

ON THE STABILITY OF A FORCE/POSITION CONTROL SCHEME FOR ROBOT MANIPULATORS

S. Chiaverini and B. Siciliano

Dipartimento di Informatica e Sistemistica, Università degli Studi di Napoli Federico II, Via Claudio 21, 80125 Napoli, Italy

Abstract. *A force/position local control scheme of the PID type with gravity compensation is shown to guarantee asymptotic stability for robot manipulators in contact with an elastically compliant environment. The scheme is built upon the foundation of the parallel control strategy which structurally provides dominance of the force control target over the position one. Stability is proved by resorting to an energy-based Lyapunov argument.*

Keywords. *Robots; force/position control; nonlinear control systems; PID control; stability; Lyapunov methods.*

INTRODUCTION

It is well known that, in order to make a robot manipulator capable of interacting with the environment, the forces arising from the contact must be properly considered. As a matter of fact, when the end effector of a position-controlled robot manipulator comes into contact with the environment, the experienced forces are treated as disturbances by the controller leading to instability phenomena. It is then opportune to design robot control strategies that can handle the interaction effects.

A survey of major force control techniques can be found in (Whitney, 1987). One can distinguish between techniques that assign a dynamic relationship between force and position variables without explicitly using force sensor feedback information, e.g. impedance control (Hogan, 1985; Kazerooni, Houpt, and Sheridan, 1986), and techniques that provide the robot with force sensor capabilities and suitably embed the force measurements into the control scheme. On the other hand, a typical manipulation task specifies both motion and force at the end effector. It becomes then necessary to manage the natural conflict between the force control loop and the position control loop.

The most widely adopted approach to force/position control of robot manipulators is the hybrid control (Raibert and Craig, 1981; Khatib, 1987; Yoshikawa, 1987; De Luca, Manes and Nicolò 1988). Distinct force and position control loops are designed and selection matrices are introduced to suitably switch from one loop to the other along each task direction. Therefore, this technique well

matches the framework of natural vs. artificial constraints introduced by Mason (1981). One intrinsic drawback of the approach is that the selection mechanism is based on the available model of the task; thus lack of knowledge about the environment may cause improper operation of the system. A recent study about stability of hybrid control is reported in (Wen and Murphy, 1990).

In the framework of force/position control techniques, a novel control strategy was proposed, namely the parallel control (Chiaverini and Sciacivico, 1988), which differs from the conventional hybrid control schemes in that it is based on feeding back both position and force errors along each task space direction. The key feature is that the force loop prevails over the position loop to manage also those cases when unplanned contacts with the environment are experienced; this is accomplished by means of an integral action on the force error. A realization of the parallel control in the case of perfect nonlinear compensation and dynamic decoupling has been discussed in (Chiaverini, 1990); however, extensive computation is required to perform on-line evaluation of compensating torques.

In this paper, we investigate the stability properties of a control scheme adopting the above strategy which is based on simple position *PD* control + gravity compensation + desired force feedforward + force *PI* control; remarkably, full model dynamic compensation is not required, thus lightening the computational burden of the control algorithm. For the purpose of the present work, we restrict our analysis to the case of an elastic con-

tact (sensor and environment). Inspired by the results on stability of *PID* position control (Arimoto and Miyazaki, 1984), an energy-based Lyapunov argument is used to show that, for given force and position set points, the force error is asymptotically driven to zero at the expense of a steady-state position error.

