

The Potential of Model-Based Control Algorithms for Improving Industrial Robot Tracking Performance

Pasquale Chiacchio, Lorenzo Sciavicco, and Bruno Siciliano

Dipartimento di Informatica e Sistemistica
Università degli Studi di Napoli "Federico II"
Via Claudio 21, 80125 Napoli, Italy

Abstract

This paper is aimed at analyzing the importance of compensation of the manipulator nonlinear dynamics in control of industrial-type robots. A dynamic analysis is carried out for a typical robot with high gear ratios which reveals that the effect of the nonlinear terms on the load at the actuators is not negligible with respect to the rotor inertia. The design of independent linear joint controllers is discussed, and the addition of an integral term in the velocity servo loop is shown to outperform the conventional Proportional+Derivative (PD) controller. Then, various model-based control algorithms are introduced which require an increasing complexity of compensation of the estimates of the dynamic terms. The above controllers are tested in discrete time by means of a number of examples both in the joint space and in the task space.

1. Introduction

In robotics a paradigm is commonly accepted that, when high gear ratios are present at the robot joints, the nonlinear coupling dynamic terms can be neglected, leading to decentralized linear controllers. Hence, independent joint controllers, e.g. Proportional+Derivative (PD), are designed for a constant joint inertia. In fact, it is argued that the actuator inertia, reported at the joint, dominates over the configuration-dependent inertial terms. Moreover, since industrial robot operational speeds are usually quite low, Coriolis/centrifugal terms are also not compensated.

Earlier works directed to improve industrial robot performance proposed to add an adaptive feedforward controller to the pure PD feedback [1], or to replace the PD controller with a lag-lead compensator [2]. Later, the importance of dynamic compensation was experimentally demonstrated not only for direct-driven manipulators [3,4], but recently also for gear-driven manipulators [5].

In order to better understand the potential of using model-based control algorithms, the nonlinear and coupling characteristics for a typical industrial manipulator with high gear ratios, the Manutec r3 robot [6], are analyzed in this work. In particular, an evaluation of the dynamic terms over the manipulator workspace is provided when the joint configuration varies; this includes the determinant of the inertia matrix and the norm of the gravitational vector.

Conventional independent joint PD linear controllers are designed, and the feedback gains are tuned for the

maximum inertia at each joint. An alternative design is proposed in this paper for the joint control servos, namely an integral action is introduced in the velocity loop which is aimed at recovering the steady-state error induced by gravity as well as at counteracting disturbances originated from coupling with other joints and parametric variations.

Successively, in order to improve robot tracking accuracy, several model-based control algorithms are designed according to progressive levels of dynamic feedback feedforward compensation: gravity; gravity and diagonal inertia; gravity and full inertia; gravity, inertia, and Coriolis/centrifugal terms (i.e. computed torque).

The performance of a discrete-time implementation of all the above controllers is tested, in simulation, for two trajectories in the joint space and one trajectory in the task space. These reveal that the controller with the integral action in the velocity servo loop outperforms the conventional PD controller. Moreover, the results confirm that also for industrial manipulators a (partial) compensation of the dynamic terms leads to reduced tracking errors.

2. Dynamic Analysis

There are several methods to derive a dynamic model of a mechanical manipulator: Newton-Euler, d'Alembert, and Lagrange [7]. The Lagrange method is used most frequently because it leads to a set of equations whose terms are easily interpreted in terms of physical quantities. The resulting model can be written in the well-known form

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau(q)$$

where q is the $n \times 1$ vector of joint variables, M is the $n \times n$ symmetric positive definite inertia matrix, $C\dot{q}$ is the $n \times 1$ vector of Coriolis/centrifugal torques, g is the $n \times 1$ vector of gravitational torques, and τ is the $n \times 1$ vector of joint actuator torques.

