

# Two-Time Scale Control for an Arm with Joint and Link Compliance

Thomas E. Alberts and Atul Kelkar

Department of Mechanical Engineering and Mechanics, Old Dominion University,  
Norfolk, VA 23529-0247, USA

Bruno Siciliano

Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli  
"Federico II" Via Claudio 21, 80125 Napoli, Italy

Dynamic analysis of the space shuttle remote manipulator system (RMS) reveals that compliance of both the joints and the links contribute to its endpoint oscillations. As such, the RMS inherently exhibits a unique two time scale behavior comprised of low frequency modes associated with joint compliance and higher frequency modes due to boom bending. Of these, the joint compliance modes are generally the most significant. The implicit two-time scale dynamic character of the RMS naturally suggests a singular perturbation approach to controller design. In this paper we consider the control problem of a flexible manipulator with flexible joints, specifically, the case where a time-scale separation exists between the joint compliance modes and the boom flexure modes. To illustrate the procedure, it is applied to a single-axis, single-link arm with compliance in both the boom and the joint. The system is parameterized to resemble the actual dynamics of the second boom of the RMS. This system retains the fundamental two-time-scale character of the RMS. A composite controller is designed for the system, which consists of a slow component aimed at controlling rigid body and joint compliance modes, and a fast component devoted to stabilizing the link compliance modes.

## Introduction

In this paper we present initial results of an investigation aimed at developing effective endpoint controllers for large flexible space manipulators similar to the Space Shuttle Remote Manipulator System (RMS). The RMS is a remotely controlled anthropomorphic multi-degree of freedom arm. It is of very lightweight construction, and as a result it exhibits much more mechanical compliance than typical terrestrial manipulators. The RMS has been used extensively by the shuttle crew for such tasks as deployment and retrieval of satellites, inspection and servicing of spacecraft and transfer of men and equipment. While it has performed reliably for all of these tasks, due to its inherent flexibility, operations are sometimes delayed while operators wait for oscillations to damp out. In the future, the RMS and similar lightweight arms are likely to play an important role in a variety of demanding on-orbit assembly and maintenance tasks including the construction of the space station. For such tasks the requirements of precise motion and short settling time from oscillation call for an effective means of end-point vibration control for these manipulators. The current control scheme for the RMS is based on independent joint control. We believe that for advanced applications it is desirable to consider model based control schemes which account for structural compliance.

Given the dynamic complexity of the RMS, the problem of controlling its motion presents a substantial challenge. The results of a recent detailed dynamic analysis [1] of the RMS reveal that, unlike most industrial manipulators, compliance of both the joints and the links contribute significantly to its endpoint oscillations. Of these two primary sources of compliance, the joint compliance is generally a stronger influence, in terms of contributing to end-point oscillations, than boom flexure. This influence has been recognized in industrial manipulators as well [2], but perhaps to a lesser degree. However, in contrast to typical industrial manipulators, the RMS also exhibits sufficient boom flexure to warrant its careful consideration during control design. In this investigation we propose an approach which exploits the inherent two time scale nature of the RMS through the use of a singular perturbation approach, which formally separates the dynamic model into slow and fast subsystems. The slow subsystem emerges as precisely the model for a rigid link manipulator with joint compliance. The fast subsystem includes the link compliance effects. The controller

then consists of a slow component and a fast component where the former is aimed at controlling rigid body and joint compliance modes, while the latter is devoted to stabilizing the link compliance modes.

A considerable body of literature has been devoted to the control of manipulators with flexible links, and recently the problem of controlling manipulators with compliant joints has been addressed seriously. To date however, with the exception of the dynamic modelling results presented by Yang and Donath [3], the authors are unaware of investigations where these two effects are considered together. The literature on flexible link manipulators is quite extensive, however we cite some recent developments that are relevant to the present investigation. Siciliano and Book [4] have developed a general singular perturbation approach with application to control of a single-link flexible manipulator. The slow subsystem, consisted of the rigid model of the manipulator. This work was eventually extended to multiple-link flexible arms [5]. Yang and Huang [6] use a singular perturbation approach to separate the time scales of a single-link flexible manipulator on a translating base. In this case a Lyapunov-based design is used to stabilize the non-linear fast subsystem. Siciliano, et al [7] have also considered a time-scale separation for a single-link flexible arm which produces a slow model subsystem comprised of the rigid body and first flexure modes. The control design employed optimal output feedback compensators. Furthermore, to achieve more accurate tracking control for the slow subsystem, an integral manifold approach was applied in [8].