MODEL OF A ROBOT MANIPULATOR IN CONTACT WITH THE ENVIRONMENT

We consider a robot manipulator formed by an open kinematic chain of n rigid links connected by actuated joints. When the manipulator operates in free space, its dynamic model is usually expressed in the joint space and attains the conventional Lagrangian form

$$B_Q(q)\ddot{q} + C_Q(q, \dot{q})\dot{q} + g_Q(q) = t \quad (1)$$

where q is the $(n \times 1)$ vector of joint variables, B_Q is the $(n \times n)$ symmetric positive definite inertia matrix, $C_Q\dot{q}$ is the $(n \times 1)$ vector of Coriolis and centrifugal generalized forces, g_Q is the $(n \times 1)$ vector of gravity generalized forces, and t is the $(n \times 1)$ vector of joint actuating generalized forces. On the other hand, when the manipulator interacts with the environment, it is more convenient to describe its dynamics in the *operational space* (Khatib, 1980) that is the space where manipulation tasks are naturally specified. The dimension of this space (m) is usually less or equal than the dimension of the joint space (n). The dynamic model in the operational space can be written as

$$B(x)\ddot{x} + C(x, \dot{x})\dot{x} + g(x) = u - f \quad (2)$$

where x is the $(m \times 1)$ vector of end-effector location, B , $C\dot{x}$, g , u are the counterparts of B_Q , $C_Q\dot{q}$, g_Q , t respectively, and f is the $(m \times 1)$ vector of contact generalized forces exerted by the manipulator on the environment; all operational space quantities are expressed in a common reference frame. When $m = n$, the vector of operational variables constitutes a set of Lagrangian generalized coordinates and B assumes the meaning of a true inertia matrix. Instead, in the case of kinematically redundant manipulators ($m < n$), B is only a pseudo inertia matrix (Khatib, 1987). Focusing on the case of non-redundant manipulators ($m = n$), the relationship between the two spaces is established through the following equations:

$$B = J^{-T}B_QJ^{-1} \quad (3a)$$

$$C\dot{x} = J^{-T}C_Q\dot{q} - B\dot{J}\dot{q} \quad (3b)$$

$$g = J^{-T}g_Q \quad (3c)$$

$$u = J^{-T}t \quad (3d)$$

where J is the $(n \times n)$ manipulator Jacobian matrix that is supposed to be non-singular.

A notable property of the dynamic model in the joint space is that the matrix

$$S_Q(q, \dot{q}) = \dot{B}_Q(q) - 2C_Q(q, \dot{q}) \quad (4)$$

is skew-symmetric (Takegaki and Arimoto, 1981). On the basis of the relations (3), it can be shown that the dynamic model in the operational space enjoys the same property; that is, the matrix

$$S(x, \dot{x}) = \dot{B}(x) - 2C(x, \dot{x}) \quad (5)$$

is skew-symmetric. This property could have been inferred by observing that in the case $m = n$, we have a set of Lagrangian generalized coordinates in the operational space, like in the joint space. Equation (5) will be useful for the stability proof in the following section.

For the purpose of the present work we restrict our attention to the case $m = n = 3$, i.e. we study only translational motion and force components.

Accurate modelling of the contact between the manipulator and the environment is usually difficult to obtain in analytic form, due to the complexity of the physical phenomena involved during the interaction. It is then reasonable to resort to a simple but significant model, relying on the robustness of the control system in order to absorb the effects of inaccurate modelling. Following these guidelines, we consider the case of an environment constituted by a rigid, frictionless and elastically compliant plane. The choice of a planar surface is motivated by noticing that it is locally a good approximation to surfaces of regular curvature. The rigidity of the contact plane allows to neglect the effects of local deformation at the contact. The total elasticity, due to end-effector force sensor and environment, is accounted through the compliance of the plane. Friction effects are neglected within the operational range of interest.

With the above assumptions, the model of the contact force considered takes on the simple form

$$f = K(x - x_0) \quad (6)$$

where x is the position of the contact point, x_0 is a point of the plane at rest, and K is a (3×3) constant symmetric *stiffness matrix* (Lončarić, 1987) that establishes a linear mapping between $(x - x_0)$ and f ; notice that eq. (6) holds only when the manipulator is in contact with the environment and all quantities are expressed in the common reference frame. Moreover we observe that:

- The contact force is orthogonal to the plane for any vector $(x - x_0)$; then, a base of $\mathcal{R}(K)$ — $\mathcal{R}(K)$ denotes the range space of matrix K — is the unit vector n orthogonal to the plane, and $\text{rank}(K) = 1 < 3$.

