Task Space Dynamic Analysis of Multiarm System Configurations

P. Chiacchio, S. Chiaverini, L. Sciavicco and B. Siciliano

The International Journal of Robotics Research 1991; 10; 708

DOI: 10.1177/027836499101000608

Communication

P. Chiacchio
S. Chiaverini
L. Sciavicco
B. Siciliano
Dipartimento di Informatica e Sistemistica
Università degli Studî di Napoli "Federico II"
80125 Napoli, Italy

Task Space Dynamic Analysis of Multiarm System Configurations

Abstract

The aim of this article is to provide a systematic method to perform dynamic analysis in the task space for a system composed of multiple arms holding a rigid object. The dynamic manipulability ellipsoid is introduced to obtain quantitative indices of the system's capability, in each configuration, of performing object accelerations along given task space directions. The ellipsoid is derived on the basis of the mapping of object accelerations onto joint driving torques via the proper kineto-static and dynamic equations of the system. The maximum joint torque limits are taken into account, and the effects of gravitational loads onto the ellipsoid are evidenced. Analysis of multiarm system configurations is carried out in a number of case studies.

1. Introduction

A great deal of attention has been directed toward the adoption of robotic systems having more than one single manipulation arm cooperating to the execution of a certain task. Pioneering works date back to the 1970s (Nakano et al. 1974; Fujii and Kurono 1975; Ishida 1977). Current research in the field covers different topics, including analysis of close kinematic chains (Luh and Zheng 1987), synchronization of multiple robots (Luh and Zheng 1986), collision avoidance between arms (Zapata, Fournier, and Dauchez 1987), coordinated control (Hayati 1986; Uchiyama et al. 1987; Nakamura et al. 1989), and programming environment (Nilakantan and Hayward 1989).

Multiple arms offer enhanced potential over a single arm if an effective coordination of their actions is accomplished. It is not possible to directly utilize the results available for single arms, but it is necessary to regard the system (arms plus the manipulated object) from an integrated standpoint. For this purpose, the formulation established in Dauchez and Uchiyama (1987) proves very adequate, as it allows a symmetric description of motion and force at the object level.

Well-established tools for analysis of single robot arm configurations are represented by the so-called manipulability ellipsoids. These give quantitative indices of the ability of the structure to arbitrarily perform force/motion in each task direction. Alternatively, a manipulator can be reconfigured to the most favorable posture to execute an assigned task by taking advantage of the above measures.

Kineto-static and dynamic manipulability ellipsoids have been introduced in the literature. The former (Yoshikawa 1985b) are based on the kineto-static mappings that (1) relate joint velocities to end-effector velocities by means of the manipulator Jacobian and (2) relate dually end-effector forces to joint torques by means of the transpose of the Jacobian. On the other hand, the latter (Asada 1983; Yoshikawa 1985a) take the arm dynamics into account and these are based on the relationship between joint actuator torques and end-effector accelerations through the manipulator Jacobian and inertia.

In view of the increasing interest in cooperative robot manipulation, we believe that the determination of suitable manipulability measures for multiple arm systems is of crucial importance to perform task space analysis of multiarm systems. Previous research contributions can be found in Lee (1989), Chiacchio et al. (1989, 1991) and Li et al. (1989), where static manipulability ellipsoids have been introduced.

The International Journal of Robotics Research,
Vol. 10, No. 6, December 1991,

The present work is aimed at providing a systematic method to perform task space analysis of multiarm systems based on the definition of a suitable dynamic manipulability ellipsoid; this is derived by expressing the joint driving forces of the multiple arms as a function of the object acceleration (Chiacchio et al. 1991). The proposed ellipsoid explicitly accounts for the joint maximum driving torques. Furthermore, unlike the previous work on dynamic manipulability ellipsoids for single arms (Yoshikawa 1985a), the static torques caused by gravitational loads are seen to produce a translation of the center of the ellipsoid. With this method, it is possible to evaluate the effects of the configuration of each single manipulator on the cooperative system. A number of case studies are developed for planar multiarm systems that illustrate the effectiveness of the approach.

2. Modeling of Multiple Single Robots

For the purpose of the present work, we regard a cooperative robot system as a system constituted by multiple arms manipulating together a common object. The aim of a task space dynamic analysis for such a system is to characterize the relationship between the driving torques available at the joints of each manipulator and the absolute accelerations achievable for the manipulated object.

It can be recognized that the derivation of the above relationship goes through the modeling of the kinematics and dynamics relative to each arm. This section is devoted to the aggregation of the kinematics and dynamics in a compact form; this turns out to be very useful for the modeling of the cooperation, which is the subject of the following section.

2.1. Kinematics

Consider a number of K manipulators; let $\mathbf{q}_i$ be the $(n_i \times 1)$ vector of joint displacements for each manipulator. These vectors can be suitably arranged in an $(N \times 1)$ joint vector $\mathbf{q}$, which is defined as

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ \vdots \\ \mathbf{q}_K \end{bmatrix}, \tag{1}$$

where $N = \sum_{i=1}^{K} n_i$ is the dimension of the extended joint space.

