A MULTILAYER APPROACH TO CONTROL OF A FLEXIBLE ARM

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ABSTRACT

The control of a flexible arm is the focus of this work. The dynamic model is obtained via an assumed mode method, and a singularly perturbed model of the system is then derived. The new feature of this model is that multiple boundary layers are considered, one for each assumed mode. A composite control strategy is then pursued. First a slow control is designed for the rigid system, then a number of fast controls (one for each layer) make the fast dynamics in the layers suitably damped. A case study is finally worked out and simulation results are presented.

INTRODUCTION

Lightweight flexible arms have been recently introduced with the goal of achieving benefits like lower arm cost, higher motion speed, better energy efficiency, improved mobility etc. [1]. Control certainly represents one key to an effective use of flexible arms. One successful way of obtaining a dynamic model for a flexible arm is based on an assumed mode method [2]. The much more complicated dynamics associated with a flexible arm, however, hardly complicates the control problem; an extended number of state variables is to be handled for control purposes. Some of the research efforts produced so far can be found in [2,3,4].

In [5] a singular perturbation approach has been first pursued to derive a new model for the system so that a composite control strategy can be applied. The same idea is also at the basis of this work. Here, the new feature is to obtain a singularly perturbed model with multiple boundary layers [6], each of them obtained in correspondence of each assumed mode. The inherent advantage with this choice is that a number of scalar systems is to be controlled. A composite control strategy is again adopted for the multilayer system. It is shown, also, that a state variable filter is used in order to reconstruct higher order derivatives of joint variables, which are needed to synthesize the fast controls.

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It has to be mentioned, however, that full state availability on the flexible variables is assumed for fast control design. A case study is finally developed and extensive simulation results are provided.

THE DYNAMIC MODEL

The flexible arm of fig. 1 is considered in this work. A solution to the flexible motion of the link can be obtained through modal analysis, under the assumption of small deflections of the link,

$$y(\eta,t) = \sum_{1}^{m}{}_{i}\ \delta_i(t)\phi_i(\eta) \tag{1}$$

where ϕ_i is the eigenfunction expressing the displacement of assumed mode i of link deflection, δ_i is the time-varying amplitude of mode i of the link and m is the number of modes used to describe the deflection of the link.

If the vibrating beam is clamped at the joint end and free ad the distal end, the orthonormal modal eigenfunctions in (1) are given by

$$\phi_i(\xi) = \sin(\beta_i\xi) - \sinh(\beta_i\xi) + \nu_i(\cos(\beta_i\xi) - \cosh(\beta_i\xi)) \tag{2}$$

$$\nu_i = \frac{\sin\beta_i + \sinh\beta_i}{\cos\beta_i + \cosh\beta_i} \qquad i = 1,\ldots,m$$

$$\beta_i^4 = \frac{\rho A(2\pi f_i)^2 L^4}{EI}$$

where:

L = beam length
A = beam cross area
E = Young's modulus
I = beam area inertia
ρ = density
f_i = frequency of ith mode.

The dynamic equations of motion for the one link flexible arm can be written in the following form [7]

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$$M(\theta,\underline{\delta})\begin{bmatrix}\ddot{\theta}\\ \hline \ddot{\underline{\delta}}\end{bmatrix}+\begin{bmatrix}f(\dot{\theta},\underline{\delta},\dot{\underline{\delta}})\\ \hline \underline{g}(\dot{\theta},\underline{\delta})\end{bmatrix}+\begin{bmatrix}0\\ \hline K\underline{\delta}\end{bmatrix}=\begin{bmatrix}u\\ \hline 0\end{bmatrix} \quad (3)$$

where

θ is the joint variable,
$\underline{\delta} = (\delta_1 \ldots \delta_m)^T$ is the vector of deflections,
$\underline{u}$ is the control torque at joint location,
M is the inertia matrix

