

# Parallel Force/Position Control of Robot Manipulators

Bruno Siciliano

Dipartimento di Informatica e Sistemistica  
Università degli Studi di Napoli Federico II  
Via Claudio 21, 80125 Napoli, Italy  
siciliano@na.infn.it

## Abstract

This paper is aimed at presenting a survey on a class of parallel force/position control schemes which have been proposed in the latest five years by the author and co-workers. Several control schemes have been devised under this framework for the case of contact with a compliant planar surface. A common feature of such schemes is that, at the equilibrium, the force can be shown to be regulated to a desired constant value at the expense of a position error which depends on environment stiffness. If tracking of end-effector position along the unconstrained task space directions is desired, a passivity-based control scheme can be used which can be naturally made adaptive with respect to manipulator dynamic parameters. On the other hand, if only regulation of position is desired, a simple PID control can be used which can be made adaptive as well in the case of imperfect gravity compensation, and can even avoid velocity measurements.

## 1 State-of-Art

Control of interaction between a robot manipulator and the environment is crucial for successful execution of a number of practical tasks where the robot's end effector has to manipulate an object or perform some operation on a surface. The specific feature of robot tasks such as polishing, deburring, or assembly, demands control also of the exchanged forces at the contact. The contact force is thus the quantity describing the state of interaction in the most complete fashion, and the interaction control problem [47] has attracted a wide number of researchers in the last decade.

Interaction control strategies can be grouped in two categories; those performing *open-loop force*

*control* and those performing *direct closed-loop force control*. The main difference between the two categories is that the former achieve indirect force control via closed-loop position control, without explicit closure of a force feedback loop; the latter, instead, offer the possibility of controlling the contact force to a desired value, thanks to the closure of a force feedback loop.

To the first category belong *compliance* (or *stiffness*) *control* [31, 33] and *impedance control* schemes [18, 20], where the position error is related to the contact force through a mechanical stiffness or impedance of adjustable parameters. A robot manipulator under impedance control is described by an equivalent mass-spring-damper system with the contact force as input. The resulting impedance may be linear or nonlinear, depending on the fact whether force feedback is used or not.

The most common strategy belonging to the second category is the *hybrid position/force control* which aims at controlling position along the unconstrained task space directions and force along the constrained task space directions. A selection matrix acting on both desired and feedback quantities serves this purpose for typically planar contact surfaces [32], whereas the explicit constraint equations have to be taken into account for general curved contact surfaces [50, 25, 26, 14, 2, 49].

An alternative strategy still in the second category is the *inner/outer position/force control* where, along each constrained task space direction, an outer force control loop is closed around the inner position control loop which is typically available in a robot manipulator [15]. By a suitable design of the force control action (typically an integral term), it is possible to achieve regulation of the contact force to a desired value. The

inclusion of an integral action in the force control loop, in fact, guarantees removal of the steady-state force error [46] and may provide robustness with respect to force measurement delays [45, 48].

A new strategy has been recently proposed, that is, the so-called *parallel force/position control* [8]. The key concept of this strategy is to combine the simplicity and robustness of the impedance and inner/outer position/force control schemes with the ability of controlling both position and force typical of the hybrid control schemes. In order to embed the possibility of controlling motion along the unconstrained task space directions, a desired position can be input to the inner loop of an inner/outer position/force control scheme. The result is two control actions, working in parallel; namely, a force control action and a position control action. In order to ensure force control along the constrained task space directions, the force action is designed so as to dominate the position action [6].

## 2 Modeling

### 2.1 Manipulator Dynamics

In order to study interaction with the environment, it is worth considering the dynamic model of a robot manipulator in the task (operational) space. This can be written in the form

$$B(x)\ddot{x} + C(x, \dot{x})\dot{x} + g(x) = u - f, \quad (1)$$

where  $x$  is the  $(m \times 1)$  vector of end-effector location,  $B$  is the  $(m \times m)$  symmetric and positive definite inertia matrix,  $C\dot{x}$  is the  $(m \times 1)$  vector of Coriolis and centrifugal generalized forces,  $g$  is the  $(m \times 1)$  vector of gravitational generalized forces,  $u$  is the  $(m \times 1)$  vector of driving generalized forces, and  $f$  is the  $(m \times 1)$  vector of contact generalized forces exerted by the end effector on the environment: all task space quantities are expressed in a common reference frame.

The end-effector direct and differential kinematics are defined by

$$x = k(q) \quad (2)$$

$$\dot{x} = \frac{\partial k(q)}{\partial q} \dot{q} = J_A(q)\dot{q} \quad (3)$$

where  $q$  is the  $(n \times 1)$  vector of joint variables and  $J_A$  is the  $(m \times n)$  manipulator analytical Jacobian matrix [34]. The  $(n \times 1)$  vector  $\tau$  of joint actuating generalized forces is computed as

$$\tau = J_A^T(q)u. \quad (4)$$

When  $m = n$  and the manipulator moves in a singularity-free region of the workspace, the vector of operational variables  $x$  constitutes a set of Lagrangian generalized coordinates and  $B$  assumes the meaning of a true inertia matrix. Instead, in the case of kinematically redundant manipulators ( $m < n$ ),  $B$  is only a pseudo inertia matrix [22].