Let $\mathbf{v}_i$ denote the $(m_i \times 1)$ vector of (linear and angular) end-effector velocities for each manipulator. This can be related to the vector of joint velocities $\dot{\mathbf{q}}_i$ by the $(m_i \times n_i)$ Jacobian matrix $\mathbf{J}_i(\mathbf{q}_i)$. According to the formalism used in (1), the $(M \times 1)$ end-effector velocity vector $\mathbf{v}$ can be defined as

$$\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \vdots \\ \mathbf{v}_K \end{bmatrix}, \tag{2}$$

where $M = \sum_{i=1}^{K} m_i$ is the dimension of an extended task space, which we refer to in the following as contact task space. Therefore the differential kinematic equation mapping the joint velocity vector $\dot{\mathbf{q}}$ onto the end-effector velocity vector $\mathbf{v}$ can be established as

$$\mathbf{v} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}}, \tag{3}$$

where

$$\mathbf{J} = \mathrm{diag}(\mathbf{J}_1 \ldots \mathbf{J}_K) \tag{4}$$

is the $(M \times N)$ extended Jacobian matrix.

In order to obtain end-effector accelerations, eq. (3) is differentiated with respect to time, leading to

$$\mathbf{a} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}, \tag{5}$$

where $\mathbf{a} = \dot{\mathbf{v}}$ is the extended vector of end-effector accelerations.

2.2. Dynamics

By adopting the Lagrangian formulation, the joint space dynamic model of a single manipulator in contact with the environment can be written in the well-known closed form

$$\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{c}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{g}_i(\mathbf{q}_i) + \mathbf{J}_i^T(\mathbf{q}_i)\mathbf{h}_i = \tau_i,$$

where $\mathbf{M}_i$ is the $(n_i \times n_i)$ symmetric positive definite inertia matrix, $\mathbf{c}_i$ is the $(n_i \times 1)$ vector of Coriolis and centrifugal torques,[1] $\mathbf{g}_i$ is the $(n_i \times 1)$ vector of gravitational torques, $\mathbf{h}_i$ is the $(m_i \times 1)$ vector of end-effector contact forces exerted on the environment, τ_i is the $(n_i \times 1)$ vector of joint driving torques, and $\mathbf{J}_i$ is the above-defined Jacobian matrix.

According to the formalism introduced in the kinematics modeling, the dynamic equations of K manipulators in the extended joint space can be suitably aggregated into the form

$$\tau = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{c}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) + \mathbf{J}^T(\mathbf{q})\mathbf{h}, \tag{6}$$

where the $(N \times N)$ matrix $\mathbf{M}$ is defined as

$$\mathbf{M} = \mathrm{diag}(\mathbf{M}_1 \ldots \mathbf{M}_K); \tag{7}$$

1. In the remainder, the terms *torque* and *force* are often used as synonyms of *generalized force*.

the $(N \times 1)$ vectors $\mathbf{c}$, $\mathbf{g}$, and τ are respectively defined as

$$\mathbf{c} = \begin{bmatrix} \mathbf{c}_1 \\ \vdots \\ \mathbf{c}_K \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} \mathbf{g}_1 \\ \vdots \\ \mathbf{g}_K \end{bmatrix}, \quad \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_K \end{bmatrix}, \tag{8}$$

and the $(M \times 1)$ vector $\mathbf{h}$ is defined as

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1 \\ \vdots \\ \mathbf{h}_K \end{bmatrix}. \tag{9}$$

The problem now is to properly relate the accelerations in (5) and the forces in (9), defined in the contact task space, to the resulting accelerations on the object being manipulated by the multiple arms.

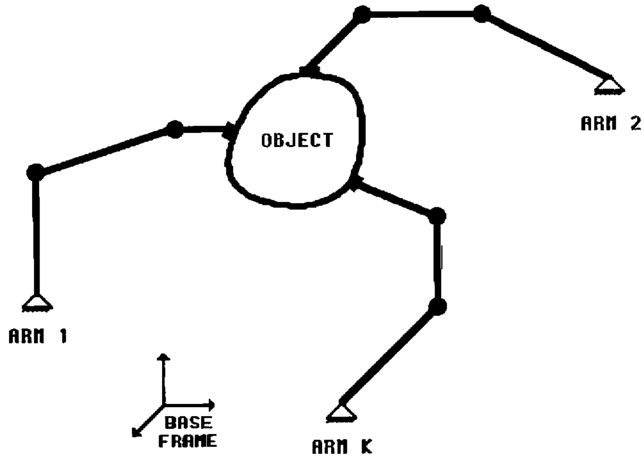

Fig. 1. A robotic system of multiple cooperating arms.

3. Modeling the Cooperation

In the previous section we considered a number of K manipulators, each having n_i joints and described with respect to an m_i-dimensional task space. If the manipulators are required to cooperate, a global object task space has to be considered; let m be the dimension thereof. Thus modeling the cooperation demands the definition of quantities characterizing the object task space, which then are to be related to the quantities characterizing the contact task space.