A survey of investigations pertaining to the control of flexible joint robots has been compiled by Spong [9]. In the conventional robotics literature, manipulators are represented as open kinematic chains of rigid bodies. Such structures allow the use of exact feedback linearization [10] techniques, among others, to achieve robust closed loop systems. However, for flexible joint manipulators, Spong pointed out that feedback linearization is possible only for certain simple configurations [11]. DeLuca [12] showed that if dynamic state feedback is considered, any flexible joint (rigid link) manipulator can also be feedback linearized. Since the slow subsystem derived in this paper consists of a rigid manipulator with flexible joints, the latter results can be employed to design a feedback linearizing control for the slow subsystem.

### Modelling

In this paper we consider a flexible manipulator with joint flexibility. The joint flexibility represents the gear train compliance exhibited by many manipulators. In the representation considered here, each link subsystem consists of an actuator inertia connected to a flexible link through a torsional spring. These components of a typical link are illustrated in Figure 1. The dynamic model can be derived using Lagrange's equations with the link deflections being modelled by the assumed modes method. For an n-link serial manipulator with n flexible joints the equations of motion are cast into the standard form:

$$M(q, \delta) \begin{bmatrix} \ddot{q} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} f_1(q, \dot{q}) \\ f_2(q, \dot{q}) \end{bmatrix} + \begin{bmatrix} g_1(q, \dot{q}, \delta, \dot{\delta}) \\ g_2(q, \dot{q}, \delta, \dot{\delta}) \end{bmatrix} + \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix} \begin{bmatrix} q \\ \delta \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix} \quad (1)$$

where

$M$  is a mass/inertia matrix,

$q = [q_1^T \ q_a^T]^T$  is the vector of joint variables  $q_1 = [\theta_1 \ \theta_2 \ \dots \ \theta_n]^T$  and actuator variables  $q_a = [\theta_{a1} \ \theta_{a2} \ \dots \ \theta_{an}]^T$ ,

$\delta = [\delta_{11} \ \delta_{12} \ \dots \ \delta_{1m_1} \ \delta_{21} \ \dots \ \delta_{2m_2} \ \dots \ \delta_{nm_n}]^T$  is the vector of deflection variables  $\delta_{ij}$  representing the  $i^{\text{th}}$  link's  $j^{\text{th}}$  mode and  $m_i$  is the number of modes representing link  $i$ ,

$f_1$  and  $f_2$  are vectors containing gravitational (only in  $f_1$ ), Coriolis and centrifugal terms;  $f_1$  is the same as it would be in the rigid link model,

$g_1$  and  $g_2$  are vectors which account for the interaction of the joint variables and their time derivatives with the link deflection variables and their time derivatives,  
 $K_1$  is a joint stiffness matrix

$$K_1 = \begin{bmatrix} K_a & -K_a \\ -K_a & K_a \end{bmatrix}$$

where  $K_a = \text{diag}(\kappa_1, \kappa_2, \dots, \kappa_n)$  is a diagonal matrix of the constant joint stiffness coefficients,  
 $K_2 = \text{diag}(k_{11}, \dots, k_{1m_1}, k_{21}, \dots, k_{2m_2}, \dots, k_{nm_n})$  is a diagonal matrix of the constant flexural stiffness coefficients,  
 $u = [\tau_1, \tau_2, \dots, \tau_n]^T$  is a control vector of generalized forces applied to the  $n$  joints.

Since the inertia matrix  $M$  is always positive definite, it can be inverted and denoted by  $H$ , which can be partitioned as follows:

$$M^{-1} = H = \begin{bmatrix} H_{11[2n \times 2n]} & H_{12[2n \times \dot{m}] } \\ H_{21[\dot{m} \times 2n]} & H_{22[\dot{m} \times \dot{m}]} \end{bmatrix} \quad (2)$$

where  $\dot{m} = m_1 + m_2 + \dots + m_n$ . Then equation (1) becomes

$$\ddot{q} = -H_{11}(\dot{f}_1 + g_1 + K_1 q - u) - H_{12}(\dot{f}_2 + g_2 + K_2 \delta) \quad (3a)$$

$$\ddot{\delta} = -H_{21}(\dot{f}_1 + g_1 + K_1 q - u) - H_{22}(\dot{f}_2 + g_2 + K_2 \delta) \quad (3b)$$