$$M = \begin{bmatrix} m_{11} & \cdots & m_{1p} & \cdots & & m_{1,m+1} \\ m_{1p} & \cdots & m_{pp} & m_{pq} & & \cdot \\ \cdot & \cdots & m_{pq} & \cdot & \cdot & \cdot \\ m_{1,m+1} & \cdots & \cdot & \cdot & \cdot & m_{m+1,m+1} \end{bmatrix} \quad (4)$$

$$m_{11} = J_0 + M_L L^2 + I_0 + M_L(\underline{\phi}_e^T\underline{\delta})^2$$

$$m_{1p} = M_L L\phi_{p-1,e} + w_{p-1} \qquad p = 2,\ldots,m+1$$

$$m_{pp} = m_b + M_L\phi_{p-1,e}^2 + J_p\phi'^2_{p-1,e}$$

$$m_{pq} = M_L\phi_{p-1,e}\phi_{q-1,e} + J_p\phi'_{p-1,e}\phi'_{q-1,e} \qquad q = p+1,\ldots,m+1$$

with

$$\underline{\phi}_e^T = (\phi_{1e} \ldots \phi_{me}), \quad \phi_{ie} = \phi_i(\xi)\Big|_{\xi=1}$$

$$\underline{\phi}'^T_e = (\phi'_{1e} \ldots \phi'_{me}), \quad \phi'_{ie} = \frac{d\phi_i(\xi)}{d\xi}\Big|_{\xi=1} \quad i = 1,\ldots,m$$

$$w_i = \rho AL^2\int_0^1 \phi_i(\xi)\xi d\xi$$

where:

m_b = beam mass
M_L = payload mass
I_0 = joint inertia
J_0 = beam inertia relative to joint
J_p = payload inertia,

f and $\underline{g}$ are nonlinear terms

$$f = 2M_L\dot{\theta}(\underline{\phi}_e^T\underline{\delta})(\underline{\phi}_e^T\dot{\underline{\delta}}) \quad (5)$$

$$\underline{g} = -M_L\dot{\theta}^2(\underline{\phi}_e\underline{\phi}_e^T)\underline{\delta}, \quad (6)$$

K is an equivalent spring constant matrix

$$K = \mathrm{diag}(k_1 \ldots k_m) \quad (7)$$

$$k_i = \frac{EI}{L^3}\int_0^1\left[\frac{d^2\phi_i(\xi)}{d\xi^2}\right]^2 d\xi.$$

Since the clamped-free assumption has been made for the vibrating beam, there is no displacement at joint location and then no control force in the lower eqs. of (3).

A SINGULARLY PERTURBED MODEL

In the following a singularly perturbed model for the system (3)-(7) will be derived. Being the inertia matrix (4) positive definite, it can be inverted and partitioned as follows:

$$M^{-1} = \begin{bmatrix} s & \underline{t}^T \\ \hline \underline{t} & H \end{bmatrix} \quad (8)$$

$$H = [\,\underline{h}_1 \ldots \underline{h}_m\,] = \begin{bmatrix}\underline{h}_1^T\\ \cdot\\ \cdot\\ \underline{h}_m^T\end{bmatrix}$$

$$\underline{t}^T = (t_1 \ldots t_m)$$

$$\underline{h}_i^T = (h_{i1} \ldots h_{im}) \qquad i = 1,\ldots,m$$

Eqs. (3) then become

$$\ddot{\theta} = -sf - \underline{t}^T\underline{g} - \underline{t}^T K\underline{\delta} + su \quad (9)$$

$$\ddot{\delta}_i = -t_i f - \underline{h}_i^T\underline{g} - \underline{h}_i^T K\underline{\delta} + t_i u \quad i = 1,\ldots,m \quad (10)$$