The relation between the end-effector contact forces and the resulting force on the object can be established following the formulation proposed in Dauchez and Uchiyama (1987) for dual-arm systems and later generalized in Chiacchio et al. (1989) for multiple arms. Furthermore, the object dynamics must be incorporated to relate external forces to object accelerations.

In what follows, we assume that the cooperative arm system manipulates an object constituted by a single rigid body of known characteristics. Also, we suppose that the end effector of each manipulator has a rigid contact with the object.

3.1. Force and Acceleration Composition

Consider the multiarm system illustrated in Figure 1. Let $\mathbf{h}_o$ denote the $(m \times 1)$ vector of external forces applied at the center of mass of the object. The force composition equation can be formally characterized as

$$\mathbf{h}_o = \mathbf{W}\mathbf{h}, \tag{10}$$

where $\mathbf{h}$ is defined in (9), and $\mathbf{W}$ is an $(m \times M)$ matrix that can be determined once the grasp geometry is assigned. The matrix $\mathbf{W}$ attains a simple expression if described with respect to a coordinate frame fixed on the object. In particular, in that frame, in the cases of tight grasp or rolling contact, the matrix $\mathbf{W}$ is constant, whereas in the case of sliding contact, the matrix $\mathbf{W}$ is variable.

Nonetheless, for our analysis it is opportune to study the absolute motion of the object with respect to a common frame attached at the base of the whole system. This implies that we have to express all quantities in this base frame. Thus the matrix $\mathbf{W}$, when referred to the base frame, will depend not only on the location of contact points, but also on the current location of the center of mass of the object.

If one desires to identify the contributions of the contact forces of each arm to the external forces on the object, the matrix $\mathbf{W}$ can be suitably partitioned as

$$\mathbf{W} = [\mathbf{W}_1 \cdots \mathbf{W}_K], \tag{11}$$

where the matrix $\mathbf{W}_i$, $i = 1, \ldots, K$, has dimension $(m \times m_i)$.

By virtue of the duality between forces and velocities that follows from the principle of virtual work in mechanics, it can be shown that

$$\mathbf{v} = \mathbf{W}^T\mathbf{v}_o \tag{12}$$

where $\mathbf{v}_o$ is the $(m \times 1)$ vector of absolute velocities of the object. Differentiating (12) with respect to time yields the acceleration composition equation in the form

$$\mathbf{a} = \mathbf{W}^T\mathbf{a}_o + \dot{\mathbf{W}}^T\mathbf{v}_o, \tag{13}$$

where $\mathbf{a}_o$ is the vector of absolute object accelerations.

3.2. Object Dynamics

The dynamic equations of motion for the held object can be expressed in the form

$$\mathbf{M}_o\mathbf{a}_o + \mathbf{c}_o + \mathbf{g}_o = \mathbf{h}_o \tag{14}$$

where $\mathbf{M}_o$ is the $(m \times m)$ object inertia matrix, $\mathbf{c}_o$ and $\mathbf{g}_o$ are, respectively, the $(m \times 1)$ vectors of velocity-dependent forces and gravitational forces, and $\mathbf{h}_o$ is given by (10).

4. Task Space Dynamic Analysis

An effective tool to perform task space dynamic analysis for a single arm is offered by the dynamic manipulability ellipsoid introduced in Yoshikawa (1985a). This gives the magnitude of the end-effector acceleration vector, in a certain task space direction, that can be obtained by applying joint driving torque vectors of fixed magnitude.

For the case of a cooperating arm system, therefore, it is appropriate to derive the mapping of the object space accelerations onto the extended joint space torques. This can be performed by suitably combining eqs. (5), (6), (10), (13), and (14).

Similarly to Yoshikawa (1985a), we regard the case when both the arms and the object are standing still ($\dot{\mathbf{q}} = \mathbf{0}$, $\mathbf{v}_o = \mathbf{0}$) as the fundamental one for considering the dynamic manipulability; this implies that $\dot{\mathbf{J}}\dot{\mathbf{q}} = \mathbf{0}$ in (5), $\mathbf{c} = \mathbf{0}$ in (6), $\dot{\mathbf{W}}^T\mathbf{v}_o = \mathbf{0}$ in (13), and $\mathbf{c}_o = \mathbf{0}$ in (14).