#### Singularly Perturbed Model

A singularly perturbed model of the system can be obtained using the procedure of Siciliano and Book [4]. Due to the dynamic nature of the system of present interest, the time scale separation is considered to occur between the flexible joint mode and the first flexure mode. Let  $\mu = 1/k$  define a perturbation parameter where  $k$  is the smallest beam flexure spring constant; accordingly, the matrix  $K_2$  becomes

$$K_2 = k\bar{K}_2$$

The system (3) can now be described in terms of the vector of elastic forces  $\zeta$  associated with link flexure defined as  $\zeta = k\bar{K}_2\delta$ . With this change, equations (3)a and (3)b become

$$\ddot{q} = -H_{11}(q, \mu \zeta) [f_1(q, \dot{q}) + g_1(q, \dot{q}, \mu \zeta, \mu \dot{\zeta}) + K_1 q - u] - H_{12}(q, \mu \zeta) [f_2(q, \dot{q}) + g_2(q, \dot{q}, \mu \zeta, \mu \dot{\zeta}) + \zeta] \quad (4a)$$

$$\mu \ddot{\zeta} = -\bar{K}H_{21}(q, \mu \zeta) [f_1(q, \dot{q}) + g_1(q, \dot{q}, \mu \zeta, \mu \dot{\zeta}) + K_1 q - u] - \bar{K}H_{22}(q, \mu \zeta) [f_2(q, \dot{q}) + g_2(q, \dot{q}, \mu \zeta, \mu \dot{\zeta}) + \zeta] \quad (4b)$$

which represent singularly perturbed model of the system under consideration. Notice that the right hand side of (4b) is scaled by the factor  $\bar{K}$ . Letting  $\mu \rightarrow 0$ , (4a) yields the equations for a manipulator with rigid links and flexible joints. If one formally sets  $\mu = 0$ , (4b) can be solved for  $\bar{\zeta}$  to obtain the steady-state solution

$$\bar{\zeta} = -H_{22}^{-1}(\bar{q}, 0)H_{21}(\bar{q}, 0)[f_1(\bar{q}, \dot{\bar{q}}) + g_1(\bar{q}, \dot{\bar{q}}, 0, 0) + K_1\bar{q} - u] - f_2(\bar{q}, \dot{\bar{q}}) - g_2(\bar{q}, \dot{\bar{q}}, 0, 0) \quad (5)$$

where the overbars indicate that the system is analyzed in the slow time scale. Now letting  $\mu = 0$ , replacing  $\zeta$  in (4a) with  $\bar{\zeta}$  as given by (5), and recognizing that by the definition of the terms  $g_1$  and  $g_2$  they are zero in the absence of link flexure, one obtains

$$\ddot{\bar{q}} = [-H_{11}(\bar{q}, 0) - H_{12}(\bar{q}, 0)H_{22}^{-1}(\bar{q}, 0)H_{21}(\bar{q}, 0)][f_1(\bar{q}, \dot{\bar{q}}) + K_1\bar{q} - u] - M_{11}^{-1}(\bar{q})[f_1(\bar{q}, \dot{\bar{q}}) + K_1\bar{q} - u] \quad (6)$$

The fast subsystem or *boundary layer* is derived by introducing a fast time scale  $\tau = t/\epsilon$ , where  $\epsilon = \sqrt{\mu}$ . Then, relative to this time scale, the system (4b) becomes:

$$\frac{d^2\eta}{d\tau^2} = -\bar{K}_z H_{22}(\bar{q}, 0)\eta + \bar{K}_z H_{21}(\bar{q}, 0)(u - \bar{u}) \quad (7)$$

where  $\eta = \zeta - \bar{\zeta}$  is a new set of fast variables. Note that the fast subsystem is a linear system parameterized in the slow variables  $\bar{q}$ , which appear constant with respect to the fast time scale.

#### Control Design

The problem is now to design a composite control for the system described by equations (6) and (7), namely

$$u = \bar{u}(\bar{q}, \dot{\bar{q}}) + u_f(q, \eta, \dot{\eta}) \quad (8)$$

with the constraint  $u_f(\bar{q}, 0, 0) = 0$ , so that  $u_f$  is inactive along the solution (5).

