

FORCE/POSITION REGULATION OF ROBOT MANIPULATORS WITH GRAVITY PARAMETER ADAPTATION

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Abstract. *The problem of force/position regulation of robot manipulators in contact with an elastically compliant surface is considered in this work. A control scheme was recently proposed which consists of a PD action on the position loop, a PI action on the force loop, together with gravity compensation and desired contact force feedforward. In this work local asymptotic stability is proved in the case of imperfect gravity compensation. The original controller is then made adaptive with respect to a suitable set of parameters in the gravity term. Numerical case studies are developed for a three-joint industrial manipulator.*

Keywords. *Robots; force/position regulation; PID control; stability; Lyapunov methods; adaptive control.*

1. INTRODUCTION

When the end-effector of a robot manipulator is constrained by the environment, a pure motion controller usually gives degraded performance and can even cause instability.

Several schemes were devised which achieve control of both end-effector position and contact force by embedding force measurements in the controller. The most widely adopted strategy is hybrid control (Raibert and Craig, 1981) in which either a position or a force is controlled along each task space direction; stability of hybrid control was addressed by Wen and Murphy (1991).

A conceptually different strategy is the parallel control (Chiaverini and Sciavicco, 1988) which offers good robustness to inaccurate contact modelling. A force/position parallel controller was recently proposed by Chiaverini and Siciliano (1991) which is based on simple position PD control + gravity compensation + desired force feedforward + force PI control. Both local asymptotic and exponential stability have been shown using the Lyapunov direct method (Chiaverini *et al.*, 1992).

In this work, the assumption of perfect gravity compensation is relaxed. First, it is shown that the closed-loop system still asymptotically converges, though to a different equilibrium state. Then, by expressing the gravity term in a linear form with respect to a vector of parameters, a suitable parameter adaptation law is incorporated into the PID force/position regulator. Conditions are derived on the feedback gains.

Simulation results are presented for the industrial robot COMAU SMART 6.10R confirming the good performance of the proposed controller with gravity parameter adaptation over the nonadap-

tive controller.

2. PID FORCE/POSITION REGULATOR

The dynamic model of a robot manipulator constrained by the environment can be effectively written in the operational space in the form

$$B(x)\ddot{x} + C(x, \dot{x})\dot{x} + g(x) = u - f, \quad (1)$$

where x is the $(m \times 1)$ vector of end-effector location, B is the $(m \times m)$ symmetric and positive definite inertia matrix, $C\dot{x}$ is the $(m \times 1)$ vector of Coriolis and centrifugal generalized forces, g is the $(m \times 1)$ vector of gravitational generalized forces, u is the $(m \times 1)$ vector of driving generalized forces, and f is the $(m \times 1)$ vector of contact generalized forces exerted by the manipulator on the environment; all operational space quantities are expressed in a common reference frame.

In the present work, the attention is restricted to the case of non-redundant non-singular manipulators with $m = n = 3$, i.e. only translational motion and force components are considered. Then, x denotes the end-effector position.

The case of an environment constituted by a rigid, frictionless and elastically compliant plane is analyzed. The model of the contact force takes on the simple form

$$f = K(x - x_0), \quad (2)$$

where x is the position of the contact point, x_0 is a point of the plane at rest, and K is the (3×3) constant symmetric stiffness matrix that can be expressed as

$$K = knn^T \quad (3)$$

where $k > 0$ is the stiffness coefficient and n is the unit vector orthogonal to the contact plane.

Consider the set points x_d and f_d (along n). The control law (Chiaverini and Siciliano, 1991)

$$u = k_P \Delta x - k_D \dot{x} + g(x) + f_d + k_F \Delta f + k_I \int_0^t \Delta f d\sigma \quad (4)$$

where $\Delta x = x_d - x$ is the position error, $\Delta f = f_d - f$ is the force error, and $k_P, k_D, k_F, k_I > 0$ are suitable feedback gains, guarantees both local asymptotic and exponential stability (Chiaverini et al., 1992) around the equilibrium point

$$x_\infty = \frac{1}{k} n n^T (f_d + k x_0) + (I - n n^T) x_d \quad (5)$$

$$f_\infty = k n n^T (x_\infty - x_0) = f_d. \quad (6)$$

The construction of the equilibrium point in a two-dimensional case is illustrated in Fig. 1.