The system (9)-(10) will be transformed now to a singularly perturbed form. One difficulty in performing this transformation is the proper choice of fast and slow variables, and the selection of one (or more) perturbation parameters. In [7] the ratio EI/L^3 in (7) was regarded as the inverse of the perturbation parameter; that choice, however, does not truly match the physical nature of the subsystem (10), since one parameter is picked for all the modes. Therefore, a more realistic approach seems to introduce m perturbation parameters for the m assumed modes as $\varepsilon_i = 1/k_i$ (i=1,...,m), performing thus a multi-time scaling with $\varepsilon_1,\ldots,\varepsilon_m$. Such doing leads not only to a boundary layer, but also to a sublayer, a subsublayer and so forth. A further advantage is gained by this choice; m scalar problems are obtained which can be solved in an explicit feedback form, thus simplifying the implementation of the control law [6]. Defining then

$$\underline{z} := K\underline{\delta} = \sum_1^m{}_i \frac{1}{\varepsilon_i}\delta_i \quad (11)$$

$$\underline{\delta} = E\underline{z} \qquad E = K^{-1} = \mathrm{diag}(\varepsilon_1 \ldots \varepsilon_m)$$

yields the eqs. of the system in singularly perturbed form, i.e.

$$\ddot{\theta} = -s(E\underline{z})f(\dot{\theta},E\underline{z},E\dot{\underline{z}}) - \underline{t}^T(E\underline{z})\underline{g}(\dot{\theta},E\underline{z}) + -\underline{t}^T(E\underline{z})\underline{z} + s(E\underline{z})u \quad (12)$$

$$\varepsilon_i\ddot{z}_i = -t_i(E\underline{z})f(\dot{\theta},E\underline{z},E\dot{\underline{z}}) - \underline{h}_i^T(E\underline{z})\underline{g}(\dot{\theta},E\underline{z}) + -\underline{h}_i^T(E\underline{z})\underline{z} + t_i(E\underline{z})u \qquad i = 1,\ldots,m \quad (13)$$

In the following it is reasonable to assume that the contributions of the deflections to the inertia matrix M in (4) are negligible, i.e. $m_{11}(E\underline{z}) = \bar{m}_{11}$. As a consequence, the quantities in (8) will not be functions of the fast variables z_i [5].

At this point the typical steps of singular

perturbation formulation can be taken. Starting from the bottom of (13), because of the presence of ε_m, the last system of (13) exhibits a boundary layer phenomenon in the fast variable z_m. If the boundary layer decays, then the dynamics of θ and $\underline{z}$ will vary slowly. Formally setting $\varepsilon_m = 0$ accomplishes a model order reduction from m + 1 to m, because the last differential equation in (13) degenerates into the algebraic transcendental equation

$$0 = -t_m f(\dot{\bar{\theta}}, \sum_1^{m-1} {}_i\varepsilon_i\bar{z}_i, \sum_1^{m-1} {}_i\varepsilon_i\dot{\bar{z}}_i) + \tag{14}$$

$$-\underline{h}_m^T \underline{g}(\dot{\bar{\theta}}, \sum_1^{m-1} {}_i\varepsilon_i\bar{z}_i) - \sum_1^{m-1} {}_i h_{mi}\bar{z}_i - h_{mm}\bar{z}_m + t_m\bar{u}_m$$

where the bar is used to indicate that the variables belong to the so-called slow subsystem with $\varepsilon_m = 0$. Being H in (8) positive definite, $h_{mm} > 0$; therefore it is possible to find the distinct quasi steady-state solution $\bar{z}_m$ to (14). Plugging this solution into the upper eqs. of (13) formally yields the slow subsystem or the reduced order subsystem

$$\ddot{\bar{\theta}} = -sf(\dot{\bar{\theta}}, \sum_1^{m-1} {}_i\varepsilon_i\bar{z}_i, \sum_1^{m-1} {}_i\varepsilon_i\dot{\bar{z}}_i) + \tag{15}$$

$$-\underline{t}^T \underline{g}(\dot{\bar{\theta}}, \sum_1^{m-1} {}_i\varepsilon_i\bar{z}_i) - \sum_1^{m-1} {}_i t_i\bar{z}_i - t_m\bar{z}_m + s\bar{u}_m$$

