

Dynamic Manipulability Ellipsoid for Cooperating Robots

P. Chiacchio, S. Chiaverini, B. Siciliano

Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli
"Federico II", Via Claudio 21, 80125 Napoli, Italy

ABSTRACT

A new definition of dynamic manipulability ellipsoid for a system composed of multiple arms holding a rigid object is provided in this paper. This is obtained by constructing the mapping of object acceleration onto joint driving torques, via the proper kinematic and dynamic equations characterizing the system. In a number of case studies, the inertial properties of cooperating robot systems are illustrated by the above ellipsoid.

INTRODUCTION

Manipulability measures have been widely recognized as an effective quantitative means for analysis, design and control of advanced robot manipulator systems. In the case of a single robot, for instance, the manipulability can be interpreted as the ability of arbitrarily changing the position and orientation of the robot's end-effector for a given arm posture. Alternatively, a manipulator can be reconfigured to the most favourable configuration to execute an assigned task, by taking advantage of the above measure.

Kinematic and dynamic manipulability measures have been introduced for single arms. Kinematic manipulability measures [1] are based on the kineto-static mappings which relate joint velocities to end-effector velocities, through the manipulator Jacobian, and dually end-effector forces to joint torques, through the transpose of the Jacobian. The concepts of velocity and force ellipsoids have been established as a comprehensive tool to characterize the robot manipulability. Isotropy design criteria and singularity avoidance are some examples where manipulability ellipsoids are of great help. On the other hand, dynamic manipulability measures [2,3] take the arm dynamics into account and then are based on the relationship between joint actuator torques and end-effector accelerations, through the manipulator Jacobian and inertia.

In view of the increasing interest in cooperative robot manipulation, we believe that the determination of suitable manipulability measures for multiple arm systems is of crucial importance to evaluate the advantages offered by cooperation. The direct extension of the well-known results for a single arm is not allowed because of the mechanical constraints imposed by the closed-chain system constituted by the robots and the manipulated object.

Recently, we have introduced force and velocity (static) manipulability ellipsoids for multiple arm system [4,5] based on a global task space description of external and internal forces as well as of absolute and relative velocities at the object level [6]. The present work is aimed to provide a new definition of dynamic manipulability

ellipsoid for cooperating robots which is derived by expressing the joint driving forces of the multiple arms as a function of the object acceleration. This allows a complete characterization of the inertial properties of the system. The effects of the configuration of each single manipulator onto the cooperative system dynamic ellipsoid are evidenced and a number of case studies are developed for planar type robots.

MODELLING OF MULTIPLE SINGLE ROBOTS

In order to obtain a description of the kinematics and dynamics of a system comprised of multiple robots holding a rigid object, it is necessary to aggregate the kinematics and dynamics of the single robots in a suitable form.

Kinematics

Consider a number of K manipulators; let $q_i \in \mathbb{R}^{n_i}$ be the joint vector for each manipulator. The joint vector $q \in \mathbb{R}^N$ can be defined as

$$q = \begin{bmatrix} q_1 \\ \vdots \\ q_K \end{bmatrix} \quad (1)$$

where $N = \sum_{i=1}^K n_i$ is the dimension of the extended joint space.

Then, let us denote by m the dimension of the common task space of interest. Let $v_i \in \mathbb{R}^m$ and $J_i(q_i) \in \mathbb{R}^{m \times n_i}$ respectively be the end-effector velocity vector and relative Jacobian matrix for each manipulator. The differential kinematic equation mapping the joint velocity vector \dot{q} onto the end-effector velocity vector $v \in \mathbb{R}^M$,

$$v = \begin{bmatrix} v_1 \\ \vdots \\ v_K \end{bmatrix} \quad (2)$$

can be established as $v = J\dot{q}$, where

$$J = \text{diag}(J_1 \dots J_K) \quad (3)$$

is the extended Jacobian matrix, being $M = Km$ the dimension of the contact task space. The above equation $v = J\dot{q}$ can be differentiated to obtain accelerations, i.e.

$$a = J(q)\ddot{q} + \dot{J}(q, \dot{q})\dot{q} \quad (4)$$

where a is the extended vector of end-effector accelerations.