The following notable properties of the task space dynamic model (1) can be established from similar properties of the joint space dynamic model.

1. The matrix  $B$  is symmetric and positive definite. If  $\lambda_m$  ( $\lambda_M$ ) denotes the minimum (maximum) eigenvalue of  $B$ , then

$$0 < \lambda_m I \leq B(x) \leq \lambda_M I, \quad (5)$$

where  $I$  is the  $(m \times m)$  identity matrix; in the case of all revolute joints, it is  $\lambda_M < \infty$ .

2. There exists a choice of the matrix  $C$  such that the matrix

$$N(x, \dot{x}) = \dot{B}(x, \dot{x}) - 2C(x, \dot{x}) \quad (6)$$

is skew-symmetric [23, 41]. This also implies that

$$\dot{B}(x, \dot{x}) = C(x, \dot{x}) + C^T(x, \dot{x}). \quad (7)$$

Further, the matrix  $C$  is upper-bounded in  $x$  and linear in  $\dot{x}$ ; hence, a constant  $0 < k_C < \infty$  exists such that

$$\|C(x, \dot{x})\| \leq k_C \|\dot{x}\|. \quad (8)$$

3. The gravity force vector  $g$  can be thought of as given by

$$g(x) = \frac{\partial U(x)}{\partial x} \quad (9)$$

where  $U(x)$  is the gravitational energy in the task space that is bounded for any  $x$ . A helpful property of the gravity term is that  $g$  has bounded partial derivatives [44], i.e.,

$$\left\| \frac{\partial g(x)}{\partial x} \right\| \leq \psi \quad (10)$$

for some  $\psi > 0$ . This also implies that

$$\|g(x_1) - g(x_2)\| \leq \psi \|x_1 - x_2\| \quad (11)$$

for any  $x_1$  and  $x_2$ .

4. The dynamic model (1) is linear in terms of a suitable set of manipulator and load constant parameters [28], i.e.,

$$B(\mathbf{x})\ddot{\mathbf{x}} + C(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{g}(\mathbf{x}) = Y(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}})\boldsymbol{\pi} \quad (12)$$

where  $Y(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}})$  is an  $(m \times p)$  matrix and  $\boldsymbol{\pi}$  is a  $(p \times 1)$  vector of parameters which depend on link masses, first moments of inertia and inertia tensors. With specific reference to the gravity term, the vector  $\mathbf{g}$  in (1) can be written in the form

$$\mathbf{g}(\mathbf{x}) = G(\mathbf{x})\boldsymbol{\theta} \quad (13)$$

where  $G(\mathbf{x})$  is an  $(m \times r)$  matrix and  $\boldsymbol{\theta}$  is an  $(r \times 1)$  vector of manipulator and load parameters; the dimension  $r$  is smaller than  $p$ , since the gravity term depends only on link masses and first moments of inertia.

The above properties will be used to derive parallel force/position control schemes other than the basic inverse dynamics parallel control.

In the remainder, the attention is restricted to the case of nonredundant nonsingular manipulators with  $m = n = 3$ , i.e., only translational motion and force components are considered; then,  $\mathbf{x}$  denotes the end-effector position.

## 2.2 Environment

Accurate modeling of the contact between the manipulator and the environment is usually difficult to obtain in analytic form, due to complexity of the physical phenomena involved during the interaction. It is then reasonable to resort to a simple but significant model, relying on the robustness of the control system in order to absorb the effects of inaccurate modeling. Following these guidelines, the case of an environment constituted by a frictionless and elastically compliant plane is analyzed. The choice of a planar surface is motivated by noticing that it is locally a good approximation to surfaces of regular curvature [7]. The total elasticity, due to end-effector force sensor and environment, is accounted through the compliance of the plane. Friction effects are neglected within the operational range of interest.

With the above assumptions, the model of the contact force considered takes on the simple form

$$\mathbf{f} = K(\mathbf{x} - \mathbf{x}_0), \quad (14)$$

where  $\mathbf{x}$  is the position of the contact point,  $\mathbf{x}_0$  is a point of the plane at rest, and  $K$  is the  $(3 \times 3)$

constant symmetric stiffness matrix of rank 1; note that Eq. (14) holds only when the end effector is in contact with the environment and all quantities are expressed in the common reference frame. It is worth considering the rotation matrix expressing the orientation of the contact frame with respect to the reference frame

$$R = (\mathbf{t}_1 \quad \mathbf{t}_2 \quad \mathbf{n}) \quad (15)$$

where  $\mathbf{n}$  is the unit vector normal to the contact plane, and  $\mathbf{t}_1, \mathbf{t}_2$  are two orthogonal unit vectors lying in the plane. In view of (15), the stiffness matrix can be written as

$$K = R \text{diag}\{0, 0, k\} R^T = k \mathbf{n} \mathbf{n}^T, \quad (16)$$

where  $k > 0$  is the stiffness coefficient.

