

DISCRETE OUTPUT DECENTRALIZED CONTROL OF ROBOTIC MANIPULATORS

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Abstract. The task of a robotic manipulator's controller consists in the control of the robot's end effector in such a way that it is able to track a desired motion trajectory. All the proposed control techniques assume that manipulator state variables (joint positions and velocities) are all measurable. In this paper a new discrete time decentralized control strategy is developed, which only makes use of joint position measurements. The boundedness on the position tracking error is obtained via a Lyapunov-based stability analysis. The trade-off existing between control computational burden and position tracking accuracy is discussed and a case study is finally worked out.

Keywords. Robotic manipulators; discrete time systems; nonlinear control; decentralized control; output feedback; Lyapunov methods.

INTRODUCTION

The control problem for a robotic manipulator is to make its end effector track a desired trajectory of motion in the task space. The dynamics of a robotic manipulator is highly nonlinear and coupled. Its computation requires in general the use of time consuming computational schemes, such as (Hollerbach, 1980) and (Luh, Walker and Paul, 1980a). A number of robot control techniques have been proposed in the latest years. See for instance the resolved acceleration control (Luh, Walker and Paul, 1980b), the adaptive control (Balestrino, De Maria and Sciavicco, 1983) and (Nicosia and Tomei, 1984), the sliding mode control (Young, 1978) and the related robust control (Slotine, 1985) to mention only a few of them.

The resolved acceleration control, along with any control technique based on the so-called 'computed torque' method, requires a very accurate computation of the robot's dynamical model. In practice, however, only estimates of the system dynamical terms are available and robustness to parameter variations may become critical. In order to overcome that drawback, adaptive control and (robust) sliding mode control have been proposed. The resulting controls are nonlinear and may effectively account for parametric uncertainties, such as imprecision on the manipulator mass properties, unknown loads, inaccuracies on the torque constants of the actuators, friction and so on.

The crucial point concerned with all the above control techniques is the assumption that full state measurements are available. As a matter of fact, joint positions can always be sensed, whereas joint velocities may eventually not. This issue is of interest, for instance, in case of direct-drive arms (Asada, Kanade and Takeyama, 1983) where, due to the lack of transmission mechanisms, a direct measure of joint velocities may be questionable. Further, the actuator dynamics is never explicitly taken into account in the derivation of the control laws. With reference to the sliding mode control, for instance, the unmodelled actuator dynamics leads to derated control performance, since in practical implementation it is not possible to use high feedback gains in the

proximity of the sliding surface. Nevertheless the direct measurement of joint accelerations, which would be needed for control of the whole system (robot + actuators), is far to be feasible.

The goal of this paper is to establish a new discrete decentralized control technique which only makes use of joint position measurements. The main idea is to build up a two-stage control. First a centralized control system accomplishes nonlinear compensation of those dominant dynamic terms whose estimates are achievable within a desired accuracy. Then a decentralized control system is designed for each degree of freedom, where all the noncompensated dynamic terms and/or the parameter uncertainties play the role of a disturbance to the system. The Lyapunov direct method is adopted to undertake a stability analysis. It allows also the quantification of the boundedness on the position tracking error, achieving thus a trade-off which may be exploited for "good" control design.

ROBOT DYNAMIC MODEL

It is well known that the dynamic model of an n degree-of-freedom (DOF) manipulator can be written in the joint space as

$$B(q)\ddot{q} + c(q, \dot{q}) + g(q) = \tau \quad (1)$$

where q is the $n \times 1$ vector of joint displacements, $B(q)$ is the $n \times n$ inertia matrix, $c(q, \dot{q})$ is the $n \times 1$ vector of centrifugal and Coriolis terms, $g(q)$ is the $n \times 1$ vector of gravity terms, and τ is the $n \times 1$ vector of joint torques.

