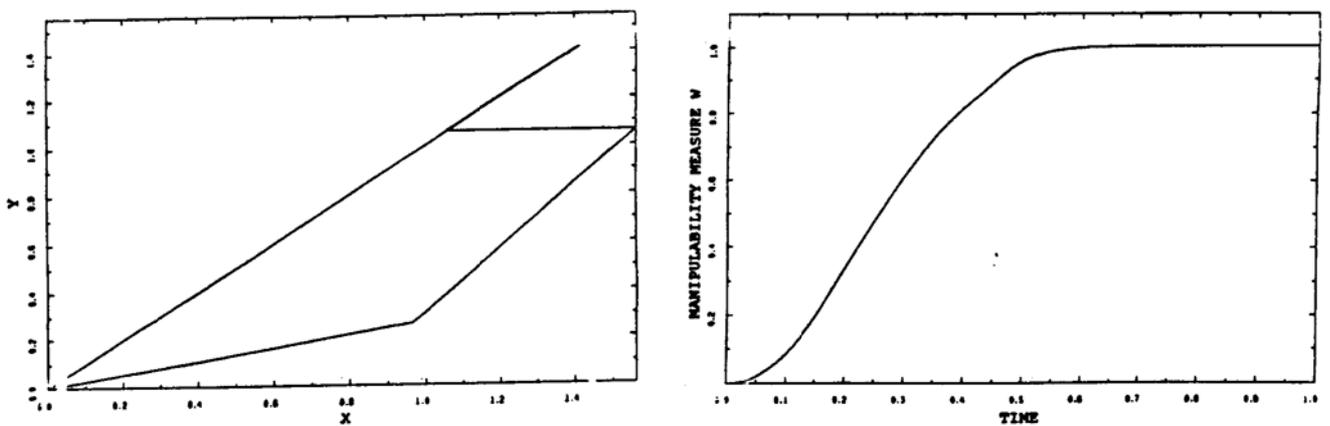


Figure 3. Arm configurations and manipulability measure w .

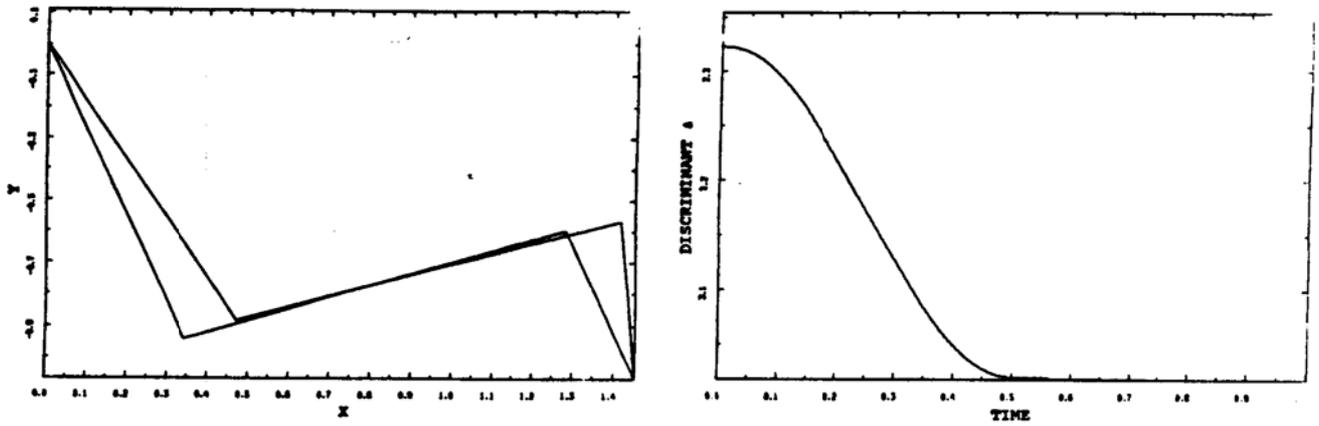


Figure 4. Arm configurations and discriminant Δ .

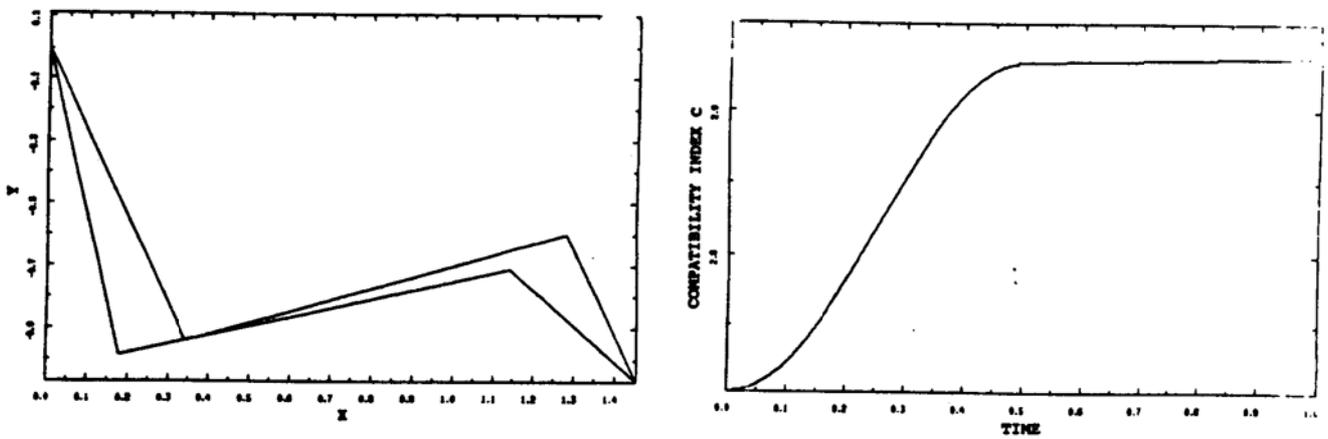


Figure 5. Arm configurations and compatibility index c .