The elastic contact model (14) and (16) suggests that a null force error can be obtained only if  $\mathbf{f}_d = f_d \mathbf{n}$ . If no information about the geometry of the environment is available, i.e., the direction of  $\mathbf{n}$  is unknown, the null vector can be assigned to  $\mathbf{f}_d$  that is anyhow in the range space of any matrix  $K$ . Analogously, it can be recognized that null position errors can be obtained only on the contact plane, while the component of  $\mathbf{x}$  along  $\mathbf{n}$  has to accommodate the force requirement specified by  $\mathbf{f}_d$ ; thus,  $\mathbf{x}_d$  can be freely reached only in the null space of  $K$ , i.e., along the unconstrained directions of the task space.

## 3 Parallel Control

The underlying philosophy of the *parallel control* approach is to combine the simplicity and robustness of the impedance control and the inner/outer position/force control with the capability of controlling both force and position of the hybrid control. This is realized by designing two control loops—one in position and one in force—acting in parallel along each task space direction. Conflicts between position and force actions are handled through a rule-based priority strategy.

A physical analysis of the interaction leads to recognizing that dominance of the force control loop over the position control loop should be achieved so as to accommodate unplanned contact forces in any situation. The most natural way to implement the sought dominance is to use a PI force control loop working in parallel to a PD position control loop. In this respect, the scheme can be regarded as an extension of an impedance control scheme (with added direct

force control capabilities) and an inner/outer position/force control scheme (with improved position control capabilities). At the same time, force and position controlled directions are not established a priori in the parallel control, as instead in the hybrid control; full sensor measurements can thus be exploited without any task-based filtering action.

The task planning provides force or position references along suitable task space directions, as in the hybrid control case. A perfect planning obviously makes the task successful, but contact is safely handled by the parallel control even in the case of planning errors. Recovery from imperfect planning is made possible thanks to the force dominance rule.

### 3.1 Inverse Dynamics

According to an *inverse dynamics* concept with contact force sensor measurements, the vector of driving forces in (1) can be synthesized as

$$\mathbf{u} = \widehat{\mathbf{B}}(\mathbf{x})\mathbf{a} + \widehat{\mathbf{C}}(\mathbf{x}, \dot{\mathbf{x}})\dot{\mathbf{x}} + \widehat{\mathbf{g}}(\mathbf{x}) + \mathbf{f}, \quad (17)$$

where the hats denote the available estimates of the dynamic terms  $\mathbf{B}$ ,  $\mathbf{C}$ ,  $\mathbf{g}$ , and  $\mathbf{f}$  is the measured contact force. Substituting control (17) into model (1), under the assumption of perfect dynamic compensation and exact force cancellation, gives

$$\ddot{\mathbf{x}} = \mathbf{a} \quad (18)$$

that is a linear decoupled system expressing a resolved acceleration.

Let  $\mathbf{x}_d$  and  $\mathbf{f}_d$  denote the desired values of position and force, respectively. According to the parallel control approach, the new control input is designed as the sum of a position control action and a force control action; namely, as [6]

$$\mathbf{a} = \mathbf{a}_x + \mathbf{a}_f \quad (19)$$

where

$$\mathbf{a}_x = \ddot{\mathbf{x}}_d + m_d^{-1}k_D\Delta\dot{\mathbf{x}} + m_d^{-1}k_P\Delta\mathbf{x} \quad (20)$$

$$\mathbf{a}_f = m_d^{-1}k_F\Delta\mathbf{f} + m_d^{-1}k_I\int_0^t \Delta\mathbf{f}d\sigma \quad (21)$$

where  $\Delta\mathbf{x} = \mathbf{x}_d - \mathbf{x}$  is the position error,  $\Delta\mathbf{f} = \mathbf{f}_d - \mathbf{f}$  is the force error,  $m_d$  is a desired mass, and  $k_P, k_D, k_F, k_I > 0$  are suitable feedback gains. Substituting (19) with (20) and (21) in (18) yields

$$m_d\Delta\ddot{\mathbf{x}} + k_D\Delta\dot{\mathbf{x}} + k_P\Delta\mathbf{x} + k_F\Delta\mathbf{f} + k_I\int_0^t \Delta\mathbf{f}d\sigma = 0 \quad (22)$$

which reveals that, thanks to the *integral* action,  $\Delta\mathbf{f}$  is allowed to prevail over  $\Delta\mathbf{x}$  at steady state.