To derive the fast subsystem or the boundary layer system, it is assumed that the slow variables are constant in the boundary layer; that is $\dot{\bar{\theta}} = 0$ and $\dot{\underline{z}} = \underline{0}$. Operating the fast state variable change around the equilibrium trajectory $z_{fm} = z_m - \bar{z}_m$, and correspondingly $u_{fm} = u - \bar{u}_m$, the fast subsystem of (13) results ($f = 0$ and $\underline{g} = \underline{0}$ on the boundary layer)

$$\varepsilon_m\ddot{z}_{fm} = -h_{mm}z_{fm} + t_m u_{fm} \tag{16a}$$

that is

$$\frac{d^2 z_{fm}}{d\tau_m^2} = -h_{mm}z_{fm} + t_m u_{fm} \tag{16b}$$

where $\tau_m = t/\varepsilon_m$ is the fast time scale.

It should be clear now that the above procedure can be iterated for the intermediate layers up to obtain the slowest scalar subsystem; it can be checked that the slowest subsystem is right the rigid system, i.e.

$$\ddot{\bar{\theta}} = \frac{1}{\bar{m}_{11}}\bar{u}_1 \tag{17}$$

where it is understood that $\bar{u}_1$ is the slowest control obtained via

$$u = \bar{u}_1 + \sum_1^m {}_i u_{fi} \tag{18}$$

As evidenced by the rigid system (17) and the m boundary layers of the kind (16), the goal now is to design the slow control $\bar{u}_1$, and then design the fast controls u_{fi} which damp out the fast dynamics of the boundary layers.

COMPOSITE CONTROL

Under the results of the previous section, the design of a feedback control u for the full system (12)-(13) can be divided into m + 1 separate designs of feedback controls $\bar{u}_1$ and u_{fi} $i = 1,\ldots,m$ for the m + 1 scalar systems, namely a multi-time scale composite control [6] as in (18), with the constraints that $u_{fi}(z_{fi}=0) = 0$ such that u_{fi} are inactive along the solutions z_i, $i = 1,\ldots,m$.

As far as the slow control is concerned, if an angular trajectory $\hat{\theta}(t)$ is to be tracked, an obvious choice for $\bar{u}_1$ would be

$$\bar{u}_1 = m_{11}v \tag{19}$$

$$v = \ddot{\hat{\theta}} + k_v(\dot{\hat{\theta}} - \dot{\theta}) + k_p(\hat{\theta} - \theta)$$

where k_p and k_v are position and velocity gains. The tracking control (19), however, is not feasible for fast controls design. More specifically, from (16b) the equation of the i-th boundary layer results

$$\frac{d^2 z_{fi}}{d\tau_i^2} = -h_{ii}z_{fi} + t_i u_{fi}. \tag{20}$$

The control u_{fi} can be thought of as

$$u_{fi} = \frac{1}{t_i}(h_{ii}z_{fi} + v_{fi}) \tag{21}$$

where v_{fi} is the new input to the system given by

$$v_{fi} = k_{pfi}z_{fi} + \sqrt{\varepsilon_i}k_{vfi}\dot{z}_{fi} \tag{22}$$

where k_{pfi} and k_{vfi} are feedback gains to be designed in order to achieve a suitably damped fast dynamics. Here, the first crucial assumption to make is that full state availability is assumed as regards the fast variables. In particular, the deflections δ_i can be obtained from strain gage measurements [8], whereas their derivatives need to be reconstructed; this is an open research issue that will soon be investigated.