Using the preceding assumption, the following equations are obtained in which the dependence on $\mathbf{q}$ is omitted for notation compactness:

$$\mathbf{a} = \mathbf{J}\ddot{\mathbf{q}} \tag{5$'$}$$

$$\tau = \mathbf{M}\ddot{\mathbf{q}} + \mathbf{g} + \mathbf{J}^T\mathbf{h} \tag{6$'$}$$

$$\mathbf{a} = \mathbf{W}^T\mathbf{a}_o \tag{13$'$}$$

$$\mathbf{M}_o\mathbf{a}_o + \mathbf{g}_o = \mathbf{h}_o \tag{14$'$}$$

These, together with (10), will yield the sought mapping. Indeed, solving (5$'$) for $\ddot{\mathbf{q}}$ and using (13$'$) for $\mathbf{a}$ gives

$$\ddot{\mathbf{q}} = \mathbf{J}^\dagger\mathbf{W}^T\mathbf{a}_o \tag{15}$$

where $\mathbf{J}^\dagger$ denotes the Moore-Penrose pseudoinverse of $\mathbf{J}$ that is defined as $\mathbf{J}^\dagger = \mathbf{J}^T(\mathbf{J}\mathbf{J}^T)^{-1}$ when $\mathbf{J}$ is full rank. Notice that $\mathbf{J}^\dagger$ yields the minimum-norm joint acceleration solution at each configuration.

On the other hand, solving (10) for $\mathbf{h}$ and using (14$'$) for $\mathbf{h}_o$ yields

$$\mathbf{h} = \mathbf{W}^\dagger(\mathbf{M}_o\mathbf{a}_o + \mathbf{g}_o). \tag{16}$$

Observe that $\mathbf{W}^\dagger$ performs equal load sharing between the multiple arms; a weighted generalized inverse can be used in lieu of $\mathbf{W}^\dagger$ if different load sharing is desired (Uchiyama and Yamashita 1989).

Finally, plugging (15) and (16) into (6$'$) gives

$$\tau = \mathbf{\Lambda}_o\mathbf{a}_o + \mu_o \tag{17}$$

where

$$\mathbf{\Lambda}_o = \mathbf{M}\mathbf{J}^\dagger\mathbf{W}^T + \mathbf{J}^T\mathbf{W}^\dagger\mathbf{M}_o \tag{18}$$

and

$$\mu_o = \mathbf{g} + \mathbf{J}^T\mathbf{W}^\dagger\mathbf{g}_o. \tag{19}$$

It should be pointed out that $\mathbf{\Lambda}_o$ is formed by the contributions of the inertias of the single manipulators and of the object inertia. This is accomplished via a sequence of suitable transformation matrices deriving from the basic equations (15) and (16) that map object accelerations onto joint accelerations and end-effector contact forces, respectively. In the same fashion, μ_o is formed by the contributions of the gravitational loads of the single manipulators and of the object; the latter is obtained via the transformation matrix derived from equation (16) that maps the object gravitational forces onto end-effector contact forces.

At this point, it is convenient to normalize the joint torques with respect to the different torque limits on the joint actuators. Let $\tau_{i,max}$, $i = 1, \ldots, N$, denote the maximum (positive) driving torque at each joint of the robotic system; without loss of generality, we suppose that the upper and lower torque limits are of equal magnitude.

The normalized torque vector can be introduced as

$$\tilde{\tau} = \mathbf{Z}\tau \tag{20}$$

where

$$\mathbf{Z} = \mathrm{diag}(Z_1 \ldots Z_N) \tag{21}$$

with $Z_i = 1/\tau_{i,max}$, $i = 1, \ldots, N$.

The unit sphere in the joint space of the normalized torques

$$\tilde{\tau}^T\tilde{\tau} = 1 \tag{22}$$

maps onto the ellipsoid in the object space of the absolute accelerations

$$\mathbf{a}_o^T\tilde{\mathbf{\Lambda}}_o^T\tilde{\mathbf{\Lambda}}_o\mathbf{a}_0 + 2\mathbf{a}_o^T\tilde{\mathbf{\Lambda}}_o^T\tilde{\mu}_o + \tilde{\mu}_o^T\tilde{\mu}_o = 1, \tag{23}$$

where $\tilde{\mathbf{\Lambda}}_o = \mathbf{Z}\mathbf{\Lambda}_o$ and $\tilde{\mu}_0 = \mathbf{Z}\mu_o$, which is defined here as the *dynamic manipulability ellipsoid* for the multiarm system.

Notice that when the effects of gravitational loads are neglected (i.e., $\mu_o = \mathbf{0}$), the ellipsoid has its center at the origin of the reference base frame. Remarkably, in this case, the result obtained in

Yoshikawa (1985a) for a single arm can be recovered. On the other hand, when gravity is considered, the center of the ellipsoid no longer coincides with the frame origin as a result of the two additional terms in (23), which are a function of μ_o.

It must be stressed that the way we account for gravitational load is conceptually different from previous works on dynamic ellipsoid for single arms. In fact, in Yoshikawa (1985a), the absolute value of the gravitational load is subtracted by the torque limit at each joint; we argue here that such method always penalizes the available torques in the joint space and does not properly describe the effects of gravity on the possible task space accelerations, as does our approach.

The preceding ellipsoid provides a systematic, direct tool to perform task space dynamic analysis of multiarm system configurations. In fact, the shape and orientation of the ellipsoid give a measure of the capability of the system to accelerate the object in a certain task space direction. Furthermore, if the system possesses some kinematic redundancies, it is possible to reconfigure it in a more effective posture to execute the given task.