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- All vectors $(\mathbf{x} - \mathbf{x}_0)$ lying on the plane do not contribute to the contact force; then, a base of $\mathcal{N}(\mathbf{K})$ — $\mathcal{N}(\mathbf{K})$ denotes the null space of matrix \mathbf{K} — is a pair of linearly independent unit vectors $(\mathbf{p}_1, \mathbf{p}_2)$ tangential to the plane.
- In force of the symmetry of \mathbf{K} , $\mathcal{R}(\mathbf{K}) \equiv \mathcal{R}(\mathbf{K}^T)$, and a convenient choice for $(\mathbf{p}_1, \mathbf{p}_2)$ is such that the columns of the matrix

$$\mathbf{R} = (\mathbf{p}_1 \quad \mathbf{p}_2 \quad \mathbf{n}) \quad (7)$$

form a set of orthonormal vectors constituting a base of \mathbb{R}^3 .

According to the above remarks, matrix \mathbf{K} can be decomposed as

$$\mathbf{K} = \mathbf{R} \text{diag}(0, 0, k) \mathbf{R}^T = k \mathbf{n} \mathbf{n}^T \quad (8)$$

where \mathbf{R} , defined in (7), is the rotation matrix from the frame attached to the plane to the reference frame; then k is the stiffness coefficient, characterizing the contact, that acts along the direction orthogonal to the plane.

PARALLEL CONTROL

We intend to derive a force/position controller in the framework of the *parallel control* approach (Chiaverini and Sciavicco, 1988). The key feature is to have a force control loop working in parallel to a position control loop along each operational space direction. The logical conflict between the two loops is managed by imposing dominance of the force control action over the position one. The potential offered by this technique compared to conventional controllers also using force feedback sensory information is discussed in full in (Chiaverini and Sciavicco, 1990).

With reference to operational space model (2), we have to synthesize the vector of actuating forces \mathbf{u} . In the previous work (Chiaverini and Sciavicco, 1988) full dynamic model compensation is performed by the law

$$\mathbf{u} = \hat{\mathbf{B}}(\mathbf{x}) \mathbf{M}_d^{-1} \ddot{\mathbf{u}} + \hat{\mathbf{C}}(\mathbf{x}, \dot{\mathbf{x}}) \dot{\mathbf{x}} + \hat{\mathbf{g}}(\mathbf{x}) + \hat{\mathbf{f}} \quad (9)$$

where the upper hat denotes the available estimates of the dynamic terms, $\mathbf{M}_d = m_d \mathbf{I}$ is a desired mass matrix — \mathbf{I} is the (3×3) identity matrix — and $\ddot{\mathbf{u}}$ is a new force input. Plugging control (9) into model (2) gives, under the assumption of perfect compensation ($\hat{\mathbf{B}} = \mathbf{B}$, $\hat{\mathbf{C}} = \mathbf{C}$, $\hat{\mathbf{g}} = \mathbf{g}$, $\hat{\mathbf{f}} = \mathbf{f}$)

$$\mathbf{M}_d \ddot{\mathbf{x}} = \ddot{\mathbf{u}} \quad (10)$$

that is a linear decoupled inertial system. Let then \mathbf{x}_d and \mathbf{f}_d respectively denote the assigned

position and force set points; $\Delta \mathbf{x} = \mathbf{x}_d - \mathbf{x}$ and $\Delta \mathbf{f} = \mathbf{f}_d - \mathbf{f}$ are the position and force errors. The new force input is designed as

$$\ddot{\mathbf{u}} = -\mathbf{K}_D \dot{\mathbf{x}} + \mathbf{K}_P \Delta \mathbf{x} + \mathbf{K}_F \Delta \mathbf{f} + \mathbf{K}_I \int_0^t \Delta \mathbf{f} d\tau, \quad (11)$$