In order to evaluate $\dot{z}_{fi}$, however, $\dot{\bar{z}}_i$ is also needed and, via (14), the first order derivative of the slow control $\bar{u}_1$ needs to be evaluated. Therefore, if the control $\bar{u}_1$ would be chosen as in (19), also $\ddot{\bar{\theta}}$ would be needed. In order to overcome this drawback, a second order state variable filter needs to be placed at the output of the rigid system. In terms of the Laplace variable s, the rigid system (17) under a control $\bar{u}_1 = m_{11}v$, results

$$s^2\theta = v. \tag{23}$$

Let

$$\hat{A}(s)\theta = \hat{u} \qquad \hat{A}(s) = \sum_{0}^{2}{}_i\, a_i s^i, \qquad a_2 = 1 \quad (24)$$

be an asymptotically stable reference model for the system (23), where $\hat{u}$ is the input to the model which allows to track the trajectory specified by $\hat{\theta}$. The goal of the design is to achieve output perfect model following, when a state variable filter is placed at the output of (23). Let then

$$\frac{1}{F(s)} = \frac{1}{\sum_{0}^{2}{}_i\, f_i s^i}, \qquad f_2 = 1 \quad (25)$$

be the transfer function of a second order asymptotically stable state variable filter. Perfect model following will be achieved if the transfer function $\theta(s)/\hat{u}(s)$ of the system plus the controller is equal to that of the reference model $1/\hat{A}(s)$. To this purpose the controller configuration of fig. 2 can be considered, where

$$G(s) = \sum_{0}^{2}{}_i\, g_i s^i, \qquad g_2 = 1 \quad (26)$$

$$D(s) = \sum_{0}^{1}{}_i\, d_i s^i. \quad (27)$$

The design goal is then reformulated as that to find the coefficients in (26)-(27) such that

$$\frac{\theta(s)}{\hat{u}(s)} = \frac{G(s)}{F(s)s^2 + D(s)} = \frac{1}{\hat{A}(s)}. \quad (28)$$

At this extent, defining the filtered values of the variables θ and $\hat{u}$ respectively as

$$\theta_f(s) = \theta(s)/F(s) \quad (29)$$

$$\hat{u}_f(s) = \hat{u}(s)/F(s) \quad (30)$$

leads to write the input v to the system as

$$v(s) = G(s)\hat{u}_f(s) - D(s)\theta_f(s). \quad (31)$$

It is easy to recognize that, accounting for (26) and (27), the first order derivative of the control v, which is needed by v_{fi} in (22), is physically available now, on condition that the desired trajectory $\hat{\theta}(t)$ is sufficiently smooth (at least up to the third order derivative).

A CASE STUDY

The control strategy outlined in the above sections has been tested via simulations for the one link flexible arm existing in the Flexible Automation Laboratory at Georgia Tech. Two assumed modes have been considered in the expansion (1) [8]. The data for the dynamic model in the form (3) can be found in [9]. It is significant to report here that the two perturbation parameters are respectively $\varepsilon_1 = 0.1805$ and $\varepsilon_2 = 5.036E-3$.

An angular trajectory is commanded from $\hat{\theta}(0) = 0°$ to $\hat{\theta}(T) = 90°$, following the smooth velocity profile $\dot{\hat{\theta}} = \alpha t^3 (T - t)^3$, $\alpha = 140 \times 90 / T^2$, T = 2s.

As far as the slow control design, two poles in -2.5 have been chosen for the reference model, i.e. $a_0 = 6.25$ and $a_1 = 5$ in (24). The poles of the filter have been all placed at -12, i.e. $f_0 = 144$, $f_1 = 24$ in (25).

Two poles in -3 have been chosen then for the two boundary layer systems of kind (20), i.e. $k_{pf1} = k_{pf2} = 9$ and $\sqrt{\varepsilon_1} k_{vf1} = \sqrt{\varepsilon_2} k_{vf2} = 6$ in (22).

Three sets of simulations have been carried out; first only the slow control has been applied to the system (3), then the slow control + the fast control for the first layer, finally the slow control + the two fast controls for the two layers. In Figs. 3-5 reported are the tracking errors at the tip of the arm, under the above three controls. It can be seen that the addition of the fast controls improves the tracking performance and the tracking error at steady-state is conveniently damped out. An interesting point is that the improvement gained by the second fast control is minimum. This is in accordance with the issue that the amplitudes of the upper modes are much smaller as compared to that of the first mode; the contribution to the tip deflection is then minimum.