In reality the manipulator is subjected to parameter uncertainties, such as imprecision on the manipulator mass properties, unknown loads, inaccuracies on the torque constants of the actuators, friction and so on. For the purpose of obtaining a decentralized control in the joint space, the nonlinear terms B and g can be redefined respectively as

$$B(q) := \hat{B} + \Delta B(q) \quad (2)$$

$$g(q) := \hat{g}(q) + \Delta g(q). \quad (3)$$

In (2) \hat{B} contains only the constant diagonal terms of $B(q)$, and $\Delta B(q)$ accounts for parameter uncertainties along with q -dependent and coupling terms of $B(q)$. In (3) $\hat{g}(q)$ is the available estimate of $g(q)$, and $\Delta g(q)$ accounts for parameter uncertainties of $g(q)$. Based on these assumptions, the dynamic model for each DOF i can be rewritten as

$$\hat{b}_{ii}\ddot{q}_i + \hat{g}_i(q) + \delta_i(q, \dot{q}) = \tau_i, \quad i=1, \dots, n \quad (4)$$

where

$$\delta_i := \sum_{\substack{j=1 \\ j \neq i}}^n b_{ij}(q)\ddot{q}_j + (b_{ii}(q) - \hat{b}_{ii})\ddot{q}_i + c_i(q, \dot{q}) + g_i(q) - \hat{g}_i(q), \quad i=1, \dots, n. \quad (5)$$

In this way n SISO systems have been obtained, allowing the design of n separate joint controllers.

CONTROLLER STRUCTURE

With reference to the dynamic model for each joint (4), the input torque τ_i is thought of as applied to the joint by an appropriate actuator system. If a d.c. motor with armature control is adopted, the torque τ_i results

$$\tau_i = s_i v_i - n_i \dot{q}_i \quad (6)$$

where s_i and n_i are usual motor constants, and v_i is the i applied voltage. Combining (4) and (6) yields

$$\hat{b}_{ii}\ddot{q}_i + n_i \dot{q}_i + \hat{g}_i(q) + \delta_i(q, \dot{q}) = v_i \quad (7)$$

where it is understood that all the terms on the left side of (7) have been appropriately scaled by s_i . The terms $\hat{g}_i(q)$ and $\delta_i(q, \dot{q})$ can be regarded as disturbances acting at the input of the linear time invariant system

$$\ddot{b}q + n\dot{q} = v \quad (8)$$

where the subscripts i 's have been conveniently dropped.

At this extent the goal is to design a digital joint controller for each DOF. In particular it must be emphasized that the discrete system corresponding to the linear time invariant continuous system (8) presents a pole at 1 in the z -plane, having the system (8) a pole at the origin in the s -plane. Consequently a preliminary joint position feedback action must be introduced in order to place the poles of the discrete system inside the unit circle.

Furthermore, in order to lighten the control signal effort, it is convenient to include in the digital control a nonlinear term which suitably compensates for the estimate of the gravity term $g(q)$ available at each sampling instant. In sum the digital control at the instant k is chosen as

$$v_k = \hat{g}(q_k) + f q_k + v_k \quad (9)$$

where q_k is the joint position measurement available at the k -th sampling instant, f is the position feedback gain at designer's disposal, and v_k is the new control input to be synthesized.

Upon these premises, the discrete time system corresponding to the system (8) can be written as

$$q_{k+1} + a_1 q_k + a_2 q_{k-1} = \quad (10)$$

$$b_1 (v_k - \delta_k) + b_2 (v_{k-1} - \delta_{k-1})$$

where a_1 , a_2 , b_1 and b_2 are the coefficients of the difference equation, depending on the sampling period, which are derived in the usual discretization process, accounting for (9); δ is the equivalent discrete disturbance originated from δ and from considering $\hat{g}(q)$ constant in the sampling period. Notice that the system (10) is a discrete time minimum phase system.