3. IMPERFECT GRAVITY COMPENSATION

In the following the requirement for perfect gravity compensation is relaxed. It will be shown that the system remains asymptotically stable, but around a different equilibrium point. To overcome this drawback, an adaptive version of the controller will be proposed in the next section.

To begin, a helpful property of the gravity term is that g has bounded partial derivatives, i.e.

$$\left\| \frac{\partial g(x)}{\partial x} \right\| \leq \delta \quad (7)$$

for some $\delta > 0$. This also implies that

$$\|g(x_1) - g(x_2)\| \leq \delta \|x_1 - x_2\|. \quad (8)$$

Consider the control law

$$u = k_P \Delta x - k_D \dot{x} + \hat{g}(x) + f_d + k_F \Delta f + k_I \int_0^t \Delta f d\sigma \quad (9)$$

where \hat{g} denotes the best available estimate of the gravity term g . A property similar to (7) holds for \hat{g} with $\hat{\delta}$ for δ .

The end-effector equilibrium position for the system (1) under the control (9) is found to satisfy

$$\hat{x}_\infty = \frac{1}{k} n n^T (f_d + k x_0) + \quad (10)$$

$$(I - n n^T) \left(x_d - \frac{1}{k_P} (g(\hat{x}_\infty) - \hat{g}(\hat{x}_\infty)) \right)$$

$$f_\infty = k n n^T (\hat{x}_\infty - x_0) = f_d. \quad (11)$$

Therefore, in the case of imperfect gravity compensation, the force set point is still attained while a different end-effector equilibrium position is reached. In fact, the components of x along the constrained direction (n) coincide, while the imperfect gravity compensation affects the components of x along the unconstrained directions.

Define the error

$$\hat{e} = \Delta x + \frac{1}{k_P} \hat{d} - \frac{1}{k_P} (g(\hat{x}_\infty) - \hat{g}(\hat{x}_\infty)), \quad (12)$$

with

$$\hat{d} = \frac{k_P}{k} n n^T (f_d + k(x_0 - x_d) + \frac{k}{k_P} (g(\hat{x}_\infty) - \hat{g}(\hat{x}_\infty))) \quad (13)$$

and

$$\dot{\hat{e}} = -\dot{x}. \quad (14)$$

Further define

$$\hat{s} = \frac{1}{k} n^T \left(\int_0^t \Delta f d\sigma - \frac{1}{k_I} \hat{d} \right). \quad (15)$$

Differentiating (15) with respect to time yields

$$\dot{\hat{s}} = n^T \dot{\hat{e}}. \quad (16)$$

The augmented system described by (1,14,16) under the control (9) can be written in the form

$$\dot{\hat{z}} = F \hat{z} + \mu \quad (17)$$

where $\hat{z} = (\dot{x}^T \ \hat{e}^T \ \hat{s})^T$,

$$F = \begin{pmatrix} -B^{-1}(C + k_D I) \\ -I \\ 0^T \end{pmatrix} \quad (18)$$

$$\mu = \begin{pmatrix} B^{-1}(k_P I + k'_F k n n^T) \ k_I k B^{-1} n \\ 0 \\ n^T \end{pmatrix}$$

with $k'_F = 1 + k_F$, and

$$\mu = \begin{pmatrix} B^{-1}(\hat{g}(x) - \hat{g}(\hat{x}_\infty) - g(x) + g(\hat{x}_\infty)) \\ 0 \\ 0 \end{pmatrix}. \quad (19)$$

Notice that $\hat{z} = 0$ is a solution of (17). Then, the following result can be stated.