For the slow control, the control techniques established by DeLuca [12] and Spong [9] for flexible joint manipulators are applicable. These techniques are in the general category of feedback linearizing controls which can be expressed as

$$\bar{u}(\bar{q}, \dot{\bar{q}}) = f_1(\bar{q}, \dot{\bar{q}}) + K_1\bar{q} + M_{11}(\bar{q})v \quad (9)$$

which reduces the system (6) to the form

$$\ddot{\bar{q}} = v \quad (10)$$

so that  $v$  is a new input to the system which can be chosen to achieve a prescribed model behavior in the slow subsystem. The control  $v$  is typically selected to cause the system (10) to follow a prescribed linear decoupled model of the form

$$\ddot{\bar{q}} + 2Z\Omega\dot{\bar{q}} + \Omega^2\bar{q} = u_r \quad (11)$$

where  $\Omega$  is a diagonal matrix of the prescribed closed loop natural frequencies,  $Z$  is a diagonal matrix of the prescribed closed loop damping ratios and  $u_r$  is an appropriately defined reference input [13] to ensure global decoupling of the closed loop system.

The fast control must be selected to ensure that the boundary layer system (7) is uniformly stable along the equilibrium trajectory  $\bar{\zeta}$  as given by (5). This can be accomplished provided that the pair

$$A = \begin{bmatrix} 0 & 1 \\ -\bar{K}_2 H_{22}(\bar{q}, 0) & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \bar{K}_2 H_{21}(\bar{q}, 0) \end{bmatrix} \quad (12)$$

is uniformly stabilizable for any slow trajectory  $\bar{q}(t)$ . Assuming that this holds, the boundary layer system (7) can be stabilized to  $\eta = 0$  ( $\zeta = \bar{\zeta}$ ) using a fast state feedback control of the type

$$u_f(\bar{q}, \eta, \dot{\eta}) = K_{pf}(\bar{q})\eta + K_{vf}(\bar{q})\dot{\eta} \quad (13)$$

#### Single-Link Case Study

A single link case study is to be performed to illustrate the procedure. The system considered is a single-axis, single-link arm with compliance in both the boom and the joint. The system is parameterized to resemble the actual dynamics of the second boom of the RMS with the joint being the RMS elbow pitch joint and the tip payload being the wrist hardware. This system retains the fundamental two-time-scale character of the full multi-link RMS model. Since we are considering space manipulators, the influence gravity is not considered. As such, the slow subsystem in this case, which is a single-link rigid manipulator with joint compliance, is a linear system. This means that for the case study, feedback linearization is not required as it would be for the multi-link case.

The model derivation for the single link arm is outlined here. Two coordinate frames are considered, the fixed inertial frame  $x_0, y_0$  and the  $x_1, y_1$  frame which rotates with the root of the flexible arm. Letting  $u_1$  denote the deflection of a differential mass  $dm$  from the  $x_1$  axis, the absolute position vector  $r_{dm}$  of the differential mass is given by:

$$r_{dm} = {}^0R_1^1 r_{dm} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} x_1 \cos \theta - u_1 \sin \theta \\ x_1 \sin \theta + u_1 \cos \theta \end{bmatrix} \quad (14)$$

Let  $u_{1E}$  denote the displacement of the arm's endpoint from the  $x_1$  axis. Then letting the position vector of the tip mass be denoted as  $r_M$  the velocity of the tip mass follows from equation A.1 as:

$$\dot{r}_M = \begin{bmatrix} -\dot{\theta} L \sin \theta - \dot{u}_{1E} \sin \theta - \dot{\theta} u_{1E} \cos \theta \\ \dot{\theta} L \cos \theta + \dot{u}_{1E} \cos \theta - u_{1E} \dot{\theta} \sin \theta \end{bmatrix} \quad (15)$$

The kinetic energy of the system is  $T = T_{link} + T_{mass} + T_{actuator}$  where

$$T_{link} = \frac{1}{2} \int_{link} \dot{r}_{dm} \cdot \dot{r}_{dm} dm = \frac{1}{2} \dot{\theta}^2 \int_{link} x_1^2 dm + \frac{1}{2} \int_{link} \dot{u}_1^2 dm + \dot{\theta} \int_{link} x_1 \dot{u}_1 dm + \frac{1}{2} \dot{\theta}^2 \int_{link} u_1^2 dm \quad (16)$$

$$T_{mass} = \frac{1}{2} M (\dot{r}_M^2 + \dot{r}_M^2) = \frac{1}{2} \theta^2 M L^2 + \frac{1}{2} M \dot{u}_{1E}^2 + \frac{1}{2} J_M \dot{u}_{1E}'^2 \quad (17)$$

with  $u_{1E}' = \left( \frac{\partial u_1}{\partial x_1} \right)_{x_1=L}$

and

$$T_{actuator} = \frac{1}{2} J_a \dot{\theta}_a^2 \quad (18)$$

The potential energy is similarly  $P = P_{link} + P_{actuator}$  where

$$P_{link} = \frac{1}{2} m g L \sin \theta + \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 u_1}{\partial x_1^2} \right)^2 dx \quad (19)$$

and

$$P_{actuator} = \frac{1}{2} \kappa (\theta_a - \theta)^2 \quad (20)$$

The modal expansion is truncated at  $m_l = 2$ , and the mode shapes employed  $\phi_i$  are the orthonormal eigenfunctions for a clamped free beam with a point mass at the distal end.