Assuming that the desired force is aligned with the normal to the contact plane and contact is not lost, the stability analysis of the system (1) under the inverse dynamics parallel control (17), (19), (20) and (21) with the environment (14) and (16) can be developed according to classical linear systems theory. In detail, plugging (14) in (22) gives

$$\begin{aligned} & m_d\ddot{\mathbf{x}} + k_D\dot{\mathbf{x}} + (k_P\mathbf{I} + k_F\mathbf{k}\mathbf{n}\mathbf{n}^T)\mathbf{x} \\ & + k_I\mathbf{k}\mathbf{n}\mathbf{n}^T\int_0^t \mathbf{x}d\sigma \quad (23) \\ & = m_d\ddot{\mathbf{x}}_d + k_D\dot{\mathbf{x}}_d + k_P\mathbf{x}_d \\ & + k_F(\mathbf{f}_d + \mathbf{k}\mathbf{n}\mathbf{n}^T\mathbf{x}_0) + k_I\int_0^t (\mathbf{f}_d + \mathbf{k}\mathbf{n}\mathbf{n}^T\mathbf{x}_0)d\sigma \end{aligned}$$

which represents a third-order linear system, whose stability can be analyzed by referring to the unforced system, i.e., by setting to zero the right-hand side of (23). According to (15), projection of the position vector on the contact frame yields

$$\mathbf{R}^T\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_n \end{pmatrix} \quad (24)$$

which leads to the system of three scalar decoupled equations

$$m_d\ddot{x}_1 + k_D\dot{x}_1 + k_Px_1 = 0 \quad (25)$$

$$m_d\ddot{x}_2 + k_D\dot{x}_2 + k_Px_2 = 0 \quad (26)$$

$$m_d\ddot{x}_n + k_D\dot{x}_n + (k_P + k_Fk)x_n \quad (27)$$

$$+ k_Ik\int_0^t x_n d\tau = 0$$

revealing that a stable behavior is ensured by a proper choice of the feedback gains  $k_P, k_D, k_F, k_I$  for the third equation. The following remarks are in order [10]:

- Stability is obtained independently of the actual normal direction to the plane; this essential feature of the parallel approach allows designing the controller based on the contact stiffness coefficient while the actual contact geometry is taken into account only at the planning level.
- The decoupled dynamics of the system (23) derives from structural properties of the parallel control scheme by virtue of the contact force measurement; this is different from the hybrid approach where a decoupled dynamics is imposed by the control law on the basis of the environment model.

## 4 Force Regulation and Position Tracking

The above parallel force/position control scheme is based on an inverse dynamics concept and, as such, is sensitive to the effects of unmodelled dynamics or disturbances.

### 4.1 Passivity-based control

For the motion control problem, it has been argued that *passivity-based* controllers are expected to have enhanced robustness with respect to inverse dynamics controllers, since they do not rely on the exact cancellation of nonlinear terms [1].

In the following, a passivity-based parallel control scheme is presented where, like in the case of motion control, the resulting control law is composed of a nonlinear model-based term and a linear compensator action. A functional expression of the reference vector is established to be used in the controller which is related both to the end-effector position error and to the contact force error. This is conceptually different from previous passivity-based hybrid force/position controllers [24] where each component of the reference vector is related either to a position error or to a force error in respect of the task space selection mechanism.

Consider the following control law

$$u = \hat{B}(x)\dot{r} + \hat{C}(x, \dot{x})r + \hat{g}(x) - k_D(\dot{x} - r) + f, \quad (28)$$

where the hats denote the estimates,  $r$  is a  $(3 \times 1)$  reference vector, and  $k_D > 0$ .

Assuming that  $\hat{B}$ ,  $\hat{C}$ ,  $\hat{g}$  have the same functional form of  $B$ ,  $C$ ,  $g$ , the control law (28) can be rewritten, in view of (12), as

$$u = Y(x, \dot{x}, r, \dot{r})\tilde{\pi} - k_D(\dot{x} - r) + f. \quad (29)$$

Setting

$$r = \dot{x} - e \quad (30)$$

$$\dot{r} = \ddot{x} - \dot{e} \quad (31)$$

where  $e$  is a suitable error vector, and combining (1) with (29) gives

$$B(x)\dot{e} + C(x, \dot{x})e + k_De = Y(\cdot)\tilde{\pi} \quad (32)$$

where  $\tilde{\pi} = \hat{\pi} - \pi$  is the parameter error vector.

From passivity theory and the skew symmetry of the matrix in (6), it is well known that if the mapping  $-e \mapsto Y(\cdot)\tilde{\pi}$  is passive and  $Y(\cdot)\tilde{\pi}$  is

bounded, then  $e \rightarrow 0$  as  $t \rightarrow \infty$  [29]. Thus, the key point is to find an error vector  $e$  related to the force and position errors, so that the force and position errors are the outputs of an exponentially stable, strictly proper filter whose input is  $e$  [16].

Let then  $x_d$  denote the *time-varying* desired end-effector position with bounded derivatives up to the second order, and  $f_d$  denote the *constant* desired force. Imposing a constant force is not a limitation from a practical viewpoint, since typical contact tasks require force regulation to a desired amount.

A keen choice for the error vector in (32) is

$$e = (s + k_x(s))\Delta x + s^{-1}k_f(s)\Delta f \quad (33)$$

where  $s$  is the Laplace variable and  $k_x(s)$ ,  $k_f(s)$  denote the transfer functions of the position and force compensators to be designed. It is worth pointing out that, in view of (30) and (31),  $k_x(s)$  and  $k_f(s)$  in (33) must have relative degree greater than or equal to 0 for the implementation of the control law (28) with only measurements of position, velocity and force.