Theorem 1. *There exists a choice of feedback gains k_P, k_D, k_F, k_I that makes the origin of the state space for the system (17-19) locally asymptotically stable.*

Sketch of proof. Consider the Lyapunov function candidate

$$V(\hat{z}) = \frac{1}{2} \hat{z}^T P \hat{z}, \quad (20)$$

where

$$P = \begin{pmatrix} B & -\rho B & 0 \\ -\rho B & (k_P + \rho k_D)I + k'_F k n n^T & k_I k n \\ 0^T & k_I k n^T & \rho k_I k \end{pmatrix} \quad (21)$$

with $\rho > 0$. Computing the time derivative of V along the trajectories of the system (17-19) gives

$$\dot{V}(\hat{z}) = \hat{z}^T \left(P F + \frac{1}{2} \dot{P} \right) \hat{z} + \hat{z}^T P \mu. \quad (22)$$

It can be shown that there exists a choice of k_P, k_D, k_F, k_I and ρ that makes V positive definite and \dot{V} negative semidefinite in the region $\{\hat{z} : \|\hat{e}\| < \hat{\Phi}\}$, i.e.

$$k_D > \rho(\lambda_M + k_C \hat{\Phi}) \quad (23)$$

$$k_P > (\delta + \hat{\delta}) \left(1 + \frac{\delta + \hat{\delta}}{4\rho(k_D - \rho(\lambda_M + k_C \hat{\Phi}))} \right) \quad (24)$$

$$\rho k'_F > k_I, \quad (25)$$

where λ_M is the maximum eigenvalue of B and k_C is so that $\|C\| \leq k_C \|\dot{x}\|$. Local asymptotic sta-

bility around $\hat{z} = 0$ follows from LaSalle invariant set theorem. ■

Compared to the stability analysis for the control (4) presented in (Chiaverini *et al.*, 1992), a more restrictive condition has to be satisfied for k_P ; this is reasonable since now the effect of imperfect gravity compensation has to be properly counteracted by k_P .

4. GRAVITY PARAMETER ADAPTATION

A well-known property of robot dynamics is that the vector g in (1) can be written in the form

$$g(x) = G(x)\theta, \quad (26)$$

where G is a $(3 \times p)$ matrix, and θ is a $(p \times 1)$ vector of manipulator and load constant parameters.

The control law (9) can then be rewritten as

$$u = k_P \Delta x - k_D \dot{x} + G\hat{\theta} + f_d + k_F \Delta f + k_I \int_0^t \Delta f d\sigma \quad (27)$$

where $\hat{\theta}$ is the vector of estimated parameters. Define $z = (\dot{x}^T \ e^T \ s)^T$ with

$$e = x_\infty - x = \Delta x + \frac{1}{k_P} d \quad (28)$$

$$d = \frac{k_P}{k} n n^T (f_d + k(x_0 - x_d)) \quad (29)$$

$$s = \frac{1}{k} n^T \left(\int_0^t \Delta f d\sigma - \frac{1}{k_I} d \right). \quad (30)$$

With these positions, the equations of the closed-loop system become

$$\dot{z} = Fz + \mu \quad (31)$$

with F as in (18), and

$$\mu = \begin{pmatrix} -B^{-1}G(x)\tilde{\theta} \\ 0 \\ 0 \end{pmatrix} \quad (32)$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the parameter error vector. The parameter estimate vector is updated according to the law

$$\dot{\hat{\theta}} = -\frac{1}{\nu} G^T \left(\dot{x} - \beta \left(\Delta x + \frac{k_I}{k_P} \int_0^t \Delta f d\sigma \right) \right) \quad (33)$$

with $\beta, \nu > 0$. The following result can be stated:

Theorem 2. *There exists a choice of feedback gains k_P, k_D, k_F, k_I and β that makes the origin of the state space for the system (31,18,32,33) asymptotically stable and the parameter estimate vector $\hat{\theta}$ bounded.* ■

