

Issues in Modelling Techniques for Control of Robotic Manipulators with Structural Flexibility

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Introduction

The adoption of lightweight materials in the construction of mechanical manipulators has lately received a great deal of attention in the robotics research community. In spite of the potential advantages of lighter designs, e.g. higher payload to structure weight ratio and faster motion, vibration induced by the structural flexibility is a major drawback.

The need for accurate dynamic models of flexible manipulators becomes crucial not only for simulation purposes but also for trajectory control with active vibration suppression. The energy-based Lagrangian approach provides a natural framework for deriving the equations of motion of mechanical systems undergoing structural deformations [1]. One critical point in modelling flexibility is the method used to obtain a finite-dimensional approximation to the distributed parameter model. In particular, attention is focused here to the assumed modes method [2].

This work analyzes some important issues that arise in conjunction with the choice of different boundary conditions for the deformation modes. The implications on the design of inversion-based controllers for flexible manipulators [3,4] are studied with reference to clamped and pinned boundary conditions [5].

Dynamic Modelling

Consider a robotic manipulator composed of a serial chain of links, some of which are flexible. The Lagrangian technique can be used to derive the dynamic model, through the computation of global kinetic and potential energy of the system [1]. Due to link flexibility, the dynamic model is indeed of distributed nature. Links are modelled as Euler-Bernoulli beams satisfying proper boundary conditions for the actuated joint and the link tip.

In order to obtain a finite-dimensional model for control and simulation purposes, a set of generalized coordinates is to be chosen. Let θ denote the $n \times 1$ vector of joint coordinates, and δ the $m \times 1$ vector of link coordinates associated with an assumed modes description of link deflections. For simplicity, we suppose to include only bending deformations limited, for each link, to the plane of rigid motion [2].

The closed-form equations of motion of the manipulator

can be written in the general form

$$\begin{pmatrix} B_{\theta\theta}(\theta, \delta) & B_{\theta\delta}(\theta, \delta) \\ B_{\theta\delta}^T(\theta, \delta) & B_{\delta\delta}(\theta, \delta) \end{pmatrix} \begin{pmatrix} \ddot{\theta} \\ \ddot{\delta} \end{pmatrix} + \begin{pmatrix} h_{\theta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \\ h_{\delta}(\theta, \delta, \dot{\theta}, \dot{\delta}) \end{pmatrix} + \begin{pmatrix} g_{\theta}(\theta, \delta) \\ g_{\delta}(\theta, \delta) \end{pmatrix} + \begin{pmatrix} 0 \\ K\delta + D\dot{\delta} \end{pmatrix} = \begin{pmatrix} I \\ Q_{\delta} \end{pmatrix} u, \quad (1)$$

where B_{ij} are blocks of the $(n+m) \times (n+m)$ positive definite symmetric inertia matrix, partitioned according to the joint and link coordinates. Similarly, h_i contain Coriolis and centrifugal forces, which can be derived via Christoffel symbols, i.e. via differentiation of the inertia matrix, and g_i denote gravitational forces. The positive definite—typically diagonal—matrices K and D describe modal stiffness and damping of flexible links, respectively. Matrix Q_{δ} , weighting the $n \times 1$ vector of joint input torques u in the lower equations, takes on different forms depending on the chosen boundary conditions for link deformation at the actuated side.

From eq. (1), flexible accelerations can be extracted as

$$\ddot{\delta} = B_{\delta\delta}^{-1}(Q_{\delta}u - (h_{\delta} + g_{\delta} + K\delta + D\dot{\delta}) - B_{\theta\delta}^T\ddot{\theta}) \quad (2)$$

which, substituted into the upper part of (1), gives

$$\begin{aligned} (B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^T)\ddot{\theta} + h_{\theta} + g_{\theta} \\ - B_{\theta\delta}B_{\delta\delta}^{-1}(h_{\delta} + g_{\delta} + K\delta + D\dot{\delta}) = Fu, \end{aligned} \quad (3)$$

where $F = I - B_{\theta\delta}B_{\delta\delta}^{-1}Q_{\delta}$ is a $n \times n$ full rank matrix. Notice that equations (3) describe the dynamics of the equivalent rigid system $B_{\theta\theta}\ddot{\theta} + h_{\theta} + g_{\theta} = u$ evaluated for $\delta \equiv 0$, modified by the effects of link flexibility.

For each link, bending deformation is usually described in terms of two alternative frames; namely, the *clamped* frame, aligned with the direction of the undeformed link at the joint location, and the *pinned* frame, pointing at the instantaneous center of mass of the deformed link. Accordingly, the joint and link coordinates attain different meanings and terms in the dynamic equations (1) assume different expressions; however, it is always possible to transform one set of coordinates into the other, e.g. $\theta_p = \theta_c + T\delta_c$ [5]. In particular, $Q_{\delta} = 0$ in the clamped case. Eigenfrequencies of the system are the same in both representations, but simplifications may occur in the model; for instance, for a single flexible link, the inertia matrix turns out to be diagonal in the pinned case, when using eigenfunctions as assumed modes of deformation.