The first term on the right-hand side of (33) is the same as in typical passivity-based motion control schemes. The presence of an *integral* action in the second term is motivated by the desire to achieve a null steady-state force error at the expense of a finite steady-state position error along the constrained task space direction, on condition that a suitable choice of  $k_x(s)$  and  $k_f(s)$  is made. It is assumed that  $k_x(s)$  does not have poles at the origin.

By virtue of (14) and (16), Eq. (33) can be effectively decomposed into the component along  $n$  and the components on the contact plane. Hence, the analysis is reduced to the following three (one + two) equations:

$$e_n = (s + k_x(s))\Delta x_n + s^{-1}k_f(s)\Delta f_n \quad (34)$$

$$e_t = (s + k_x(s))\Delta x_t - s^{-1}k_f(s)f_{dt}, \quad (35)$$

where, with obvious notation, Eq. (34) involves the normal components of the force and position vectors, while Eq. (35) involves the components on the plane. Notice that the contact model (14) and (16) implies  $f_t = 0$ ; hence it is reasonable to choose  $f_{dt} = 0$ . As emphasized in the previous section, if the direction of  $n$  is unknown, then it is advisable to choose  $f_d = 0$ .

The position along the normal direction is given by

$$x_n = k^{-1}\Delta f_n + k^{-1}f_{dn} + x_{0n}. \quad (36)$$

From (34) and (36), it follows that

$$\Delta f_n = k(g(s)e_n + h(s)(x_{dn} - k^{-1}f_{dn} - x_{0n})) \quad (37)$$

where

$$g(s) = \frac{s}{s^2 + k_x(s)s + kk_f(s)} \quad (38)$$

$$h(s) = \frac{s(s + k_x(s))}{s^2 + k_x(s)s + kk_f(s)} \quad (39)$$

Further, Eq. (35) with  $f_{dt} = 0$  gives

$$\Delta x_t = \frac{1}{s + k_x(s)} e_t \quad (40)$$

Notice that, since  $k_x(s)$  and  $k_f(s)$  have relative degree greater than or equal to 0, both  $g(s)$  and  $1/(s + k_x(s))$  have relative degree 1, whereas  $h(s)$  is proper. Therefore,  $k_x(s)$  and  $k_f(s)$  must be chosen so that the transfer functions  $g(s)$  and  $1/(s + k_x(s))$  are both exponentially stable. In that case, from (39), also  $h(s)$  is exponentially stable.

On the other hand, the second term on the right-hand side of (37) plays the role of a disturbance on the force error along the normal. Since  $h(s)$  possesses at least a zero at the origin, the effect of a constant disturbance is rejected.

It can be demonstrated that, in the *known parameter case* ( $\tilde{\pi} = 0$ ), if  $k_x(s)$  and  $k_f(s)$  are chosen so that both  $g(s)$  and  $1/(s + k_x(s))$  are exponentially stable and the poles of  $k_x(s)$  and  $sk_f(s)$  have all negative real parts, then  $\Delta x_t, \Delta \dot{x}_t \rightarrow 0$  as  $t \rightarrow \infty$  and all signals in the system remain bounded. Further, if  $x_{dn}$  is a constant, then  $\Delta f_n \rightarrow 0$  as  $t \rightarrow \infty$ . The proof can be found in [39] and is based on typical passivity arguments.

On the other hand, in the *unknown parameter case* ( $\tilde{\pi} \neq 0$ ), it is sufficient to add to the control law (28), the parameter *adaptive law*

$$\dot{\hat{\pi}} = -\Gamma^{-1}Y^T(\cdot)e \quad (41)$$

where  $\Gamma$  is a  $(p \times p)$  symmetric, positive definite matrix. It can be shown that the same result is recovered as in the known parameter case.

The above results are based on a suitable choice of  $k_x(s)$  and  $k_f(s)$ . The simplest choice is to take them as constants, i.e.,  $k_x(s) = \lambda_1$  and  $k_f(s) = \lambda_2$  [37] with  $\lambda_1, \lambda_2 > 0$ . This ensures that the transfer functions (38) and (39) are always exponentially stable independently of  $k$ , although the actual value of the stiffness coefficient affects the performance of the system. By virtue

of this choice, from (30) and (31) the reference vector  $r$  and its derivative become

$$r = \dot{x} - e = \dot{x}_d - \lambda_1 \Delta x - \lambda_2 \int_0^t \Delta f d\sigma \quad (42)$$

$$\dot{r} = \ddot{x} - \dot{e} = \ddot{x}_d - \lambda_1 \Delta \dot{x} - \lambda_2 \Delta f \quad (43)$$

It can be shown that the control law (28) with (42) and (43) leads to the following equilibrium trajectory

$$x_e = (I - nn^T)x_d + nn^T(k^{-1}f_d + x_0) \quad (44)$$

$$\dot{x}_e = (I - nn^T)\dot{x}_d = \dot{x}_d \quad (45)$$

$$\ddot{x}_e = (I - nn^T)\ddot{x}_d = \ddot{x}_d \quad (46)$$

$$f_e = k nn^T(x_e - x_0) = f_d \quad (47)$$

on condition that  $x_{dn}$  is a constant. A Lyapunov stability proof of the closed-loop system around the above trajectory can be found in [38]. Notice that the above trajectory is the same that can be obtained with an inverse dynamics control with perfect dynamic compensation.