The goal now is to design the digital control v_k . The discrete reference model can be defined as

$$\hat{q}_{k+1} + a_1 \hat{q}_k + a_2 \hat{q}_{k-1} = \quad (11)$$

$$b_1 \hat{v}_k + b_2 \hat{v}_{k-1}$$

where \hat{q}_k is the desired joint position at k -th sampling instant. Subtracting (10) from (11) yields

$$e_{k+1} + a_1 e_k + a_2 e_{k-1} = \quad (12)$$

$$b_1 (\hat{v}_k - v_k + \delta_k) + b_2 (\hat{v}_{k-1} - v_{k-1} + \delta_{k-1})$$

where $e_k := \hat{q}_k - q_k$ is the joint position tracking error. Arranging the system (12) in state space form gives

$$e_{k+1} = A e_k + \tilde{B} (\hat{u}_k - u_k + d_k) \quad (13)$$

where

$$e_k := \begin{bmatrix} e_{k-1} \\ e_k \end{bmatrix}, \quad \hat{u}_k := \begin{bmatrix} \hat{v}_{k-1} \\ \hat{v}_k \end{bmatrix} \quad (14)$$

$$u_k := \begin{bmatrix} v_{k-1} \\ v_k \end{bmatrix}, \quad d_k := \begin{bmatrix} \delta_{k-1} \\ \delta_k \end{bmatrix}$$

$$A := \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix}, \quad \tilde{B} := \begin{bmatrix} 0^T \\ \tilde{b}^T \end{bmatrix}, \quad \tilde{b} := \begin{bmatrix} b_2 \\ b_1 \end{bmatrix}$$

A feedforward control action is first introduced in v_k as

$$v_k = \hat{v}_k + \gamma_k \quad (15)$$

The control input to be synthesized is then

$$c_k := \begin{bmatrix} \gamma_{k-1} \\ \gamma_k \end{bmatrix}. \quad (16)$$

At this extent, in order to synthesize the control signal γ_k , the Lyapunov direct method is followed. Let

$$v_k = e_k^T Q e_k, \quad Q > 0 \quad (17)$$

be a positive definite Lyapunov function. Then one obtains

$$\Delta v_k = v_{k+1} - v_k = e_{k+1}^T Q e_{k+1} - e_k^T Q e_k \quad (18)$$

which, accounting for (13)-(16), gives

$$\Delta v_k = e_k^T (A^T Q A - Q) e_k - 2(c_k - d_k)^T \tilde{B}^T Q A e_k + (c_k - d_k)^T \tilde{B}^T Q \tilde{B} (c_k - d_k). \quad (19)$$

Partitioning Q as

$$Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = [q_1 \quad q_2], \quad q_2^T = (q_{21} \quad q_{22}) \quad (20)$$

yields

$$\Delta v_k^T = -e_k^T P e_k - 2c_k^T b q_2^T A e_k + 2d_k^T b q_2^T A e_k + q_{22}(c_k^T b - d_k^T b)^2 \quad (21)$$

where

$$A^T Q A - Q = -P, \quad P > 0, \quad (22)$$

being A strictly stable. Finally the control v_k can be chosen as

$$v_k = -\frac{b_2}{b_1} v_{k-1} + \frac{1}{b_1 q_{22}} q_2^T A e_k \quad (23)$$

which gives also

$$\Delta v_k = -e_k^T (P + A^T q_2 q_2^T A / q_{22}) e_k + q_{22} (d_k^T b)^2 \quad (24)$$

It is easily seen from (24) that the first term on the right side is negative definite, whereas the second term is positive definite. This fact clearly points out the trade-off at designer's disposal. The smaller the disturbance and the sampling time, the smaller the joint position tracking error. From a geometrical viewpoint, if one seeks the ellipsis $v_k = \text{const.}$ which is externally tangent to the ellipsis $\Delta v_k = 0$, indeed, a region which contains the maximum error will be obtained. It can be checked that an estimate of the upper bound on e_k is given by

$$e_{kmax} = q_{22} (d_k^T b) (q_{11} / (\det(Q) (q_{22} - q_{11})))^{1/2} \quad (25)$$

The maximum tracking error, in fact, can be minimized by a suitable choice of the matrix P in (22).

CASE STUDY

In order to test the effectiveness of the proposed control technique, the three DOF robotic manipulator referred to in (Nicosia and Tomei, 1984) has been chosen to work out a set of numerical simulations.