In practice, the model should also account for the viscous and static friction torques; these have to be experimentally determined, indeed. Furthermore, it has been implicitly assumed that the electrical time constant associated with the manipulator drive system is small compared to the mechanical time constant, and then the system is accurately represented by a second-order model.

In designing joint control servos for industrial manipulators, one should consider the equation of motion at each joint. This can be extracted from the above model as

$$(J_{L,i} + \rho_i^2 J_{R,i})\ddot{q}_i + \tau_{m,i} + \tau_{n,i} = \tau_i \quad i = 1, \dots, n$$

where $J_{L,i}$ is the moment of inertia of the arm at joint, $J_{R,i}$ is the moment of inertia of the actuator rotor, ρ_i is the gear ratio, $\tau_{m,i}$ is the inertia torque due to the interaction with other joints, $\tau_{n,i}$ is the torque due to nonlinear terms (gravity and Coriolis/centrifugal), and τ_i is the effective torque at the joint.

Because of typical high gear ratios of industrial robots, it is tacitly assumed that the constant rotor inertia dominates over the arm inertia that varies with the joints. Moreover, since the robot usually operates at relatively low speeds, the effect of Coriolis/centrifugal terms is neglected. These affirmations lead to the design of linear independent joint controllers of PD type, eventually with gravity compensation.

In order to verify the correctness of the above approximations, an analysis is carried out in the following which is aimed at studying the variability of inertia and gravity as a function of the arm configuration. We have concentrated our investigation on the Manutec r3 (Fig. 1), a gear-driven industrial robot with a PUMA-like kinematic structure. Only the first three joints are considered as they provide the significant part of the manipulator dynamics. The dynamic model and all relevant data are taken from [6].

We have chosen to study the determinant of the inertia matrix and the norm of the gravity vector; it can be seen that those quantities are a function of only q_2 and q_3 , in force of the particular kinematic structure. The results reported in Figs. 2 and 3 reveal wide variations of these indices over the joint ranges. This does not allow to ignore a priori the potential offered by controllers which account for the dynamic model of the robot, as it is done instead in the common industrial practice. These conclusions, indeed, are in agreement with those derived for the PUMA robot reported in [5].

3. Control Design

As the basic step for designing a robot control system, simple independent PD joint controllers with feedforward velocity term have been considered first, i.e.

$$\tau = \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) \quad (\text{a})$$

where \mathbf{e} is the error between the desired and the actual joint position ($\mathbf{e} = \mathbf{q}_d - \mathbf{q}$), and the positive diagonal feedback matrices, \mathbf{K}_P and \mathbf{K}_D , determine the dynamic system response.

On the basis of the considerations in the preceding section, dynamic compensation is introduced in the following with the purpose of obtaining higher performance. This compensation relies on accurate estimates of the terms in the dynamic model, and can be performed either feedforward or feedback: The former is computationally advantageous (off-line), since the dynamic terms are evaluated for the desired joint trajectory \mathbf{q}_d . The latter is carried out on-line with the updates of the dynamic terms for the actual joint trajectory \mathbf{q} , and then is apparently more robust to disturbances and parametric variations.

Different levels of complexity are considered:

PD with feedback compensation of gravitational torques

$$\tau = \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{g}(\mathbf{q}) \quad (\text{b})$$

PD with feedback compensation of gravitational torques and diagonal inertia

$$\tau = \widehat{\mathbf{M}}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}) \quad (\text{c})$$

where $\ddot{\mathbf{q}}_d$ is the desired feedforward acceleration, and $\widehat{\mathbf{M}}$ denotes the diagonal matrix that can be extracted from \mathbf{M} .