Notice that the above expressions (42) and (43) are similar in nature to the reference trajectories proposed in [42] and [2] which were derived in the joint space, though. Differently from those works, however, here no selection between force-controlled and position-controlled components of  $r$  and  $\dot{r}$  is accomplished by the control law (28). Interestingly enough, the control law (28) with (42) and (43) can be regarded as an extension of the impedance controller in [21] in that a desired force  $f_d$  different from zero can be specified.

Another simple choice for the force action is  $k_f(s) = \lambda_2 + \lambda_3/s$  which determines an additional zero of  $h(s)$  at the origin and thus ensures a null steady-state force error along the normal in case of a constant  $\dot{x}_{dn}$ . Nevertheless, the presence of a double integrator on the force error may lead to an oscillatory behavior in the force response. More complex choices for  $k_x(s)$  and  $k_f(s)$  are feasible if suitable filtering actions on the position and force errors are sought. In such cases more accurate estimates of the stiffness coefficient are required to tune the coefficients of the compensators.

In summary, both in the known and in the unknown parameter case, it is possible to design a passivity-based control scheme (with adaptive law) which guarantees *tracking* of the end-effector position along the unconstrained directions with *regulation* of the contact force along the constrained direction.

## 5 Force and Position Regulation

The parallel control scheme presented in the above section requires complete knowledge of the manipulator dynamic model in order to ensure tracking of the desired end-effector position along the constrained task space directions. Nevertheless, if the actual motion along the contact surface is not relevant but only a desired contact force is to be maintained, a computationally lighter control scheme can be devised which ensures *force regulation* along the constrained task space directions and *position regulation* along the unconstrained task space directions.

### 5.1 PID control

Consider the *constant* set points  $x_d$  and  $f_d = f_d n$ , and the control law

$$u = k_P \Delta x - k_D \dot{x} + \hat{g}(x) + f_d + k_F \Delta f + k_I \int_0^t \Delta f d\sigma \quad (48)$$

where  $k_P, k_D, k_F, k_I > 0$ . This controller corresponds to position PD action + gravity compensation + desired force feedforward + force PI action. Notice that the use of gravity compensation is the only model-based requirement and is inherited from ordinary PD position control to recover steady-state position errors [43].

Remarkably, it can be shown that in the case of perfect gravity compensation ( $\hat{g} = g$ ) the equilibrium for the system (1) under control (48) is described by the same position in (44) and force in (47) [9].

The study of stability for the force/position regulation case requires methods from nonlinear systems theory. To this purpose, an energy-based argument inspired by the kind of Lyapunov functions used for stability of PID position control [3] can be pursued. It should be emphasized that the Lyapunov method is used only as a means to prove stability of the closed-loop system, and not to derive the control law in a constructive manner; the control law, in fact, has been postulated above on the basis of physical considerations related to the parallel approach in a problem of interaction with an elastically compliant planar surface.

The key point is to find a state description for the system which is suitably augmented to take into account the interaction force in respect of the constraints imposed by the contact. Such a

description should lead to a Lyapunov function composed of a potential energy term related to the deviation from the equilibrium contact position, a kinetic energy term related to the system rate of motion, as well as a term related to the energy stored along the normal direction to the plane due to the integral force action. This is accomplished by considering the  $(7 \times 1)$  state vector [11].

$$z = (\dot{x}^T \quad \epsilon^T \quad w)^T, \quad (49)$$

where

$$\epsilon = x_e - x = \Delta x + d n \quad (50)$$

$$w = k^{-1} n^T \left( \int_0^t \Delta f d\sigma - k_P k_I^{-1} d n \right), \quad (51)$$

with

$$d = k^{-1} f_d + x_{0n} - x_{dn} \quad (52)$$

being a constant quantity taking into account the effects of the environment contact force and the desired force set point along the constrained task space direction. It is important to remark that  $z = 0$  corresponds to the equilibrium (44) and (47), as can be easily verified. Also, note that the following relations hold:

$$n^T \epsilon = k^{-1} n^T \Delta f \quad (53)$$

$$\dot{\epsilon} = -\dot{x} \quad (54)$$

$$\dot{w} = n^T \epsilon. \quad (55)$$

The augmented system described by (1), (54), and (55) under the control (48) with  $\hat{g} = g$  can be written in the standard compact homogeneous form

$$\dot{z} = F z \quad (56)$$

where

$$F = \begin{pmatrix} -B^{-1}(C + k_D I) & B^{-1}(k_P I + k'_F k n n^T) \\ -I & 0 \\ 0^T & n^T \\ & k_I k B^{-1} n \\ & 0 \\ & 0 \end{pmatrix} \quad (57)$$

with  $k'_F = 1 + k_F$ . Notice that some handy reductions—using the structural properties of  $K$  in (16) and the definition of  $w$  in (51)—have been performed to derive (57).