With reference to the dynamic model in the form (1), the data are:

$$b_{11} = A_1 + A_2 \cos^2 q_2 + A_3 \cos^2 (q_2 + q_3) + A_4 \cos q_2 \cos (q_2 + q_3) \quad (26)$$

$$b_{12} = b_{13} = 0$$

$$b_{22} = A_5 + A_4 \cos q_3$$

$$b_{23} = A_6 + A_7 \cos q_3$$

$$b_{33} = A_8$$

$$c_1 = C_{12} \dot{q}_1 \dot{q}_2 + C_{13} \dot{q}_1 \dot{q}_3 \quad (27)$$

$$c_2 = -C_{12} \dot{q}_1^2 / 2 + C_{25} \dot{q}_2 \dot{q}_3 - C_{25} \dot{q}_3^2 / 2$$

$$c_3 = -C_{13} \dot{q}_1^2 / 2 - C_{25} \dot{q}_2^2 / 2$$

$$C_{12} = -A_2 \sin 2q_2 - A_3 \sin 2(q_2 + q_3) + A_4 \sin (q_3 + 2q_2)$$

$$C_{13} = -A_3 \sin 2(q_2 + q_3) - A_4 \cos q_2 \sin (q_2 + q_3)$$

$$C_{25} = -A_4 \sin q_3$$

$$g_1 = 0 \quad (28)$$

$$g_2 = B_1 \cos q_2 + B_2 \cos (q_2 + q_3)$$

$$g_3 = B_2 \cos (q_2 + q_3)$$

Three different values of the manipulator's payload have been assumed: 0, 5, 10. In Table 1 the numerical values of coefficients in (26)-(28) for the three different payloads are given.

Table 1. Numerical values of coeffs. in (26)-(28)

| | Payload mass | | |
|----------------|--------------|-----------|-----------|
| | 0 | 5 | 10 |
| A ₁ | 23.3803 | 23.3803 | 23.3803 |
| A ₂ | 9.2063 | 10.4563 | 11.7063 |
| A ₃ | 2.4515 | 3.7015 | 4.9515 |
| A ₄ | 5.4 | 7.9 | 10.4 |
| A ₅ | 82.399 | 84.899 | 87.399 |
| A ₆ | 2.6274 | 3.8774 | 5.1274 |
| A ₇ | 2.7 | 3.95 | 5.2 |
| A ₈ | 25.7778 | 27.0278 | 28.2778 |
| B ₁ | -189.1708 | -213.6748 | -238.1788 |
| B ₂ | -52.9286 | -77.4326 | -101.9366 |

The manipulator is actuated by d.c. motors; with reference to (6) the data are:

$$s_1 = 29.59 \quad s_2 = 25.42 \quad s_3 = 29.59 \quad (29)$$

$$n_1 = 814.36 \quad n_2 = 1163.32 \quad n_3 = 814.36.$$

All the above values are understood to be in SI units.

The trajectory to track in the joint space is described in Table 2; a trapezoidal velocity profile (acceleration, cruise, deceleration) has been chosen. The values are in degrees.

Table 2. Desired trajectory parameters

| joint | q _i | q _f | \ddot{q} | \dot{q}_{max} |
|-------|----------------|----------------|------------|-----------------|
| 1 | 0 | 90 | 26.15 | 5.23 |
| 2 | 90 | 135 | 78.5 | 3.925 |
| 3 | 45 | 90 | 78.5 | 3.925 |

The nonlinear compensating term $\hat{g}(q_k)$ in (9) has been introduced in the control v_k ; the payload of 5 has been selected as the nominal one. The position feedback gain in (9) has been derived in order to get two coincident poles for the equivalent discrete linear time invariant system (10).

An inverse model technique has been adopted so that, given the desired joint trajectory, the input to the reference model \hat{v}_k in (15) can be determined. More precisely, since the zero of the discrete reference model lies in the left hand plane (inside the unit circle), the actual input \hat{v}_k has been conveniently picked up as the average of two consecutive samples so as to smoothen its alternating shape.

In (22), moreover, it has been chosen P = I.

The solution sampling time adopted is 5 ms.