Potentially, the dynamic manipulability ellipsoid can also be employed as an aid in the design of a multiarm system in which dimensions of the arm structures and their mass distributions are optimized to obtain, for instance, dynamic isotropic configurations over the cooperative robotic system work space.

Furthermore, a scalar manipulability measure can be derived by computing the volume of the ellipsoid and then using the result to detect singular configurations of the system (the volume becomes zero) (i.e., when it is not possible to accelerate the object in some direction).

Finally, it is possible to analyze the effects of the configuration of each single manipulator on the cooperative system, as done for the static case in Chiacchio et al. (1989); numeric studies are carried out in the following section.

5. Case Studies

A number of case studies are developed in this section to provide further insight into the potential offered by the multiarm system dynamic ellipsoid for task space analysis of system configurations.

Planar robotic systems composed of two or three arms manipulating a rigid object are considered. All the quantities are referred to a base frame, which is located at the base of one arm. For each arm, we have taken the three-degree-of-freedom geometry of the MANUTEC R3 industrial robot (Otter and Türk 1988), which can be obtained by considering the two parallel pitch joints of the shoulder and of the elbow and the pitch joint of the spherical wrist when the roll joint of the forearm is locked in the zero position.

For the sake of simplicity, in all case studies, the dynamic analysis is performed in a two-dimensional global object task space ($m = 2$), and only linear object accelerations are of interest. Similarly, the end-effector task space of each arm is assumed to be two-dimensional ($m_i = 2$), and only pure forces are considered. Thus the matrices $\mathbf{W}_i$ in (11) are (2×2) identity matrices.

Therefore for the two-arm system we have a six-dimensional extended joint space ($N = n_1 + n_2 = 6$) and a four-dimensional contact space ($M = m_1 + m_2 = 4$). On the other hand, for the three-arm system we have a nine-dimensional extended joint space ($N = n_1 + n_2 + n_3 = 9$) and a six-dimensional contact space ($M = m_1 + m_2 + m_3 = 6$).

The three-link planar structure is characterized by the Jacobian matrix whose elements are:

$$J_{11} = -l_1 s_1 - l_2 s_{12} - l_3 s_{123}$$

$$J_{12} = l_2 s_{12} - l_3 s_{123}$$

$$J_{13} = -l_3 s_{123}$$

$$J_{21} = l_1 c_1 + l_2 c_{12} + l_3 c_{123}$$

$$J_{22} = l_2 c_{12} + l_3 c_{123}$$

$$J_{23} = l_3 c_{123}$$

by the symmetric inertia matrix whose elements are:

$$\begin{aligned} M_{11} = {} & 2 l_1 l_{c3} m_3 c_{23} + 2 l_1 (l_{c2} m_2 + l_2 m_3) c_2 \\ & + 2 l_2 l_{c3} m_3 c_3 + l_{c1}^2 m_1 + l_1^2 (m_2 + m_3) \\ & + l_{c2}^2 m_2 + (l_2^2 + l_{c3}^2) m_3 + I_1 + I_2 + I_3 \end{aligned}$$

$$\begin{aligned} M_{12} = {} & l_1 l_{c3} m_3 c_{23} + l_1 (l_{c2} m_2 + l_2 m_3) c_2 + 2 l_2 l_{c3} m_3 c_3 \\ & + l_{c2}^2 m_2 + (l_2^2 + l_{c3}^2) m_3 + I_2 + I_3 \end{aligned}$$

$$M_{13} = l_1 l_{c3} m_3 c_{23} + l_2 l_{c3} m_3 c_3 + l_{c3}^2 m_3 + I_3$$

$$M_{22} = 2 l_2 l_{c3} m_3 c_3 + l_{c2}^2 m_2 + (l_2^2 + l_{c3}^2) m_3 + I_2 + I_3$$

$$M_{23} = l_2 l_{c3} m_3 c_3 + l_{c3}^2 m_3 + I_3$$

$$M_{33} = l_{c3}^2 m_3 + I_3$$

and by the gravitational force vector whose elements are:

$$\begin{aligned} g_1 = {} & g_0 [l_{c3} m_3 c_{123} + (l_{c2} m_2 + l_2 m_3) c_{12} \\ & + (l_{c1} m_1 + l_1 m_2 + l_2 m_3) c_1] \end{aligned}$$

$$g_2 = g_0 [l_{c3} m_3 c_{123} + (l_{c2} m_2 + l_2 m_3) c_{12}]$$

$$g_3 = g_0 l_{c3} m_3 c_{123}$$

where l_i is the link length, l_{ci} is the distance of the link center of mass from the joint, m_i is the link mass, I_i is the link moment of inertia about the joint axis, g_0 is the value of the gravity acceleration, and $s_{i..j}$, $c_{i..j}$ are the shorthand notations for $\sin(q_i + \cdots + q_j)$, $\cos(q_i + \cdots + q_j)$.