where \mathbf{K}_P , \mathbf{K}_D , \mathbf{K}_F , \mathbf{K}_I are suitable gain matrices that are to be chosen in order to make the system described by eqs. (10,11,6) asymptotically stable. These matrices are taken as $\mathbf{K}_P = k_P \mathbf{I}$, $\mathbf{K}_D = k_D \mathbf{I}$, $\mathbf{K}_F = k_F \mathbf{I}$, $\mathbf{K}_I = k_I \mathbf{I}$ with $k_P, k_D, k_F, k_I > 0$ to preserve a decoupled and isotropic behaviour of the system (Chiaverini, 1990). Control (11) is formed by a *PD* action on the position variables and of a *PI* action on the force variables so as to realize the above required force dominance. It is important to emphasize that no exact information about stiffness matrix \mathbf{K} is required by the control, but only a rough estimate of k is used to suitably tune the feedback gains.

Since the task is prescribed in terms of a position set point \mathbf{x}_d and of a force set point \mathbf{f}_d , there is obviously no guarantee that the simultaneous achievement of both of them be compatible with the task geometry. In other words, the presence of the manipulator and of the environment imposes constraints between position and force variables, and thus there is no control scheme that can take the system towards given set points if those violate the physical constraints.

As emphasized in (Chiaverini and Sciavicco, 1988, 1990), the force dominance requirement met by control (11) leads system (10,6) to an equilibrium state characterized by a null force error and a constant position error. Further clarification is in order and is provided below.

Looking at the properties of elastic contact model (6), the only possibility of obtaining a null force error is to assign a set point $\mathbf{f}_d \in \mathcal{R}(\mathbf{K})$. On the other hand, this is consistent with the fact that the considered environment can generate reaction forces only along the direction of \mathbf{n} . If no information about the geometry of the environment is available, i.e. \mathbf{n} is unknown, the null vector can be assigned for \mathbf{f}_d that is anyhow in the range space of any matrix \mathbf{K} . Thus, in the remainder, we assume $\mathbf{f}_d \in \mathcal{R}(\mathbf{K})$.

Adopting similar reasoning about the contact model, we can recognize that there is no problem to obtain a null position error in the plane of $(\mathbf{p}_1, \mathbf{p}_2)$, while the component of \mathbf{x} along \mathbf{n} has to accommodate the force requirement specified by \mathbf{f}_d . Thus, \mathbf{x}_d can be freely reached only in $\mathcal{N}(\mathbf{K})$.

It can be shown that the resulting equilibrium state for system (10,11,6) is

$$\mathbf{x}_\infty = \mathbf{K}^{-1} (\mathbf{f}_d + \mathbf{K} \mathbf{x}_0) + (\mathbf{I} - \mathbf{K}^{-1} \mathbf{K}) \mathbf{x}_d \quad (12)$$

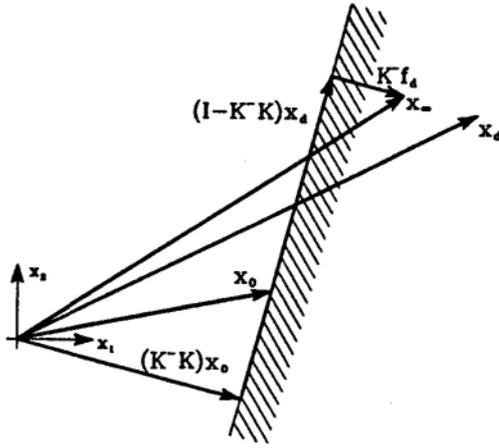


Fig. 1. Construction of the equilibrium state in a two-dimensional case.

$$f_{\infty} = K(x_{\infty} - x_0) = f_d \quad (13)$$

which merely reflects the above considerations. The matrix K^{-} indicates a generalized inverse of matrix K which, in view of expression (8), can be written in the simple form

$$K^{-} = R \text{diag}(0, 0, 1/k) R^T = (1/k)nn^T. \quad (14)$$

For further use, we remark that the products

$$KK^{-} = K^{-}K = R \text{diag}(0, 0, 1) R^T = nn^T \quad (15)$$

do not affect vector components in $\mathcal{R}(K)$ while cut off vector components outside $\mathcal{R}(K)$.