On the basis of the above augmented state space description, suitable Lyapunov function candidates can be constructed to derive local stability results around the origin of the state space

in (49). The key feature of such functions is the introduction of off-diagonal terms and positive constants which are remarkably not used by the control law. These constants serve as additional degrees of freedom to satisfy conditions on the feedback gains guaranteeing stability of the system (56) and (57). Two major results can be established.

- *Local asymptotic stability* can be demonstrated by choosing the following Lyapunov function:

$$V = \frac{1}{2} z^T P z, \quad (58)$$

where

$$P = \begin{pmatrix} B & -\rho B \\ -\rho B & (k_P + \rho k_D)I + k'_F k n n^T \\ 0^T & k_I k n^T \\ & 0 \\ & k_I k n \\ & \rho k_I k \end{pmatrix} \quad (59)$$

with  $\rho > 0$ .

- *Local exponential stability* can be demonstrated by choosing the following Lyapunov function:

$$W = \frac{1}{2} z^T Q z, \quad (60)$$

where

$$Q = \begin{pmatrix} B & -\delta B \\ -\delta B & (k_P + \delta k_D)I + k'_F k n n^T \\ -\gamma n^T B & (k_I k + \gamma k_D) n^T \\ & -\gamma B n \\ & (k_I k + \gamma k_D) n \\ \delta k_I k + \gamma(k_P + k'_F k) \end{pmatrix}, \quad (61)$$

with  $\delta, \gamma > 0$ .

The stability proof exploits the skew symmetry of the matrix in (6) and the properties (5), (7) and (8). Technical details are omitted for brevity and can be found in [12]. It is worth reporting here that  $k_P$  is not involved by the conditions on the feedback gains that guarantee local asymptotic stability, and then is available to meet further design requirements during the unconstrained phase of the task. On the other hand, local exponential stability is more demanding and in fact leads to more complex conditions on the feedback gains involving also  $k_P$ . Ensuring exponential stability, though, provides a performance

measure of the system rate of convergence to the equilibrium.

In the case of imperfect gravity compensation ( $\hat{g} \neq g$ ), the equations of the system (1) under control (48) become

$$\dot{z} = Fz + \mu \quad (62)$$

with  $z$  as in (49),  $F$  as in (57), and

$$\mu = \begin{pmatrix} -B^{-1}G(x)\tilde{\theta} \\ 0 \\ 0 \end{pmatrix} \quad (63)$$

where  $\tilde{\theta} = \theta - \hat{\theta}$  is the parameter error vector in (13).

Remarkably, local asymptotic stability holds also in the case of imperfect gravity compensation [35], and the proof exploits the properties in (9), (10), and (11). It can be shown that a different equilibrium is reached for the system (1) under the control (48) with  $\hat{g} \neq g$ , i.e.,

$$\hat{x}_e = k^{-1} n n^T (f_d + k x_0) \quad (64)$$

$$+(I - n n^T)(x_d - k_P^{-1}(g(\hat{x}_e) - \hat{g}(\hat{x}_e)))$$

$$f_e = f_d. \quad (65)$$

In this case, the force set point is still attained while a different end-effector equilibrium position is reached compared to the case of perfect gravity compensation. More specifically, a comparison between (44) and (64) reveals that the components of  $x$  along the constrained task space direction  $n$  coincide, while the imperfect gravity compensation only affects the components of  $x$  along the unconstrained task space directions  $t_1, t_2$ .

In order to counteract imperfect estimation of the gravity term, the control law (48) can be made adaptive with respect to the vector of parameters in (13) by adding the parameter estimate update law

$$\dot{\hat{\theta}} = -\frac{1}{\nu} G^T(x) \left( \dot{x} - \delta \left( \Delta x + k_I k_P^{-1} \int_0^t \Delta f d\sigma \right) \right) \quad (66)$$

with  $\nu > 0$ .

Local asymptotic stability around the original equilibrium (44) and (47) can be proven [36]. It is worth mentioning here that the proof is based on the Lyapunov function candidate

$$W' = \frac{1}{2} z^T Q z + \frac{1}{2} \nu \tilde{\theta}^T \tilde{\theta}, \quad (67)$$

in which noticeably  $Q$  is the same as in (61). Differently from the local exponential stability

case,  $\gamma$  is not available to satisfy conditions on the feedback gains but it is keenly chosen to render the adaptation law a function of physically measurable quantities already used in the control law (48), i.e., position, velocity and force measurements.

## 5.2 Output feedback

The implementation of the control law (48) requires joint velocity measurements for the computation of the end-effector velocity vector  $\dot{x}$  through the differential kinematics equation (3). A number of robot manipulators are endowed with joint position sensors (encoders or resolvers) only, and joint velocities have to be reconstructed. Hence, it is worth considering the problem of designing a stable force/position regulator *without velocity measurements*.

The regulator presented above can be enhanced in an output feedback setting by relaxing the requirement of joint velocity measurements. Velocity can be reconstructed through a suitable linear filtering action, similarly to the case of motion controllers in [4, 30], on condition that an extra term is introduced in the control law.