Dynamics

In accordance to what developed above, let $\tau_i, c_i, g_i \in R^{n_i}$ respectively be the vectors of joint driving torques, Coriolis/centrifugal forces, and gravitational forces for each manipulator. Also, let $M_i \in R^{n_i}$ be the inertia matrix, and $h_i \in R^m$ the vector of end-effector contact forces. The dynamic equation into the extended joint space can be derived as

$$\tau = M(q)\ddot{q} + c(q, \dot{q}) + g(q) + J^T(q)h \quad (5)$$

where the vectors $\tau, c, g \in R^N$ are

$$\tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_K \end{bmatrix}, \quad c = \begin{bmatrix} c_1 \\ \vdots \\ c_K \end{bmatrix}, \quad g = \begin{bmatrix} g_1 \\ \vdots \\ g_K \end{bmatrix}; \quad (6)$$

the matrix $M \in R^{N \times N}$ is

$$M = \text{diag}(M_1 \dots M_K); \quad (7)$$

and the vector $h \in R^M$ is

$$h = \begin{bmatrix} h_1 \\ \vdots \\ h_K \end{bmatrix}. \quad (8)$$

In the following, the dependence of various terms on the joint configuration will be often omitted for notation compactness.

MODELLING OF COOPERATING ROBOT SYSTEM

If the robots are assumed as tightly grasping the rigid object, the relation between the end-effector contact forces and the resulting force on the object can be established following the formulation proposed in [6] for dual robot systems, and later generalized in [5] for multiple robots. The object dynamics must then be incorporated to relate external forces to object accelerations.

Force and acceleration composition

Let then $h_o \in R^m$ denote the vector of external forces applied at the center of mass of the object, where m is the dimension of the object task space. The force composition equation is obtained as

$$h_o = Wh \quad (9)$$

where $W \in R^{m \times M}$ is the grasp matrix given by

$$W = [W_1 \dots W_K]. \quad (10)$$

The grasp sub-matrices $W_i \in R^{m \times n_i}, i = 1, \dots, K$ are

$$W_i = \begin{bmatrix} I & O \\ R_i & I \end{bmatrix} \quad (11)$$

where I and O respectively denote identity and null matrices of appropriate dimensions, and R_i are the matrices performing the vector product $r_i \times f_i = R_i f_i$ with r_i illustrated in Fig. 1 (f_i indicates the pure force component of h_i).

At this point, in force of the duality between forces and velocities which follows from the principle of virtual work in mechanics, it can be shown that

$$\mathbf{v} = \mathbf{W}^T \mathbf{v}_o \quad (12)$$

where $\mathbf{v}_o \in \mathbb{R}^m$ is the vector of absolute velocities of the object. Differentiating (12) yields the velocity composition equation in the form

$$\mathbf{a} = \mathbf{W}^T \mathbf{a}_o + \dot{\mathbf{W}}^T \mathbf{v}_o \quad (13)$$

where \mathbf{a}_o is the vector of absolute object accelerations.

Object Dynamics

The dynamic equations of motion for the held object can be expressed in the form

$$\mathbf{M}_o \mathbf{a}_o + \mathbf{c}_o + \mathbf{g}_o = \mathbf{h}_o \quad (14)$$

where $\mathbf{M}_o \in \mathbb{R}^{m \times m}$ is the object inertia matrix, and $\mathbf{c}_o, \mathbf{g}_o \in \mathbb{R}^m$ respectively are the vectors of velocity dependent forces, and gravitational forces.

DYNAMIC MANIPULABILITY ELLIPSOID

The dynamic manipulability ellipsoid for a single robot gives the magnitude of the end-effector acceleration vector, in a certain task space direction, that can be realized by applying joint driving torque vectors of fixed magnitude [3].