Figs. 1 thru 3 show some of the simulation results obtained. First the joint position tracking errors for the most significant cases of off-nominal

payloads (0, 10) are reported in figs. 1 and 2 respectively. They are seen to maintain bounded along the trajectory and practically vanish at steady-state. Fig. 3 then shows the three control torques for a payload of 10. They are seen to be smooth and of reasonable magnitude.

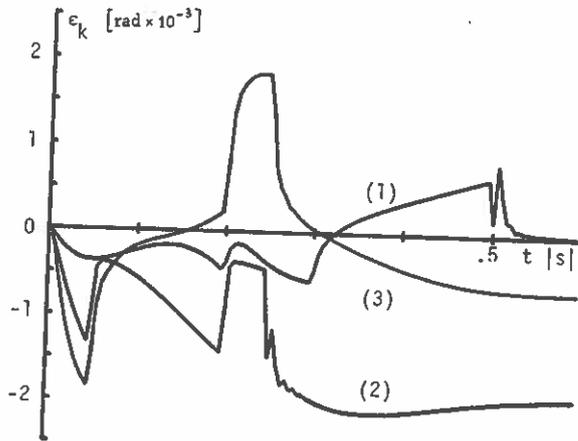


Fig. 1. Joint position tracking errors (0 kg.)

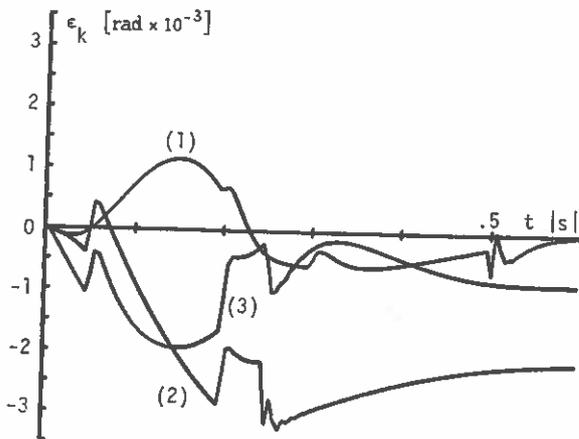


Fig. 2. Joint position tracking errors (10 Kg.)

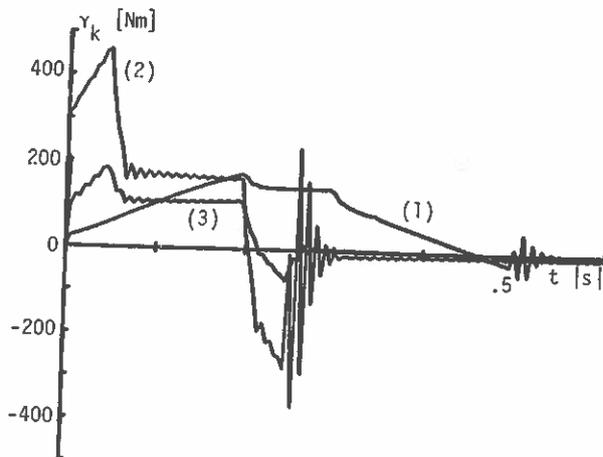


Fig. 3. Control torques (10 Kg.)

CONCLUSIONS

A new discrete control strategy has been developed for robot tracking control in the joint space. The two main features of the control are:

-) decentralization (n SISO joint control systems)
-) output measurements (only joint positions).

The control is made decentralized by rearranging the manipulator's dynamic model so as to isolate the constant diagonal terms in the inertia matrix. Then a suitable nonlinear term compensates for the available estimate of the gravity term. One might also think of compensating for the available estimates of the dominant configuration-dependent terms in the inertia matrix, further lightening thus the control effort. To this purpose the desired joint accelerations are understood to be used in lieu of the actual joint accelerations. The digital control is then synthesized via an usual model reference technique but, due to the particular choice of the model, only joint position measurements are used to form the feedback control. Simulation results have finally shown the effectiveness of the proposed control technique, also when off-nominal payloads are assumed.

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