Sketch of Proof. Consider the Lyapunov function candidate

$$W(z, \tilde{\theta}) = \frac{1}{2} z^T Q z + \frac{1}{2} \nu \tilde{\theta}^T \tilde{\theta}, \quad (34)$$

where

$$Q = \begin{pmatrix} B \\ -\beta B \\ -\gamma n^T B \end{pmatrix} \quad (35)$$

$$\begin{pmatrix} -\beta B & -\gamma B n \\ (k_P + \beta k_D)I + k'_F k n n^T & (k_I k + \gamma k_D) n \\ (k_I k + \gamma k_D) n^T & \beta k_I k + \gamma(k_P + k'_F k) \end{pmatrix}$$

with $\gamma > 0$. Computing the time derivative of W along the trajectories of system (31,18,32) gives

$$\dot{W}(z, \tilde{\theta}) = z^T \left(QF + \frac{1}{2} \dot{Q} \right) z \quad (36)$$

$$- (\dot{x}^T - \beta e^T - \gamma s n^T) G \tilde{\theta} + \nu \dot{\tilde{\theta}}^T \tilde{\theta}.$$

Taking

$$-\dot{\tilde{\theta}} = \dot{\hat{\theta}} = -\frac{1}{\nu} G^T (\dot{x} - \beta e - \gamma s n) \quad (37)$$

allows to cancel the terms in (36) depending on $\tilde{\theta}$. It is crucial to remark that, in view of (28,30), the choice

$$\gamma = \frac{\beta k_I k}{k_P} \quad (38)$$

makes the update law (37) equal to (33). It can be shown that there exists a choice of k_P, k_D, k_F, k_I , and β , with γ as in (38), that makes W positive definite and \dot{W} negative semidefinite in the region $\{z : \|e(0)\| \leq \Psi_1 < \infty, |s(0)| \leq \Psi_2 < \infty\}$, i.e.

$$k_D > \beta \left(k_C \left(\Psi_1 + \frac{\Psi_2 k_I k}{k_P} \right) + \right. \quad (39)$$

$$\left. \lambda_M \left(1 + \frac{\lambda_M (k_I k)^2}{k_P^3} \right) \right)$$

$$k_P > \beta (2\beta \lambda_M - k_D) \quad (40)$$

$$k_P + k'_F k > \frac{2\beta k_I k \lambda_M}{k_P} \quad (41)$$

$$\beta k'_F > k_I \left(1 + \frac{\beta k_D}{k_P} \right)^2. \quad (42)$$

The negative semidefiniteness of \dot{W} implies that $\tilde{\theta}$ is bounded and then $\hat{\theta}$ is also bounded. Local asymptotic stability around $z = 0$ follows from LaSalle invariant set theorem. ■

Notice that the constant β is not used in the control law (27) but affects the dynamics of the adaptation mechanism in (33). The other constant γ is keenly used to render the adaptation law (37) a function of physically measurable quantities already used in the control law (27), i.e. position, velocity and force measurements.

5. CASE STUDY

The proposed controllers were tested in a case study on the first three joints of the industrial robot COMAU SMART 6.10R. A payload of 10 kg mass concentrated at 0.2 m from the arm tip was added; this mass was assumed to be the only relevant parameter θ in (26).

A step motion from $x = (1.100 \ 0 \ 0)^T$ [m] to the set point $x_d = (1.120 \ 0 \ 0)^T$ [m] was commanded to the manipulator's tip. The set point $f_d = 0$ was assigned for the tip force. The geometry of the contact plane is characterized by $n = (1 \ 0 \ 0)^T$ and $x_0 = (1.115 \ 0 \ 0)^T$ [m]; the stiffness coefficient is $k = 10^5$ [N m⁻¹].

Four sets of simulations were carried out at a sampling time of 2 ms with the following controllers:

- control law (4) without force action ($k_F = k_I = 0$);
- control law (4) with force action;
- control law (9) with non-compensated load mass;
- control law (27) with initially non-compensated load mass and adaptive law (33).

The PD position feedback gains for all cases were set to $k_P = 10^5$ [$N\ m^{-1}$], $k_D = 10^4$ [$N\ s\ m^{-1}$] so as to guarantee a well-damped behaviour for the unconstrained motion of the manipulator.