It can be shown that the system remains asymptotically stable around the equilibrium (44) and (47) even if  $\dot{x}$  in the control law (48) is replaced with a vector  $v$  obtained by filtering the position vector  $x$ , i.e.,

$$v = \text{diag} \left\{ \frac{\beta s}{s + \alpha} \right\} x, \quad (68)$$

where  $\alpha, \beta > 0$  and  $s$  is the Laplace variable.

Consider the control law

$$u = k_P \Delta x - k_D v + g(x) + f_d + k_F \Delta f + k_I \int_0^t \Delta f d\sigma - k_\phi \phi \quad (69)$$

where  $k_\phi > 0$  and  $\phi$  is obtained as

$$\phi = \text{diag} \left\{ \frac{\beta}{s + \alpha} \right\} \Delta f. \quad (70)$$

Assuming  $v(0) = 0$  in (68) and  $\phi(0) = \beta \Delta f(0)$  in (70), by virtue of (14), the following equality holds

$$\dot{\phi} = -Kv. \quad (71)$$

In order to study stability of the system (1) with (14) and (68)–(71), the same error vector as in (50) can be considered. Substituting (69) into (1), and accounting for (50) and (54), gives

$$B(x)\ddot{e} + C(x, \dot{x})\dot{e} + k_P e - k_D v + y n = 0 \quad (72)$$

where

$$y = n^T \left( k'_F \Delta f + k_I \int_0^t \Delta f d\sigma - k_\phi \phi + k_P d n \right). \quad (73)$$

Differentiating (73) with respect to time, and taking into account (53), (54), (71), and (16), gives

$$\dot{y} = k n^T (k'_F \dot{e} + k_I e + k_\phi v). \quad (74)$$

In view of (54), the time domain equivalent of (68) is

$$\dot{v} = -\alpha v - \beta \dot{e}. \quad (75)$$

Eqs. (72), (74), and (75) provide a state space representation of the closed-loop system (1), (68), (69), and (71) in terms of the  $(10 \times 1)$  state vector

$$z' = (\dot{e}^T \quad e^T \quad v^T \quad y)^T. \quad (76)$$

The system equations can be rewritten in the standard compact homogeneous form:

$$\dot{z}' = H z' \quad (77)$$

with

$$H = \begin{pmatrix} -B^{-1}C & -k_P B^{-1} & k_D B^{-1} & -B^{-1}n \\ I & O & O & 0 \\ -\beta I & O & -\alpha I & 0 \\ k k'_F n^T & k k_I n^T & k k_\phi n^T & 0 \end{pmatrix} \quad (78)$$

Local asymptotic stability of the system (77) and (78) around the origin of the state space can be demonstrated by choosing the following Lyapunov function:

$$V' = \frac{1}{2} z'^T S z' \quad (79)$$

where

$$S = \begin{pmatrix} B & \rho_1 B & \rho_1 \rho_2 B & 0 \\ \rho_1 B & k_P I & O & 0 \\ \rho_1 \rho_2 B & O & \beta^{-1} k_D I & 0 \\ 0^T & 0^T & 0^T & (k'_F k)^{-1} \end{pmatrix} \quad (80)$$

with

$$\rho_1 = k'_F{}^{-1} k_I \quad (81)$$

$$\rho_2 = k_P^{-1} k_D. \quad (82)$$

As in the previous case of full state feedback, the stability proof exploits the skew symmetry of the matrix in (6) and the properties (5), (7) and (8); in particular, the gain of the additional term in (69) is to be chosen as

$$k_\phi = \rho_2 k_I = k_P^{-1} k_D k_I. \quad (83)$$

Technical details are omitted for brevity and can be found in [40]. It is worth reporting here that the the conditions on the feedback gains that guarantee local asymptotic stability can be satisfied independently of the value of the environment stiffness  $k$ .

## 6 Future Work

This paper has surveyed a number of force/position control schemes for a robot manipulator interacting with a compliant planar surface. The schemes have been developed in the so-called parallel framework which exploits full force and position measurements. Contact force regulation along the constrained task space directions and either regulation or tracking of end-effector position along the unconstrained task space directions can be achieved.

It is worth pointing out that the advantageous feature of the presented schemes consists in the simplicity of the control laws which do not contain explicit information on the environment. It should be clear, however, that the geometry and the mechanical characteristics of the environment influence the performance of the manipulator during the contact under such kind of controllers. Extensive simulation results can be found in the papers in the list of References. On the other hand, initial experimental results of parallel control have been performed and will be described in forthcoming papers, e.g., in [13].

Two existing limitations of the approach, which anyhow are common also to the majority of force/position control schemes, are:

- All control schemes have been derived on the assumption that the manipulator's end effector has already come in contact with the environment and, once contact is established, it is not lost. Impact phenomena may occur which deserve careful consideration [19], and there is a need for global analysis of control schemes including the transition from non-contact into contact and vice versa, e.g., [27, 5].
- Only translational motion and linear force have been considered. Hence, there is a need of extending the control schemes to the full-dimensional task space. Preliminary results on a stability analysis of 6-degree-of-freedom parallel force/position control can be found in [17].

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