For the case of cooperating robots, therefore, it is appropriate to derive the mapping of the object space accelerations onto the extended joint space torques. This can be performed by suitably combining eqs. (4), (5), (9), (13) and (14).

Similarly to [3], we regard the case when both the arms and the object are standing still ($\dot{\mathbf{q}} = \mathbf{0}$, $\mathbf{v}_o = \mathbf{0}$) as the fundamental one for considering the dynamic manipulability; this implies that $\mathbf{J}\dot{\mathbf{q}} = \mathbf{0}$ in (4), $\mathbf{c} = \mathbf{0}$ in (5), $\dot{\mathbf{W}}^T \mathbf{v}_o = \mathbf{0}$ in (13), and $\mathbf{c}_o = \mathbf{0}$ in (14). Furthermore, without loss of generality, we do not include the gravitational forces, i.e. $\mathbf{g} = \mathbf{0}$ in (5), and $\mathbf{g}_o = \mathbf{0}$ in (14).[†] In sum, we consider the following simplified equations:

$$\mathbf{a} = \mathbf{J}\ddot{\mathbf{q}} \quad (4')$$

$$\boldsymbol{\tau} = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{J}^T \mathbf{h} \quad (5')$$

$$\mathbf{a} = \mathbf{W}^T \mathbf{a}_o \quad (13')$$

$$\mathbf{M}_o \mathbf{a}_o = \mathbf{h}_o \quad (14')$$

[†] In general, it is possible to express the above mapping for a new set of variables — joint torques and object accelerations, respectively — obtained by subtracting the absolute values of the neglected terms from the original variables.

which, together with (9), will yield the sought mapping. Indeed, solving (4') for \ddot{q} and using (13') for a gives

$$\ddot{q} = J^T W^T a_o \quad (15)$$

On the other hand, solving (9) for h and using (14') for h_o yields

$$h = W^T M_o a_o \quad (16)$$

Finally, plugging (15) and (16) into (5') gives

$$r = A_o a_o \quad (17)$$

where

$$A_o = M J^T W^T + J^T W^T M_o \quad (18)$$

As one could expect, A_o is formed by the contribution of the inertias of the single manipulators and of the object inertia. This is accomplished via a sequence of suitable transformation matrices deriving from the basic equations (15) and (16) that map the object acceleration onto joint accelerations and end-effector contact forces, respectively.

At this point, it can be concluded that the unit sphere in the extended joint space

$$r^T r = 1 \quad (19)$$

maps onto the object space ellipsoid

$$a_o A_o^T A_o a_o = 1 \quad (20)$$

which is defined as the *dynamic manipulability ellipsoid* for the system.

Notice that in the case of a single arm and the center of mass coincident with the end-effector location, the result obtained in [3] is recovered. In other words, the dynamic manipulability ellipsoid derived in (20) is a generalization of the one in [3] for the case of multiple cooperating robot tightly grasping a rigid object.

Suitable manipulability measures can be derived by considering the volume of the ellipsoid — this is given by the determinant of $A_o^T A_o$ except for a constant coefficient — and then used to detect singular configurations of the system (when the volume becomes zero). Also, assigned an object task, e.g. mating mechanical parts, it is possible to derive quantitative indices of the task compatibility of the system in any joint configuration, and then exploit the eventual redundant degrees of freedom to reconfigure the system in a more dexterous posture to execute that task.

CASE STUDIES

Two case studies have been worked out to illustrate the effectiveness of the dynamic ellipsoid for cooperating robots; a two-arm planar system and a three-arm planar system. The dynamic ellipsoids of the single arms and the global dynamic ellipsoid are presented (in the same scale) to evidence the effects of the configuration of each single arm onto the system. A two-dimensional task space is assumed, i.e. only linear object

accelerations are of interest. The reference frame is always located at the base of the first arm.