PD with feedback compensation of gravitational torques and full inertia

$$\tau = \mathbf{M}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}) \quad (\text{d})$$

PD with feedback compensation of gravitational torques, Coriolis/centrifugal torques, and full inertia, i.e. a true computed torque control,

$$\tau = \mathbf{M}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \quad (\text{e})$$

In case of feedforward compensation, the last three controllers are modified into:

$$\tau = \widehat{\mathbf{M}}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}_d) \quad (\text{f})$$

$$\tau = \mathbf{M}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}_d) \quad (\text{g})$$

$$\tau = \mathbf{M}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e})] + \mathbf{g}(\mathbf{q}_d) + \mathbf{C}(\mathbf{q}_d, \dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d \quad (\text{h})$$

In practical design of industrial servo control systems for D.C. drives, it is customary to introduce an integral action in the velocity loop in order to recover steady-state errors and reject constant disturbance torques. Thus, it seems interesting to us to follow this approach for robot control system design and compare its performance with conventional PD servo loop.

Specifically, the control torque (a) is modified into

$$\tau = \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt \quad (\text{a}')$$

Analogously to the compensations proposed above, we can derive the following controllers

$$\tau = \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt + \mathbf{g}(\mathbf{q}) \quad (\text{b}')$$

$$\tau = \widehat{\mathbf{M}}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}) \quad (\text{c}')$$

$$\tau = \mathbf{M}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}) \quad (\text{d}')$$

$$\tau = \mathbf{M}(\mathbf{q})[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}) + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} \quad (e')$$

$$\tau = \widehat{\mathbf{M}}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}_d) \quad (f')$$

$$\tau = \mathbf{M}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}_d) \quad (g')$$

$$\tau = \mathbf{M}(\mathbf{q}_d)[\ddot{\mathbf{q}}_d + \mathbf{K}_D(\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) + \mathbf{K}_I \int (\dot{\mathbf{e}} + \mathbf{K}_P \mathbf{e}) dt] + \mathbf{g}(\mathbf{q}_d) + \mathbf{C}(\mathbf{q}_d, \dot{\mathbf{q}}_d)\dot{\mathbf{q}}_d \quad (h')$$

4. Case Studies

In order to investigate the performance of all the controllers presented in the above section, a number of case studies were developed and are described in the following.

The software package SIMNON was used to simulate the controlled Manutec r3 manipulator. The controllers were implemented in discrete time by utilizing the bilinear transformation from s -domain to z -domain, i.e. $s = \frac{2}{T} \frac{z-1}{z+1}$ where the sampling period T is chosen as 2 ms. The velocity profile for the tested trajectories is

$$v = \begin{cases} \frac{v_{\max}}{2} \left(1 - \cos \frac{3\pi t}{t_f}\right) & 0 \leq t < \frac{t_f}{3} \\ v_{\max} & \frac{t_f}{3} \leq t < \frac{2t_f}{3} \\ \frac{v_{\max}}{2} \left(1 - \cos \frac{3\pi}{t_f} \left(t - \frac{t_f}{3}\right)\right) & \frac{2t_f}{3} \leq t < t_f \\ 0 & t \geq t_f \end{cases}$$

where $t_f = 3d/2v_{\max}$, being d the total displacement. This smooth profile was chosen in order to have initial and final zero accelerations and a cruise time (at maximum velocity) of a third of the total traveling time.

The linear controllers (a) and (a') were tuned for an ideal, purely inertial system with decoupled joints, absence of gravity, and inertia equal to the maximum value of the total joint inertia: These are $J_1 = 79 \text{ kg m}^2$, $J_2 = 119 \text{ kg m}^2$, and $J_3 = 22 \text{ kg m}^2$; incidentally, we remark that J_3 is a constant. The dynamic requirements for the servo loop at each joint were to have a bandwidth of 20 rad/s and an overshoot of 6%, for both controllers. The designs resulted into the following feedback gains:

$$k_{P,i} = 14.38, k_{D,i} = 26.20J_i$$

$$k_{P,i} = 11.40, k_{D,i} = 22.85J_i, k_{I,i} = 130.6J_i$$

for controllers (a) and (a'), respectively. As mentioned above, these controllers were then transformed into their equivalent discrete-time forms.