Inversion Control

Trajectory tracking in nonlinear systems is usually achieved by input-output *inversion control* techniques [3,4]. Under the assumption of stability of the resulting closed-loop system, *exact* reproduction of smooth desired output trajectories is guaranteed.

In what follows, only the case of joint output trajectory is considered and the effects of clamped vs. pinned representation on the synthesis of the control law are assessed. Let then a denote the desired joint acceleration. Setting $\ddot{\theta} = a$ in (3) and solving for u yields the feedback law

$$u = F^{-1}((B_{\theta\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}B_{\theta\delta}^T)a + h_{\theta} + g_{\theta} - B_{\theta\delta}B_{\delta\delta}^{-1}n_{\delta}), \quad (4)$$

where $n_{\delta} = h_{\delta} + g_{\delta} + K\delta + D\dot{\delta}$. In the pinned case, a computationally efficient expression for F^{-1} is [6]

$$F^{-1} = I + B_{\theta\delta}(B_{\delta\delta} - Q_{\delta}B_{\theta\delta})^{-1}Q_{\delta}. \quad (5)$$

The control (4) transforms the closed-loop system into the input-output linearized form

$$\ddot{\theta} = a \quad (6a)$$

$$\begin{aligned} \ddot{\delta} = & -(I + Q_{\delta}F^{-1}B_{\theta\delta})(B_{\delta\delta}^{-1}(B_{\theta\delta}^T a + n_{\delta})) \\ & + B_{\delta\delta}^{-1}Q_{\delta}F^{-1}u_r \end{aligned} \quad (6b)$$

where $u_r = B_{\theta\theta}a + h_{\theta} + g_{\theta}$ would be the computed torque control for the equivalent rigid system.

At this point, one can recognize that in the clamped case eq. (4) becomes

$$\begin{aligned} u_c = & (B_{\theta_c\theta_c} - B_{\theta_c\delta_c}B_{\delta_c\delta_c}^{-1}B_{\theta_c\delta_c}^T)a + h_{\theta_c} + g_{\theta_c} \\ & - B_{\theta_c\delta_c}B_{\delta_c\delta_c}^{-1}n_{\delta_c}, \end{aligned} \quad (4')$$

where subscript c denotes clamped quantities, and eq. (6) remarkably simplifies to

$$\ddot{\theta}_c = a \quad (6a')$$

$$\ddot{\delta}_c = -B_{\delta_c\delta_c}^{-1}(B_{\theta_c\delta_c}^T a + n_{\delta_c}). \quad (6b')$$

Notice that the joint variables which track now the desired trajectory are indeed the clamped ones. The feasibility of this control approach is based on the stability of (6b'). For, it is sufficient that the associated zero-dynamics [3] is asymptotically stable. This dynamics is obtained by forcing to zero the output θ_c of the nonlinear system. Accordingly, one has

$$\ddot{\delta}_c = -B_{\delta_c\delta_c}^{-1}(h_{\delta_c} + g_{\delta_c} + K\delta_c + D\dot{\delta}_c). \quad (7)$$

In the absence of gravity terms ($g_{\delta_c} = 0$), it can be shown via a Lyapunov argument that $\delta_c = \dot{\delta}_c = 0$ is an asymptotically stable equilibrium of (7). In fact, defining the candidate

$$V = \frac{1}{2}\delta_c^T K\delta_c + \frac{1}{2}\dot{\delta}_c^T B_{\delta_c\delta_c}\dot{\delta}_c \quad (8)$$

leads to

$$\begin{aligned} \dot{V} = & \delta_c^T K\dot{\delta}_c + \dot{\delta}_c^T B_{\delta_c\delta_c}\ddot{\delta}_c + \frac{1}{2}\dot{\delta}_c^T \dot{B}_{\delta_c\delta_c}\dot{\delta}_c \\ = & -\delta_c^T (h_{\delta_c} + D\dot{\delta}_c) + \frac{1}{2}\dot{\delta}_c^T \dot{B}_{\delta_c\delta_c}\dot{\delta}_c \\ = & -\dot{\delta}_c^T D\dot{\delta}_c \leq 0, \end{aligned} \quad (9)$$

using (7) and the skew symmetry of matrix $\dot{B}_{\delta_c\delta_c} - 2S_{\delta_c\delta_c}$, with $S_{\delta_c\delta_c}\dot{\delta}_c = h_{\delta_c}$ [7]. Asymptotic stability then follows from LaSalle's theorem.

Similar arguments can be used in the pinned case, although the developments are much more involved and omitted here for brevity.

Since closed-loop stability is verified in both cases, the design of a trajectory controller is completed by specifying

$$a = \ddot{\theta}_{des} + K_D(\dot{\theta}_{des} - \dot{\theta}) + K_P(\theta_{des} - \theta), \quad (10)$$

where $K_P > 0$, $K_D > 0$ allow pole placement in the open left-hand complex half plane for the linear system (6a).

The above design allows reproduction of trajectories defined at the joint level in flexible manipulators. The choice of an output at the link level—ultimately the tip point—is a challenging topic that deserves further investigation. The single-link case with clamped coordinates has been studied in [4].

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