For the two-arm system, we have considered two three-degree-of-freedom equal arms; each arm has equal links of length 0.4 m, mass 1 Kg, moment of inertia 0.03 Kg-m^2 , center of mass at link midpoint. The object is a disk of radius 0.15 m and mass 0.05 Kg. In the first example of Fig. 2, the ellipsoids for the single arms indicate the existence of preferred directions to execute linear accelerations at the end-effectors. As one could derive from the above theory, the global ellipsoid can be interpreted as a "rough" intersection of the two ellipsoids, properly scaled by the object mass. And, indeed, we have chosen a light object to render the volume of the ellipsoid "visible". The second example (Fig. 3) is aimed at analyzing the inertial performance of the system when the two arms are in near-isotropic configurations.

For the three-arm system, we have added an arm with the same characteristics as the other two. The example of Fig. 4 demonstrates the effect of the ellipsoid of the third arm onto the global ellipsoid; in particular the preferred direction evidenced by the third ellipsoid is reflected in the global ellipsoid, in spite of the near-isotropy shown by the ellipsoids of the two other arms.

CONCLUSIONS

The dynamic manipulability ellipsoid for cooperating robots holding a rigid object has been defined in this paper. It describes the system capability of performing object accelerations along given object space directions for joint torques belonging to a given set.

The case studies presented have validated the theory and shown the inertial contributions of the single arms to the global system. To the purpose, it can be noticed that, when multiple robots cooperate, the dynamic manipulability of the system cannot improve on the manipulability of the robot with the least favourable inertial configuration. The inertial characteristics of the object further penalize the overall performance.

The dynamic manipulability ellipsoid, together with the force and velocity ellipsoids already proposed by the authors, seem to be very useful both for the analysis of a given multi-arm system and for the planning of optimal postures, depending on the task to be performed.

REFERENCES

- [1] T. Yoshikawa, "Manipulability of robotic mechanisms," *Int. J. Rob. Res.*, Vol. 4, No. 1, pp. 3-9, 1985.
- [2] H. Asada, "A geometrical representation of manipulator dynamics and its application to arm design," *Trans. ASME J. Dyn. Syst. Meas. Contr.*, Vol. 105, No. 3, pp. 131-135, 1983.
- [3] T. Yoshikawa, "Dynamic manipulability of robot manipulators," *J. Rob. Syst.*, Vol. 2, No. 2, pp. 113-124, 1985.
- [4] P. Chiacchio, S. Chiaverini, L. Sciavicco and B. Siciliano, "On the manipulability of dual cooperative robots," *NASA Conf. Space Telerobotics*, Pasadena, CA, Jan.-Feb. 1989.

- [5] P. Chiacchio, S. Chiaverini, L. Sciavicco and B. Siciliano, "Global task space manipulability ellipsoids for multiple arm systems," submitted for publication to *IEEE Trans. Rob. Autom.*, 1989.
- [6] P. Dauchez and M. Uchiyama, "Kinematic formulation for two force-controlled cooperating robots," *3rd Int. Conf. Advanced Robotics*, Versailles, France, Oct. 1987.

Acknowledgements. We would like to thank Prof. L. Sciavicco for helpful discussions and technical insights regarding this work. The financial support of CNR under research contract PFR-89.00514.67 is acknowledged.