The object is a disk of radius r_o characterized by the symmetric inertia matrix whose elements are:

$$M_{0,11} = m_o$$

$$M_{0,12} = 0$$

$$M_{0,22} = m_o$$

and by the gravitational force vector whose elements are:

$$g_{o1} = 0$$

$$g_{o2} = m_o g_o$$

where m_o is the object mass.

The numeric values of the arm and object parameters are the following (in SI units):

Link 1:

$l_1 = 0.5 \quad l_{c1} = 0.205 \quad m_1 = 56.5 \quad I_1 = 2.58 \quad \tau_{1,max} = 1890$

Link 2:

$l_2 = 0.73 \quad l_{c2} = 0.32 \quad m_2 = 28.7 \quad I_2 = 1.67 \quad \tau_{2,max} = 540$

Link 3:

$l_3 = 0.2 \quad l_{c3} = 0.023 \quad m_3 = 5.2 \quad I_3 = 0.0125 \quad \tau_{3,max} = 160.5$

Object:

$m_o = 15 \quad r_o = 0.15$

Notice that the object mass has been chosen as the maximum load that can be lifted by a single MANUTEC R3 arm (Otter and Türk 1988).

In each of the following illustrations, the ellipsoids are plotted in the same scale (i.e., with reference to the axes of the larger ellipsoid in the picture).

In the first case study, a two-arm system is analyzed in a symmetric configuration. The effect of gravitational loads has been neglected here ($\mu_o = \mathbf{0}$; i.e., the system is supposed to be in the horizontal plane). The dynamic ellipsoid for each single arm holding the object has been computed in order to evidence the composition of the single arm ellipsoids into the ellipsoid for the overall system. The results in Figure 2 show the existence of preferred directions to perform linear accelerations at the end effectors of each arm. The ellipsoid of the two-arm

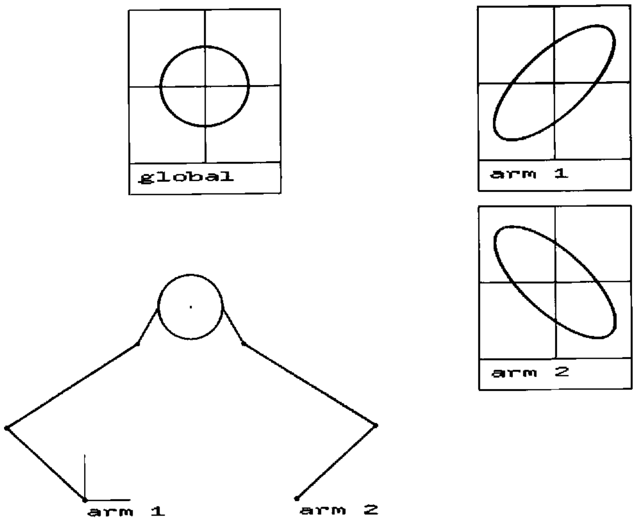

Fig. 2. Dynamic manipulability ellipsoid for the two-arm system in a symmetric configuration.

system instead reveals that the system is in a near-isotropic configuration because of the constraints imposed by the cooperation.

In the second case study, a third arm is added to the previous system in the same configuration. The shape of the ellipsoid for the single third arm reflects on the shape of the global ellipsoid (Fig. 3). This indicates that the system is no longer in a near-isotropic configuration, and the figure shows preferred directions to perform object accelerations that are, in turn, dictated by the third arm.

The final case study is aimed at analyzing the

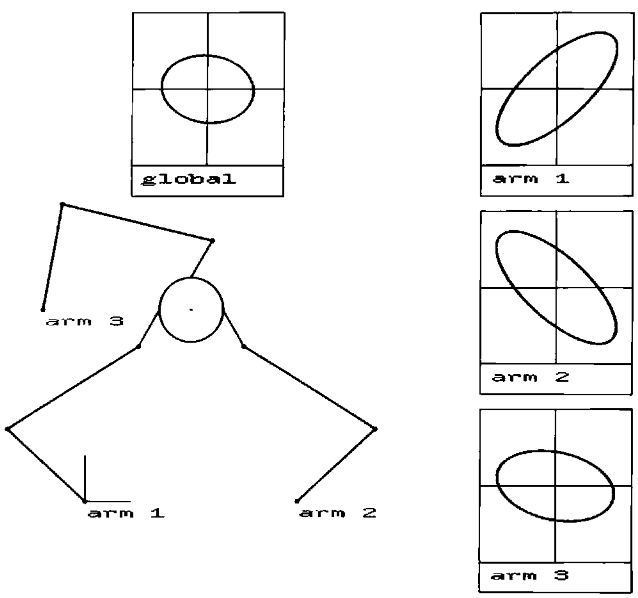

Fig. 3. Dynamic manipulability ellipsoid for the three-arm system.