An example of construction of the equilibrium state in a two-dimensional case is illustrated in Figure 1.

STABILITY OF A NEW SCHEME

The parallel control scheme based on laws (9,11) achieves given contact force and position set points even in the case of unplanned contact with the environment. Our goal is to design a robust control law that guarantees the same steady-state performance without requiring complete knowledge of manipulator dynamic model; this drastically reduces the computational burden of the control algorithm and then is more suitable for real-time implementation.

We show below that system (2) under a control law based on simple position PD control + gravity compensation + desired force feedforward + force PI control converges asymptotically to the same equilibrium state (12,13) as for the system under control (9,11).

In particular, we choose the following control law

$$u = -K_D \dot{x} + K_P \Delta x + g(x) + f_d + K_F \Delta f + K_I \int_0^t \Delta f d\tau. \quad (16)$$

where K_P , K_D , K_F , K_I have the same structure as in law (11) and $f_d \in \mathcal{R}(K)$.

Let define

$$e = x_{\infty} - x \quad (17)$$

which, by virtue of (12,6), can be also written as

$$e = (I - K^{-}K) \Delta x + K^{-} \Delta f = \Delta x + K_P^{-1} d \quad (18)$$

where

$$d = K_P K^{-} (f_d + K(x_0 - x_d)) \quad (19)$$

is a constant vector. For later use, notice that

$$K^{-} K e = K^{-} \Delta f \quad (20)$$

From (17) it is

$$\dot{e} = -\dot{x}. \quad (21)$$

Further, let define

$$s = R^T \left(\int_0^t K^{-} \Delta f d\tau - K_I^{-1} K^{-} d \right) \quad (22)$$

that is a vector expressed in the frame attached to the contact plane. Being $f_d \in \mathcal{R}(K)$, vector (22) is of the kind

$$s = (0 \ 0 \ s_n)^T. \quad (23)$$

Deriving (22) with respect to time and accounting for (20,23), it turns out

$$\dot{s}_n = n^T e. \quad (24)$$

At this point, let consider the (7×1) augmented state vector

$$z = (\dot{x} \ e \ s_n)^T. \quad (25)$$

The augmented system described by eqs. (2,21,24) under control (16) can be written in the standard compact form:

$$\dot{z} = Fz \quad (26)$$

where

$$F = \begin{pmatrix} -B^{-1}(C + k_D I) & B^{-1}(k_P I + k'_F k n n^T) & k_I k B^{-1} n \\ -I & 0 & 0 \\ 0^T & n^T & 0 \end{pmatrix} \quad (27)$$

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with $k'_F = 1 + k_F$; \mathbf{O} denotes the (3×3) null matrix and $\mathbf{0}$ the (3×1) null vector. Notice that some handy reductions, using the structural properties of \mathbf{K} in eqs. (7,8,15) and the definition of s_n in (23), have been performed to derive (27).

The main result of the work can be stated as:

Theorem. *If the following assumption holds:*

$$\left\| \sum_{i=1}^3 \epsilon_i \frac{\partial \mathbf{B}}{\partial x_i} \right\| \leq \Phi < \infty, \quad (28)$$

then there exists a choice of feedback gains k_P , k_D , k_F , k_I that makes the origin of the space state for system (26,27) asymptotically stable. ■