For the controllers (b) and (b'), the linear part of the design is the same as for (a) and (a'). For the remaining controllers, the values of J_i are imposed to be 1, since compensation of the joint inertia occurs.

Two trajectories were assigned in the joint space by moving one joint at time, with the purpose of analyzing the dynamic interaction between the joints. Also, the moving joint is required to reach its velocity limit.

In the first trajectory, q_{1d} goes from -2.8 rad to 2.8 rad with a maximum velocity of 3 rad/s, while q_{2d} is kept at 0.78 rad and q_{3d} at 0 rad. This trajectory allows to evaluate the effects of Coriolis/centrifugal torques on the joints 2 and 3. In Fig. 4a are reported the joint tracking errors when the controllers (a-h) were applied. The results show that: For joint 1, a sole compensation of the self-inertia, controllers (c,f), drastically reduces the error. For joints 2 and 3, a bias on the error occurs if gravity is not compensated and this results into a steady-state error, controller (a); the performance of controllers (b,c,f) are equivalent since the two joints have maximum self-inertias; a compensation of off-diagonal terms of inertia matrix, controllers (d,g), reduces the errors while compensation of also velocity dependent terms practically annuls the errors, controllers (e,h). Notice that, for this case study, as well as in the following ones, no appreciable difference can be observed between corresponding feedback and feedforward compensations. On the other hand, when controllers (a'-h') were applied, the same kind of considerations as above can be made (Fig. 4b). It can be recognized that the addition of an integral term in the velocity loop not only eliminates steady-state errors induced by gravity, controller (a'), but has also a remarkable effect on the reduction of tracking errors due to non-perfect compensation which play the role of disturbances. These advantages are obtained, however, at the expenses of slightly higher settling times.

In the second trajectory, q_{2d} goes from 0 rad to 1.5 rad with a maximum velocity of 1.5 rad/s, while q_{1d} is kept at 0 rad and q_{3d} at 0 rad. This trajectory allows to evaluate the effects induced on joint 3 by the motion of joint 2. In Figs. 5a and 5b are reported only the tracking errors for joints 2 and 3, since joint 1 is not affected by this type of motion. Similar global conclusions can be drawn as for the previous case study. In particular, in force of the particular given trajectory, the Coriolis/centrifugal terms on joint 3 are not relevant and then compensation of full inertia and gravity is sufficient to obtain best tracking accuracy, controllers (d,g,d',g').

The last case study was worked out in the task space, so that all the three joints are interested by the motion. The end-effector (EE) trajectory is a straight line from (0.3, -0.5, 1.0) m to (0.3, 0.5, 1.0) m with a maximum velocity of 0.9 m/s. The inverse kinematic functions were computed in order to obtain reference joint trajectories. It could be seen that the assigned trajectory results into a large displacement for joint 1 which reaches its velocity limit. Figs. 6a and 6b show the norm of EE tracking errors, respectively with the two classes of controllers: (a-h) and (a'-h'). The behavior of the tested controllers confirms most of the results derived above. Nevertheless, an interesting point to outline is that compensation of inertia — either diagonal, controllers (c,f), or full, con-

trollers (d,g) — reduces the tracking error during the acceleration/deceleration phases, but increases it with respect to the non-compensating controller (b) during the cruise period. This effect is somewhat mitigated with the introduction of the integral action, controllers (c',f') and (d',g').

Conclusions

This work was aimed at investigating the performance of industrial robot controllers with compensation of nonlinear dynamic terms. Different levels of both feedback and feedforward compensation were considered ranging from simple gravity compensation to full computed torque. For the design of the linear part of the controllers, besides the conventional PD feedback action, an integral term was introduced in the velocity loop, as done in industrial practice for servo drive systems. The dynamic analysis of inertia and gravity, together with the simulation results, for a typical gear-driven industrial robot clearly demonstrated the potential of model-based control algorithms for improving robot tracking performance. The effects of each type of compensation were extensively studied in a number of case studies. The results showed that the addition of the integral action eliminates steady-state errors, even without gravity compensation, and proves advantageous for reducing tracking errors by about 50%, compared with standard PD controllers.