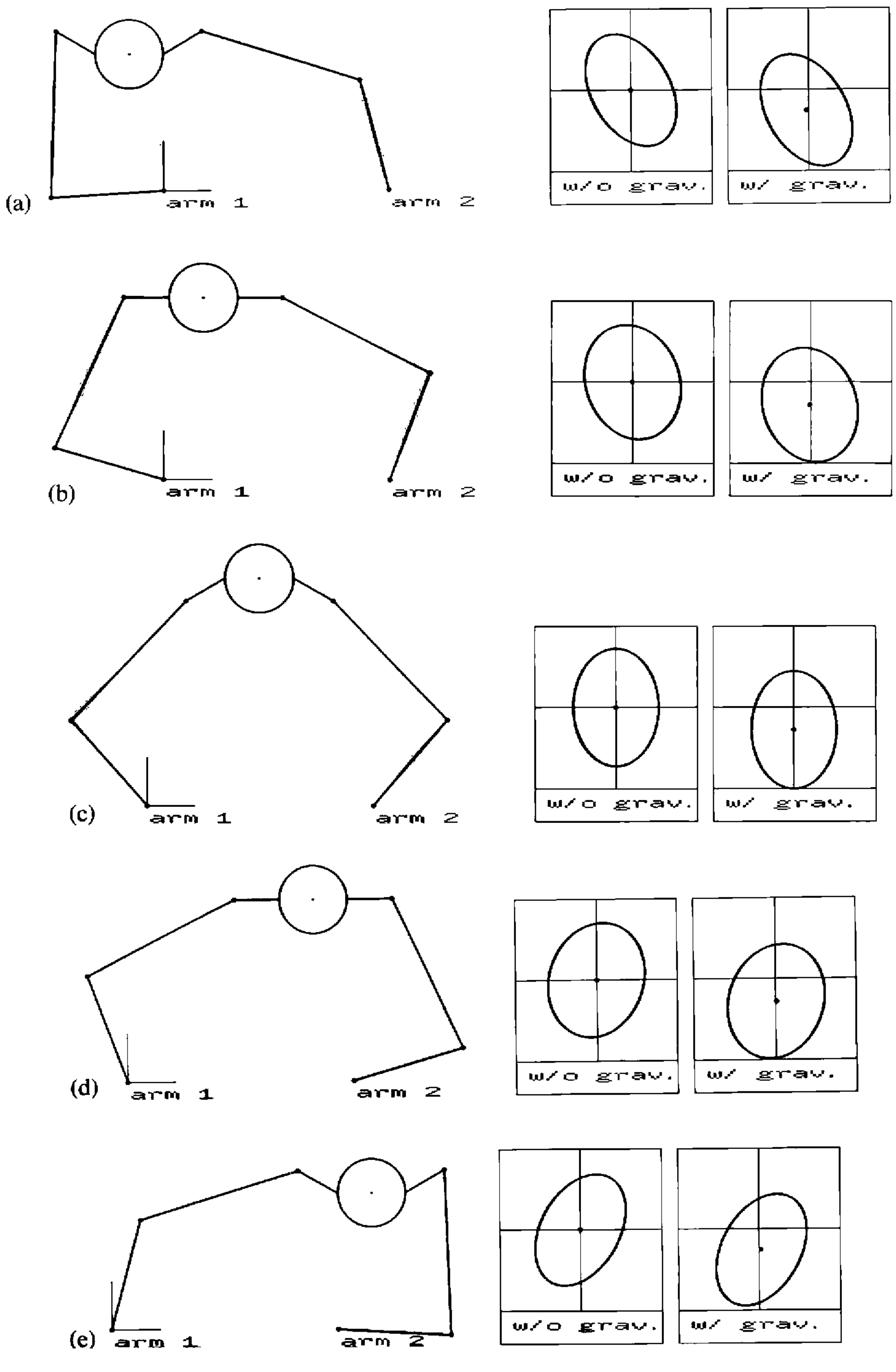

Fig. 4. Effects of gravitational loads on the dynamic manipulability ellipsoid for the two-arm system in five different configurations.

effects of gravitational loads on a two-arm system. Five different configurations have been considered, with and without the inclusion of gravitational loads (Fig. 4). As anticipated in theory, the center of the ellipsoid translates when gravity is considered. The displacement is dominant in the downward direction; in fact, the "asymmetric" effect of gravity in the vertical direction implies that feasible object accelerations are decreased upward and increased downward. Notice also that the center of the ellipsoid lies exactly on the vertical axis only in the case of a symmetric system configuration (Fig. 4C), as one could have expected. Instead, in the other configurations a slight horizontal shift also occurs; this effect is symmetric with regard to each pair of configurations (Fig. 4*A* vs. Fig. 4*E* and Fig. 4*B* vs. Fig. 4*D*, respectively).

6. Conclusions

The dynamic manipulability ellipsoid for cooperating arms holding a rigid object has been introduced in this article as an effective tool to perform task space analysis of system configurations. It describes the system capability of performing object accelerations along given task space directions for joint torques belonging to a given set.