CONCLUSIONS

A singular perturbation approach has been developed for control of a lightweight flexible arm. The flexibility distributed along the structure has been modeled by an assumed mode method. The novelty is that a multi-boundary layer structure has been obtained for the system, in virtue of the definition of the perturbation parameters as the inverses of the spring constants for each assumed mode. A composite control strategy has been then adopted leading to the design of a slow control for the rigid system + a number of fast controls for the boundary layers taken into consideration. The actual slow controller requires a state variable filter in order to have practically implementable fast controllers. A case study has been finally worked out, showing the effectiveness of the proposed multi-time scale control strategy.

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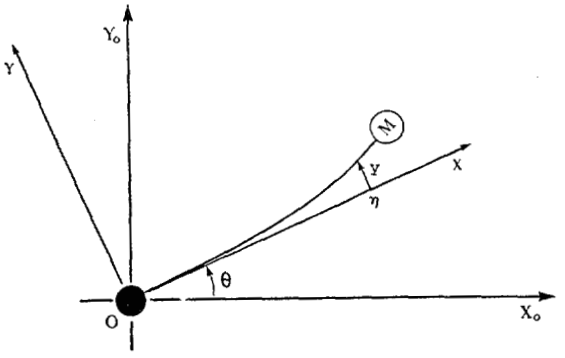

Fig. 1. The flexible arm.

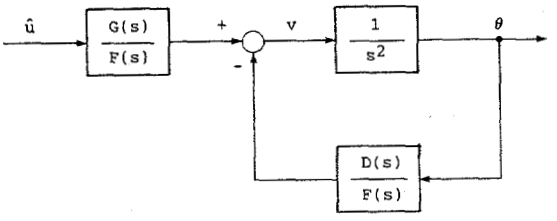

Fig. 2. The linear tracking controller.

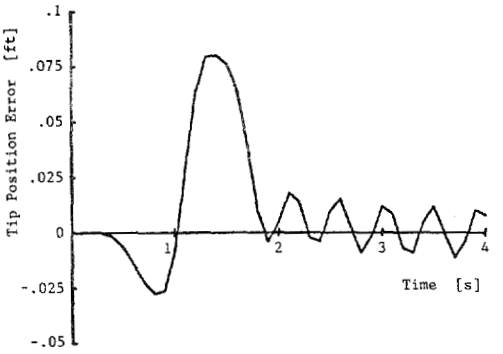

Fig. 3. Tip position error (slow control).

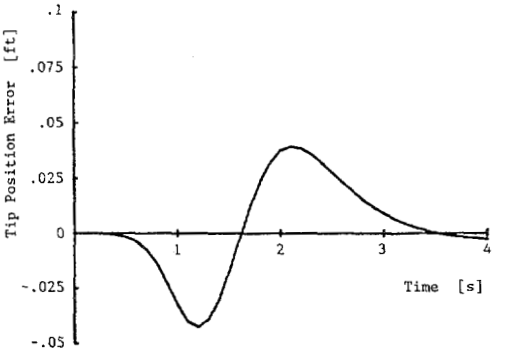

Fig. 4. Tip position error (composite control: slow + fast-1st layer).

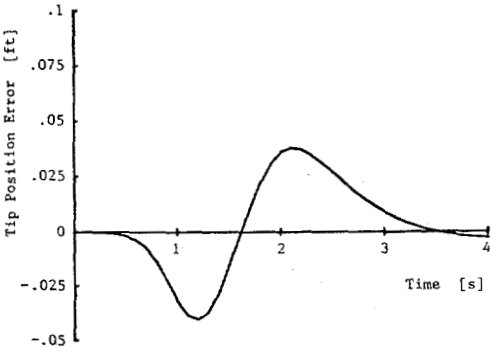

Fig. 5. Tip position error (composite control: slow + fast-1st layer + fast-2nd layer).