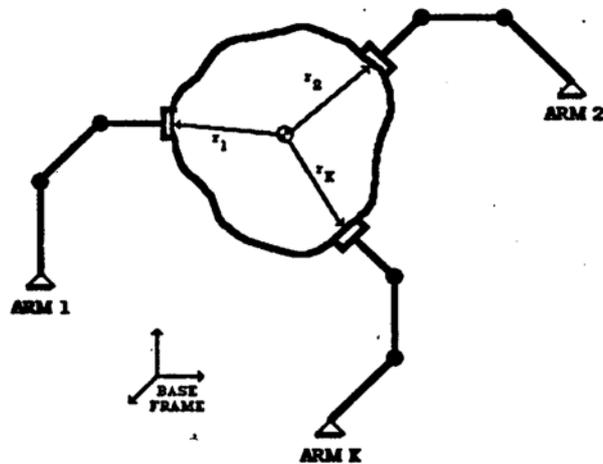


Fig. 1 — A multiple cooperating robot system

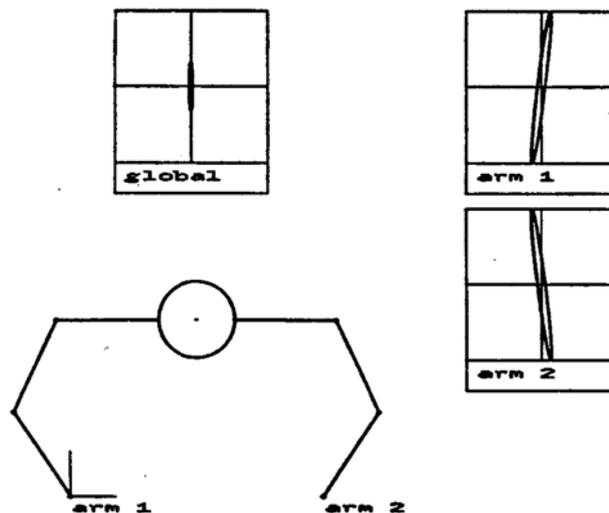


Fig. 2 — Dynamic manipulability ellipsoid for the two-arm system (first example)

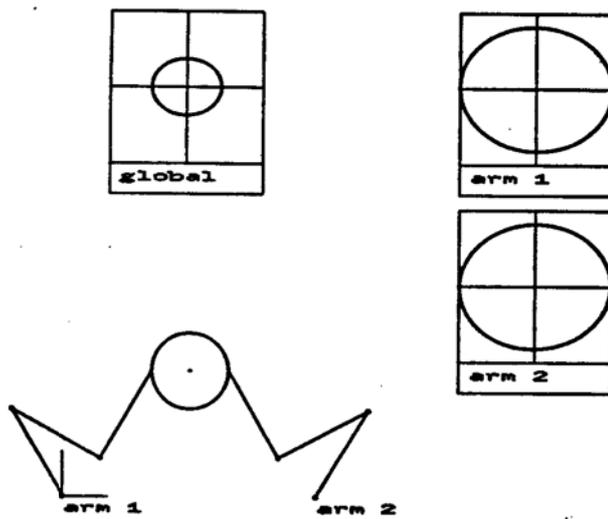


Fig. 3 — Dynamic manipulability ellipsoid for the two-arm system (second example)

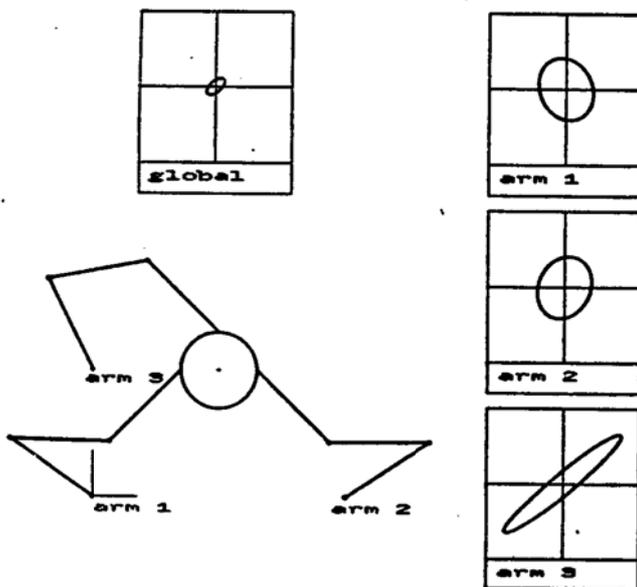


Fig. 4 — Dynamic manipulability ellipsoid for the three-arm system