Acknowledgements

This work was supported by *Consiglio Nazionale delle Ricerche* under grant n. 89.00514.67-PFR.

References

- [1] R.M. Goor, "A new approach to minimum time robot control," *Proc. ASME Winter Annual Meeting: Robotics and Manufacturing Automation*, Miami Beach, FL, PED-Vol. 15, pp. 1-11, Nov. 1985.
- [2] Y. Chen, "Frequency response of discrete-time robot systems — Limitations of PD controllers and improvements by lag-lead compensation," *Proc. 1987 IEEE Int. Conf. on Robotics and Automation*, Raleigh, NC, pp. 464-472, Mar.-Apr. 1987.
- [3] C.H. An, C.G. Atkeson, and J.M. Hollerbach, *Model-Based Control of a Robot Manipulator*, MIT Press, Cambridge, MA, 1988.
- [4] P.K. Khosla and T. Kanade, "Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulators," *Int. J. of Robotics Research*, Vol. 7, No. 1, pp. 18-28, 1988.
- [5] M.B. Leahy, Jr. and G.N. Saridis, "Compensation of industrial manipulator dynamics," *Int. J. of Robotics Research*, Vol. 8, No. 4, pp. 73-84, 1988.
- [6] M. Otter and S. Türk, *The DFVLR Models 1 and 2 of the Manutec r3 Robot*, DFVLR-Mitt. 88-13, 1988.
- [7] K.S. Fu, R.C. Gonzales, and C.S.G. Lee, *Robotics: Control, Sensing, Vision, and Intelligence*, McGraw-Hill, New York, 1987.