The PI force feedback gains for cases b,c,d were set to $k_F = 9$, $k_I = 24$ [s^{-1}] so as to achieve a satisfactory behaviour during the constrained motion. With the above values, it was possible to satisfy the design conditions on the feedback gains for the various controllers.

The numerical results for the above four cases are illustrated in Fig. 2 in terms of the time history of the x and z components of position, the contact force (purely along x), and the estimated load mass which is significant only for case d.

The x -position and the force reveal that, without force feedback (case a), finite steady-state errors occur both for x -position and force; instead, with force feedback (cases b,c,d), the x -position and force behaviour is essentially the same for all controllers and null steady-state errors are guaranteed. On the other hand, the z -position behaviour differs for each controller. It is driven to zero at steady-state in all cases except c; in fact, in this case, the imperfect gravity compensation affects the vertical component of the tip position. Remarkably, the adaptation mechanism (case d) works satisfactorily to recover the initially non-compensated load mass, and the performance favourably compares with that of the controller with perfect compensation (case b).

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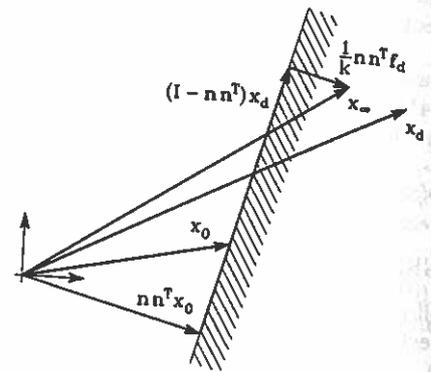


Fig. 1. Construction of the equilibrium point in a two-dimensional case.

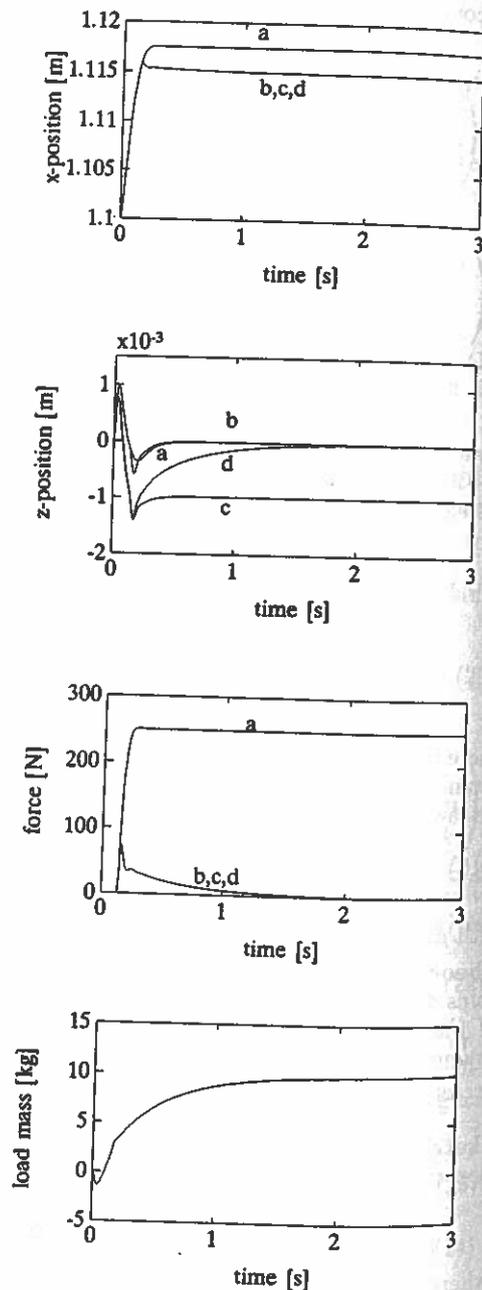


Fig. 2. Time history of x -position and z -position, contact force and load mass estimate.