Proof. We consider the energy-based positive definite Lyapunov function candidate

$$V = \frac{1}{2} \mathbf{z}^T \mathbf{P} \mathbf{z} \quad (29)$$

where

$$\mathbf{P} = \begin{pmatrix} \mathbf{B} & -\rho \mathbf{B} & \mathbf{0} \\ -\rho \mathbf{B} & k_P \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T & k_I k \mathbf{n} \\ \mathbf{0}^T & k_I k \mathbf{n}^T & \rho k_I k \end{pmatrix}. \quad (30)$$

Then, we compute the time derivative of V along the trajectories of system (26,27), i.e.

$$\dot{V} = \mathbf{z}^T \left(\mathbf{P} \mathbf{F} + \frac{1}{2} \dot{\mathbf{P}} \right) \mathbf{z}. \quad (31)$$

The core of the quadratic form in (31) can be evaluated in two steps:

$$\mathbf{P} \mathbf{F} = \begin{pmatrix} -(\mathbf{C} + k_D \mathbf{I}) + \rho \mathbf{B} & & \\ \rho(\mathbf{C} + k_D \mathbf{I}) - (k_P \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T) & & \\ -k_I k \mathbf{n}^T & & \\ & k_P \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T & k_I k \mathbf{n} \\ -\rho(k_P \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T) + k_I k \mathbf{n} \mathbf{n}^T & & -\rho k_I k \mathbf{n} \\ & \rho k_I k \mathbf{n}^T & 0 \end{pmatrix} \quad (32)$$

$$\frac{1}{2} \dot{\mathbf{P}} = \begin{pmatrix} (1/2) \dot{\mathbf{B}} & -(\rho/2) \dot{\mathbf{B}} & \mathbf{0} \\ -(\rho/2) \dot{\mathbf{B}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0}^T & \mathbf{0}^T & 0 \end{pmatrix} \quad (33)$$

Then, combining (32,33) and extracting the symmetric part gives

$$\dot{V} = \dot{\mathbf{x}}^T (-k_D \mathbf{I} + \rho \mathbf{B}) \dot{\mathbf{x}} + \rho \mathbf{e}^T (k_D \mathbf{I} + \mathbf{C} - \dot{\mathbf{B}}) \dot{\mathbf{x}} + \mathbf{e}^T (k_I k \mathbf{n} \mathbf{n}^T - \rho(k_P \mathbf{I} + k'_F k \mathbf{n} \mathbf{n}^T)) \mathbf{e} \quad (34)$$

in which we have taken advantage of the skew-symmetry property of (5). Next, we observe that

from the Lagrangian operational space model one can derive the relation

$$(\mathbf{C} - \dot{\mathbf{B}}) \dot{\mathbf{x}} = -\frac{1}{2} \frac{\partial (\dot{\mathbf{x}}^T \mathbf{B} \dot{\mathbf{x}})}{\partial \mathbf{x}}, \quad (35)$$

and then, manipulating the term containing (35) and exploiting (24), function (34) becomes

$$\begin{aligned} \dot{V} &= -\dot{\mathbf{x}}^T (k_D \mathbf{I} - \rho \mathbf{B}) \dot{\mathbf{x}} + \rho k_D \mathbf{e}^T \dot{\mathbf{x}} \\ &\quad - \frac{\rho}{2} \dot{\mathbf{x}}^T \left(\sum_{i=1}^3 \epsilon_i \frac{\partial \mathbf{B}}{\partial x_i} \right) \dot{\mathbf{x}} - \rho k_P \mathbf{e}^T \mathbf{e} \\ &\quad - k(\rho k'_F - k_I) s_n^2. \end{aligned} \quad (36)$$

By virtue of assumption (28) and of the inequality

$$\lambda_m \mathbf{I} \leq \mathbf{B} \leq \lambda_M \mathbf{I}, \quad (37)$$

where λ_m and λ_M respectively denote the minimum and maximum eigenvalue of \mathbf{B} , we have

$$\begin{aligned} \dot{V} &\leq -(\dot{\mathbf{x}}^T \quad \mathbf{e}^T) \begin{pmatrix} (k_D - \rho \lambda_M - \rho \Phi/2) \mathbf{I} & \\ & -(\rho k_D/2) \mathbf{I} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{e} \end{pmatrix} \\ &\quad - k(\rho k'_F - k_I) s_n^2. \end{aligned} \quad (38)$$