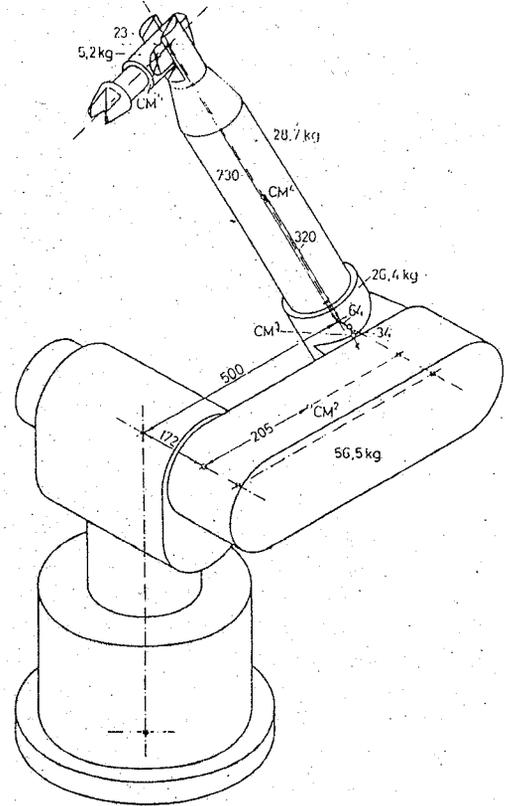


Fig. 1 — The Manutec r3 robot

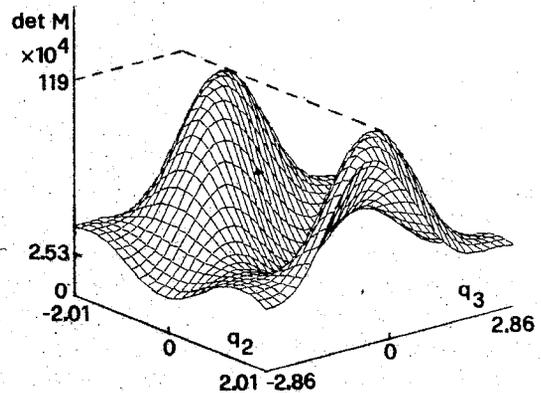


Fig. 2 — Determinant of inertia matrix vs joint variables

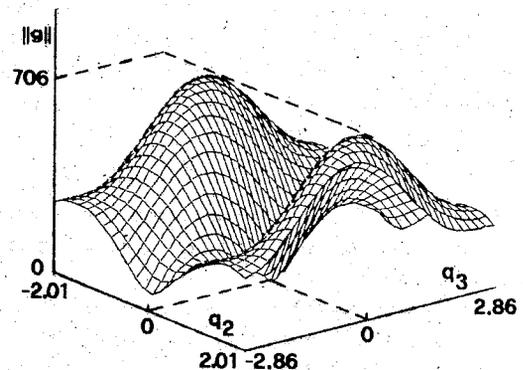


Fig. 3 — Norm of gravity vector vs joint variables

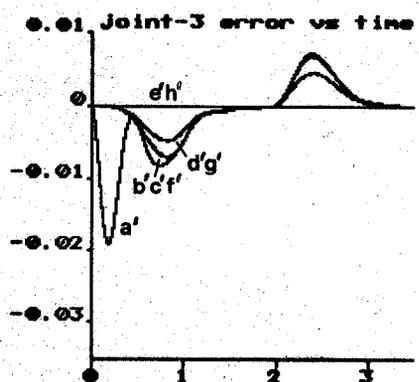
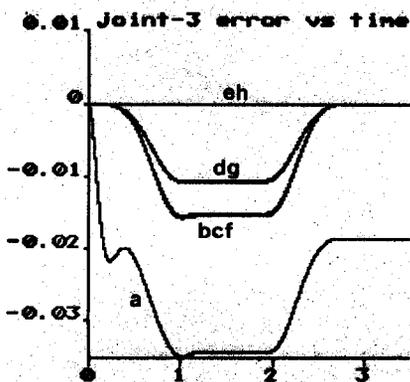
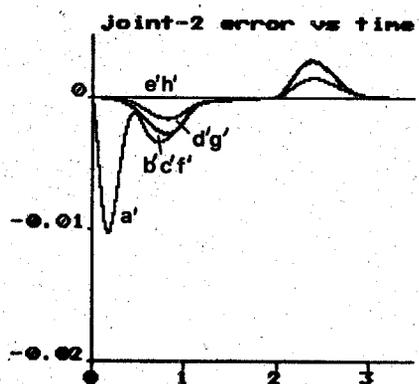
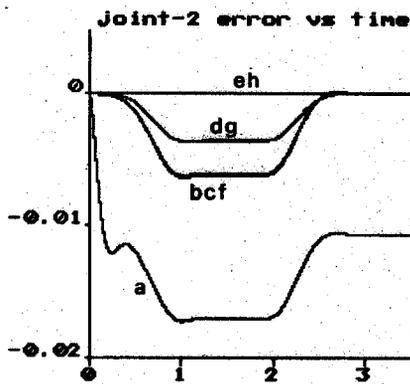
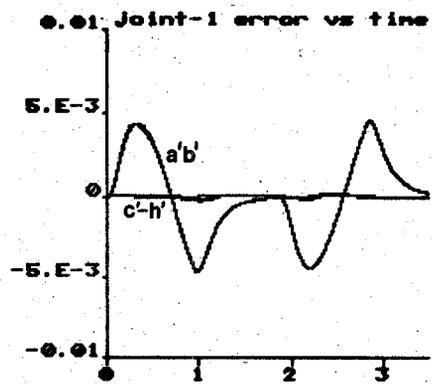
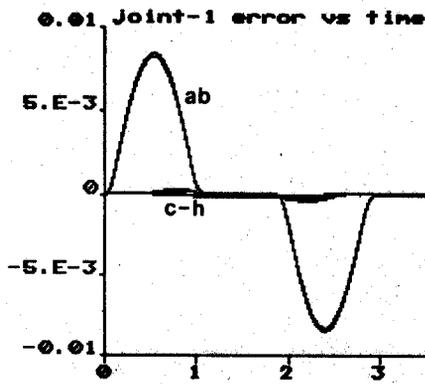