The matrix coefficients of the equation of motion are given as:

$$M(q, \delta) = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \quad (21)$$

$$\begin{aligned} m_{11} &= J_0 + M L^2 + M (\phi_{1E} \delta_1 + \phi_{2E} \delta_2)^2 \\ m_{12} &= 0 \\ m_{13} &= M L \phi_{1E} + w_1 \\ m_{14} &= M L \phi_{2E} + w_2 \\ m_{22} &= J_a \\ m_{23} &= m_{24} = 0 \\ m_{33} &= m + M \phi_{1E}^2 + J_M \phi_{2E}'^2 \\ m_{34} &= M \phi_{1E} \phi_{2E}' + J_M \phi_{1E}' \phi_{2E}' \\ m_{44} &= m + M \phi_{2E}^2 + J_M \phi_{2E}'^2 \end{aligned} \quad (22)$$

$$f_1 - f_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (23)$$

$$g_1(\theta, \varepsilon, \delta) = \begin{bmatrix} 2M_L \theta [(\phi_{1E} \delta_1 + \phi_{2E} \delta_2) \phi_{1E} \delta_1 + (\phi_{1E} \delta_1 + \phi_{2E} \delta_2) \phi_{2E} \delta_2] \\ 0 \end{bmatrix} \quad (24)$$

$$g_2 = \begin{bmatrix} -M \theta^2 (\phi_{1E}^2 \delta_1 + \phi_{1E} \phi_{2E} \delta_2) \\ -M \theta^2 (\phi_{2E}^2 \delta_2 + \phi_{1E} \phi_{2E} \delta_1) \end{bmatrix} \quad (25)$$

where

$$\begin{aligned} \phi_{iE} &= \phi_{iE}(\xi)_{\xi=1} \quad i = 1, 2 \\ \phi'_{iE} &= \left. \frac{d\phi_i(\xi)}{d\xi} \right|_{\xi=1}, \quad i = 1, 2 \\ w_i &= \rho A L^2 \int_0^1 \phi_i(\xi) \xi d\xi, \quad i = 1, 2, \\ \kappa_{iE} &= \frac{EI}{L^3} \int_0^1 \left[ \frac{d^2 \phi_i(\xi)}{d\xi^2} \right] d\xi, \quad i = 1, 2. \end{aligned} \quad (26)$$

The resulting model has two oscillatory modes associated primarily with boom flexure and an additional mode due to joint compliance.

#### Summary

This paper considers the control of manipulators with flexible links and joints, and particularly the case where the modes due to joint compliance are substantially lower in frequency than the boom flexure dominated modes. We have considered singular perturbation approach, in which time-scale separation produces a slow subsystem model which is equivalent to that of a manipulator with rigid links and flexible joints. In this case, existing results for flexible joint manipulators can be applied to control the slow subsystem. At the time of this writing, simulation results are not available, however that work is currently in progress. It is interesting that some practical examples have a joint oscillation frequency that is relatively close to the first boom flexure frequency. In such a case it may be appropriate to consider a slow subsystem which includes the first boom flexure mode as well as the joint compliance mode. In this case the slow subsystem control design would be more difficult, as the results pertaining to flexible joint, rigid link manipulators do not apply directly. Finally, we point out that a potentially attractive approach would be to rely on designed-in passive damping [14, 15] for stabilization, of the fast subsystem. Such approaches can provide stabilization which is quite robust to model frequency uncertainty, which is of particular concern in the case of flexible manipulators since the frequencies are configuration dependent.

#### Appendix - Tables of Symbols and Parameter Values

$L$	= boom length
$m$	= boom mass
$M$	= payload mass
$J_0$	= boom inertia about joint (including hub inertia)
$J_a$	= actuator inertia (as if on gearbox output side)
$A$	= boom cross sectional area
$E$	= Young's modulus
$I$	= boom area moment of inertia
$f_i$	= frequency of $i^{\text{th}}$ mode
$\rho$	= boom density
$\kappa$	= joint stiffness

The parameters chosen represent an idealized model of the second tubular graphite fiber composite boom of the RMS. The joint considered is the elbow pitch joint. The compliance of the gearbox and its housing are modelled as a single discrete torsional spring. Other gearbox dynamics are not represented. The payload mass  $M$  corresponds to that of the RMS wrist joints and end effector hardware, as if they were a point mass rigidly distal end of the boom. According to this model, the values used for the above parameters are the following.