The theory has been validated in a number of case studies for planar multiarm systems. The contributions of the single arm ellipsoids to the global system ellipsoid have been evidenced. Further, the effects of gravitational loads of the arms and object have been analyzed in different configurations.

In conclusion, we believe that the dynamic manipulability ellipsoid, together with the force and velocity ellipsoids we have recently proposed, not only allows the user to completely analyze multiarm system configurations, but may also be utilized to define suitable manipulability constraints on the reconfiguration of the system in optimal task-oriented postures. This will be the subject of further investigation.

Acknowledgments

The work reported in this paper is based on research partially supported by Consiglio Nazionale delle Ricerche under contract 90.00355.PF67.

References

Asada, H. 1983. A geometrical representation of manipulator dynamics and its application to arm design. *Trans. ASME J. Dyn. Sys. Meas. Control* 105(3):131–135.

Chiacchio, P., Chiaverini, S., Sciavicco, L., and Siciliano, B. 1989 (Pasadena, CA, Jan.–Feb.). On the manipulability of dual cooperative robots. *Proc. NASA Conf. Space Telerobotics*, pp. 351–360.

Chiacchio, P., Chiaverini, S., and Siciliano, B. 1990a (Cracow, July). Dynamic manipulability ellipsoid for cooperating robots. *Prep. 8th CISM-IFToMM Symp. Theory on Practice of Robots and Manipulators.*

Chiacchio, P., Chiaverini, S., Sciavicco, L., and Siciliano, B. 1991. Global task space manipulability ellipsoids for multiple arm systems. *IEEE Trans. Robot. Automat.* In press.

Dauchez, P., and Uchiyama, M. 1987 (Versailles, Oct.). Kinematic formulation for two force-controlled cooperating robots. *Proc. 3rd Int. Conf. Advanced Robotics.* Bedford, U.K.: IFS, pp. 457–467.

Fujii, S., and Kurono, S. 1975 (Newcastle-upon-Tyne, Sept.). Coordinated computer control of a pair of manipulators. *Proc. 4th IFToMM World Congress*, pp. 411–417.

Hayati, S. 1986 (San Francisco, April). Hybrid position/force control of multi-arm cooperating robots. *Proc. IEEE Int. Conf. on Robotics and Automation.* New York: IEEE, pp. 82–89.

Ishida, T. 1977 (Aug.). Force control in coordination of two arms. *Proc. 5th Int. Conf. on Artificial Intelligence*, pp. 717–722.

Lee, S. 1989. Dual redundant arm configuration optimization with task-oriented dual arm manipulability. *IEEE Trans. Robot. Automat.* 5(1):78–97.

Li, Z., Hsu, P., and Sastry, S. 1989. Grasping and coordinated manipulation by a multifingered hand. *Int. J. Robot. Res.* 8(4):33–50.

Luh, J. Y. S., and Zheng, Y. F. 1986 (Athens, Dec.). An interactively hierarchical control scheme for two coordinating industrial robots. *Proc. 25th IEEE Conf. on Decision and Control.* New York: IEEE, pp. 1265–1266.

Luh, J. Y. S., and Zheng, Y. F. 1987. Constrained relations between two coordinated industrial robots for motion control. *Int. J. Robot. Res.* 6(3):60–70.

Nakamura, Y., Nagai, K., and Yoshikawa, T. 1989. Dynamics and stability in coordination of multiple robotic mechanisms. *Int. J. Robot. Res.* 8(2):44–61.

Nakano, E., Ozaki, S., Ishida, T., and Kato, I. 1974 (Tokyo, Nov.). Cooperational control of the anthropomorphous manipulator MELARM. *Proc. 4th Int. Symp. on Industrial Robots*, pp. 251–260.

Nilakantan, A., and Hayward, V. 1989. The synchronization of multiple manipulators in Kali. *Robot. Autonomous Sys.* 5(4):345–358.

Otter, M., and Türk, S. 1988. *The DFVLR Models 1 and 2 of the Manutec r3 Robot.* DFVLR-Mitteilung 88-13.

Uchiyama, M., Iwasawa, N., and Hakomori, K. 1987 (Raleigh, NC, Mar.–Apr.). Hybrid position/force control scheme for the coordination of two robots. *Proc. IEEE Int. Conf. on Robotics and Automation.* New York: IEEE, pp. 1242–1247.

Uchiyama, M., and Yamashita, T. 1989 (Montreal, June). Asymmetric hybrid control of positions and forces of a dual arm robot to share loads. *Prep. 1st Int. Symp. on Experimental Robotics.* Lecture Notes in Control and Info. Sci., 139 (V. Hayward, O. Khatib, eds.). Berlin: Springer-Verlag, pp. 100–115.

Yoshikawa, T. 1985a. Dynamic manipulability of robot manipulators. *J. Robot. Sys.* 2(2):113–124.

Yoshikawa, T. 1985b. Manipulability of robotic mechanisms. *Int. J. Robot. Res.* 4(1):3–9.