Fig. 4a — Joint errors for controllers (a-h)

Fig. 4b — Joint errors for controllers (a'-h')

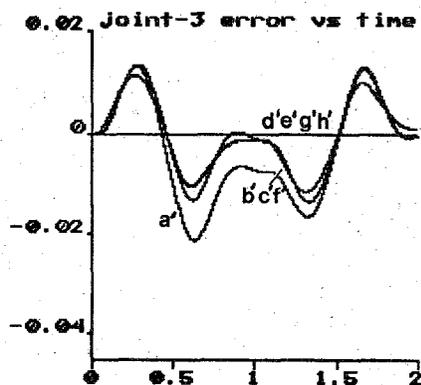
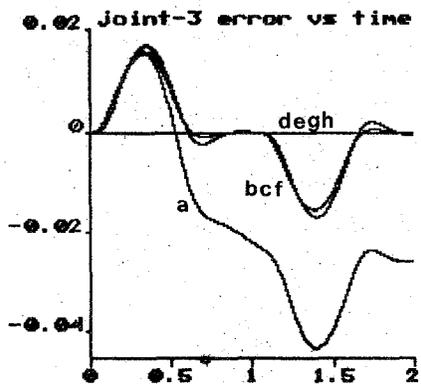
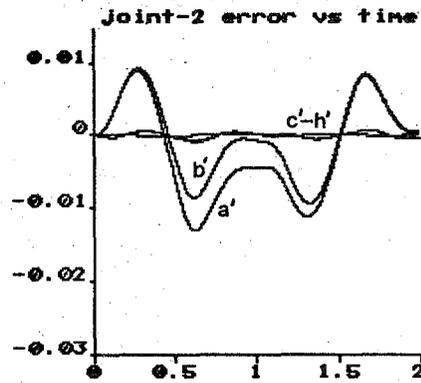
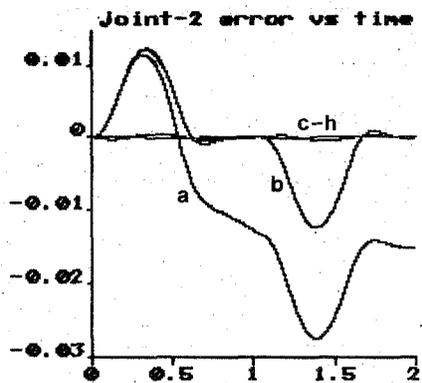


Fig. 5a — Joint errors for controllers (a-h)

Fig. 5b — Joint errors for controllers (a'-h')

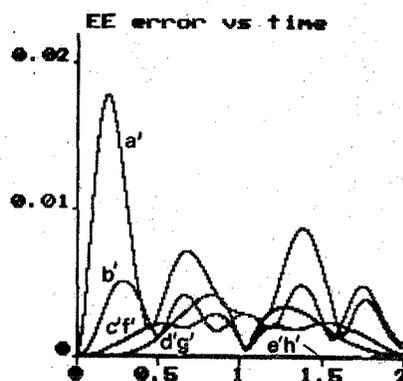
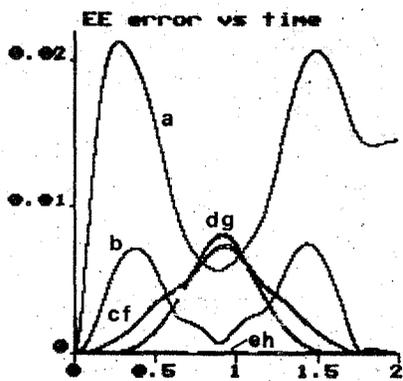


Fig. 6a — Norm of EE errors for controllers (a-h)

Fig. 6b — Norm of EE errors for controllers (a'-h')