On the other hand, function candidate (29), accounting for (30) and (24), can be written as

$$\begin{aligned} V &= \frac{1}{2} \dot{\mathbf{x}}^T \mathbf{B} \dot{\mathbf{x}} - \rho \dot{\mathbf{x}}^T \mathbf{B} \mathbf{e} + \frac{1}{2} k_P \mathbf{e}^T \mathbf{e} \\ &\quad + \frac{1}{2} k'_F k s_n^2 + k_I k s_n \dot{s}_n + \frac{\rho}{2} k_I k s_n^2, \end{aligned} \quad (39)$$

or, in compact form,

$$\begin{aligned} V &= \frac{1}{2} (\dot{\mathbf{x}}^T \quad \mathbf{e}^T) \begin{pmatrix} \mathbf{B} & -\rho \mathbf{B} \\ -\rho \mathbf{B} & k_P \mathbf{I} \end{pmatrix} \begin{pmatrix} \dot{\mathbf{x}} \\ \mathbf{e} \end{pmatrix} \\ &\quad + \frac{k}{2} (s_n \quad \dot{s}_n) \begin{pmatrix} \rho k_I & k_I \\ k_I & k'_F \end{pmatrix} \begin{pmatrix} s_n \\ \dot{s}_n \end{pmatrix}. \end{aligned} \quad (40)$$

Expressions (38,40) reveal that, for ρ sufficiently small, there exists a choice of k_P , k_D , k_F , k_I that makes $V \geq 0$ and $\dot{V} \leq 0$. In particular, we have $\dot{V} < 0$, $\forall \dot{\mathbf{x}} \neq \mathbf{0}, \mathbf{e} \neq \mathbf{0}, \dot{s}_n \neq 0$ while $\dot{V} = 0$ implies $\dot{\mathbf{x}} = \mathbf{0}, \mathbf{e} = \mathbf{0}, \dot{s}_n = 0$. Therefore, s_n is a constant and $\dot{\mathbf{x}} = \mathbf{0}$; from the first three equations of (26,27) we get $k_I k s_n \mathbf{B}^{-1} \mathbf{n} = \mathbf{0}$ and then $s_n = 0$, too.

In sum, the only equilibrium state satisfying $\dot{V} = 0$ is $\mathbf{z} = (\mathbf{0}^T \quad \mathbf{0}^T \quad 0)^T$. Hence, due to LaSalle's theorem (La Salle, 1960), this state is asymptotically stable.

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DISCUSSION

In order to gain insight in the above stability proof, we remark the following issues:

- Satisfaction of assumption (28) is not critical since it is reasonable to assume that both Δx and d are bounded and derivatives of the terms of B are bounded too.
- It can be recognized that the set of conditions to be satisfied by k_P , k_D , k_F , k_I are:

$$k_D - \rho\lambda_M - \rho\Phi/2 > \rho k_D^2/4k_P \quad (41)$$

$$\rho(1 + k_F) > k_I \quad (42)$$

$$k_P > \rho^2\lambda_M^2 \quad (43)$$

Remarkably ρ is a free parameter that is not used in control law (16). About the gains, notice that: From (41) and (43) it is convenient to choose large values of k_P . From (41) a large value of k_D is needed. From (42) a large value of k_F and a small value of k_I are required.

- We stress that stiffness matrix K is not used at all in control law (16). However, the choice of the feedback gains is indirectly influenced by k via the bound in (28); in any case, a conservative choice can be made.

ACKNOWLEDGEMENTS

We would like to thank Prof. Lorenzo Sciavicco for useful suggestions. The research work reported in this paper was supported by *Consiglio Nazionale delle Ricerche* under contract 90.00355.PF67.

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