$L$	= 278 in
$m$	= 0.499 lbf.s <sup>2</sup> /in
$M$	= 0.569 lbf.s <sup>2</sup> /in
$J_0$	= 12,855 lbf.s <sup>2</sup> in
$J_a$	= 66,400 lbf.s <sup>2</sup> in
$EI$	= $9.93 \times 10^8$ lbf.in <sup>2</sup>
$f_1$	= 2.261 Hz
$f_2$	= 24.72 Hz
$\rho A$	= 0.0018 lbf.s <sup>2</sup> /in <sup>2</sup>
$\kappa$	= $9.45 \times 10^6$ in.lbf./rad

#### References

1. Alberts, T.E, Xia, H. and Chen, Y., "Dynamic analysis to evaluate viscoelastic passive damping augmentation for the Space Shuttle Remote Manipulator System", 1990 ASME Winter Annual Meeting, Dallas, TX, Nov. 1990.
2. Sweet, L.M. and Good, M.C., "Redefinition of the Robot Motion Control Problem", *IEEE Control Systems Magazine*, Vol.5, No.3, 1985.
3. Yang, G.-B. and Donath, M., "Dynamic Model of a One-Link Robot Manipulator with Both Structural and Joint Flexibility", *Proc. 1988 IEEE Int. Conf. on Robotics and Automation*, Philadelphia, PA, pp.476-481.
4. Siciliano, B. and Book, W.J., "A singular perturbation approach to control of lightweight flexible manipulators", *International Journal of Robotics Research*, Vol.7, No.4, pp.79-90.

5. Siciliano, B., Prasad, J.V.R., and Calise, A.J., "Design of a Composite Controller for a Two-Link Flexible Manipulator", this conference, January, 1991.
6. Yang, L.-F., and Huang, J.-K., "Two-time-scale control designs for large flexible structures", *Proc. 1990 SPIE Aerospace Sensing Conference*, Orlando, Florida, April 17-19, 1990.
7. Calise, A.J., Prasad, J.V.R. and Siciliano, B., "Design of Optimal Feedback Compensation in Two-Time Scale Systems", *IEEE Trans. on Automatic Control*, Vol.35, No.4, April 1990, pp. 488-492.
8. Siciliano, B., Book, W.J., and De Maria, G., "An Integral Manifold Approach to Control of a One Link Flexible Arm", *Proc. of the 25<sup>th</sup> Conference on Decision and Control*, Athens, Greece, Dec. 1986.
9. Spong, M.W., "Control of Flexible Joint Robots: A Survey", University of Illinois at Urbana-Champaign Coordinated Science Laboratory Report #UILU-ENG-90-2203DC-116, February 1990.
10. Kreutz, K., "On Manipulator Control by Exact Linearization", *IEEE Trans. on Automatic Control*, Vol.34, No.7, July, 1989, pp.763-767.
11. Spong, M.W., "Modeling and Control of Elastic Joint Robots", 1986 ASME Winter Annual Meeting, DSC-Vol.3, *Robotics: Theory and Application*, Anaheim CA, 1986.
12. DeLuca, A., "Dynamic Control Properties of Robot Arms with Joint Elasticity", *Proc. 1988 IEEE Int. Conf. on Robotics and Automation*, Philadelphia, PA, pp. 1574-1580.
13. Spong, M.W. and Vidyasagar, M., *Robot Dynamics and Control*, John Wiley & Sons, 1989.
14. Alberts, T.E., Love, L.J., Bayo, E., "Experiments with Endpoint Control of a Flexible Link Using the Inverse Dynamics Approach and Passive Damping", *Proceedings of the 1990 American Control Conference*, San Diego, CA, May 23-25, 1990, pp. 350-355.
15. Alberts, T.E., Chen, Y. and Xia, H., "On the Effectiveness of Section Length Optimization for Constrained Viscoelastic Layer Damping Treatments", Presented at the SPIE Aerospace Sensing Conference, April 16-20, 1990, SPIE volume: *Advances in Optical Structures*, SPIE Vol. 1303, ISBN 0-8194-0354-7, 1990.