



# A New Approach for Simplifying Multi-Degree of Freedom Haptic Device Dynamics Model

Ahmad Mashayekhi<sup>1</sup> · Abbas Karami<sup>2</sup> · Bruno Siciliano<sup>3</sup>

Received: 29 November 2022 / Accepted: 18 March 2023 / Published online: 8 May 2023  
© The Author(s), under exclusive licence to Springer Nature B.V. 2023

## Abstract

A haptic device (HD) is an interface used for simulating a virtual environment (VE) for its operator. While simulating a VE, the HD should be stable; otherwise, it can damage itself or its operator. Usually, HDs are multi-degree-of-freedom serial manipulators with sensor quantization and friction in their joints. Hence, the HD dynamics is complex and its analytical stability analysis is complicated. During simulating of the VE for the operator, stylus movements are small. In the previous studies, the multi-DOF nonlinear dynamics of the HD was replaced with simple dynamics in which mass and viscous values are constant. However, there were neither analytical methods to determine the values of the mentioned parameters in the simplified model nor studying the accuracy of this simplification is studied. In this paper, a novel and general approach is employed for simplifying a multi-degree-of-freedom haptic device dynamics during arbitrary motion around the operating point, and its accuracy in the prediction of the stable simulation of the VE is discussed. Meanwhile, sensor quantization and Coulomb friction are considered in the model. This method is evaluated through simulation for stability analysis of the PHANTOM 1.5 and KUKA Light Weight Robot IV (LWR IV) as haptic interfaces in various situations.

**Keywords** Haptic device · Stability · Multi-DOF dynamic · Simplification

## 1 Introduction

Nowadays, HDs are employed on various tasks, including human-robot interaction [1], surgical applications [2, 3], and specially teleoperation applications [4–8]. A good review of haptic bilateral teleoperation systems can be found in [9].

A general sketch of the haptic system is shown in Fig. 1. As shown in this figure, an operator moves the end-effector (stylus) of a HD, while its position is read by HD's sensors ( $x_H$ ). Based on it, a virtual object is simulated for the operator

(mostly with 3D pictures) and based on the VE's impedance parameters, a desired force is calculated ( $f_d$ ) and applied to the operator's hand ( $f_{VE}$ ). In the case of a complete transparency, these two forces are the same. Transparency and stability in applying the desired force to the operator are critical issues for these devices.

A HD can simulate touching an entity in the VE for the user. To this end, the respective forces/torques raised from touching the virtual entity are applied by the HD to the user's hand.

The left loop shows a human who moves the HD stylus. Human behaviour is one of the factors for studying HD stability. Instabilities in these devices occur at frequencies higher than 100 Hz. On the other hand, it has been proven that the energy actively added by the operator to the haptic interface is at frequencies lower than 10 Hz [10]. Consequently, the operator is considered as a passive element, and the human finger, wrist, and hand are usually modelled by passive elements of mass-spring-damper [11–13], which makes the haptic system more stable [14, 15].

Compliant behaviour of a haptic device is required in its interaction with the VE to ensure safe interaction [16]. The HD stability is studied by Minsky et al. in [17]. This stability

✉ Ahmad Mashayekhi  
Mashayekhi@sirjantech.ac.ir;  
Ahmad.Mashayekhi@yahoo.com

Abbas Karami  
karami@sutech.ac.ir

<sup>1</sup> Head of the Mechanical Engineering Department,  
Sirjan University of Technology, Sirjan 7813733385, Iran  
<sup>2</sup> Department of Mechanical Engineering, Shiraz University  
of Technology, Shiraz, Iran  
<sup>3</sup> Department of Electrical Engineering and Information  
Technology, University of Naples Federico II,  
80125 Naples, Italy

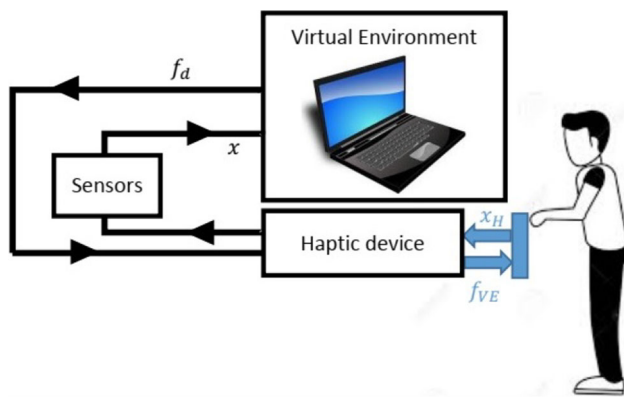


Fig. 1 A haptic system scheme

analysis of a HD is performed through linear modelling for the robot and a continuous model for the VE. Other works such as [14, 18, 19] studied the stability of HDs by considering the human operator. The acquired results show an improvement in the stability in the presence of the operator's hand.

It is shown that dissipated energy due to Coulomb friction is more than generated energy due to quantization in most commercial public haptic devices [19, 20]. So in all the previous stability analyses, these two nonlinear effects are omitted by each other. As a result, there is no explicit formula for finding a stable domain for a HD considering quantization and both viscous and Coulomb friction. Moreover, HD passivity is studied in [20–22]. There are some passivity criteria, which relate the mentioned parameters together, such as [19, 20], but the passive operation range is conservative [18, 23]. Virtual damping and time delay have a considerable effect on HD stability. Unstable behaviour in large time delays has been reported in [24, 25]. Recently stability boundary of a 1-DOF HD with the LuGre friction model is studied in [26], using the new determined describing function of the LuGre friction model.

While the aforementioned papers considered approximated 1-DOF haptic device with constant dynamic parameters for studying HD stability, neither there is a method to show how the nonlinear and multi-degree-of-freedom dynamic of the HD can be simplified to a constant mass, viscous and Coulomb friction, nor the accuracy of this assumption is studied. Most of the previous works employed the intrinsic dynamics of the only active joint involved in the movement of the stylus while simulating the VE. For example, in [22, 27], a 1-DOF HD for validating the theory via experiment is employed. Moreover, in [14, 23, 28], just one joint of a multi-DOF HD was active, and all other joints were locked during simulating a VE.

On the other hand, stability analysis of a multi-DOF multilateral HD through a new numerical straightforward approach is employed in references such as [29–31]. Also,

in [32] stability of a linearized multi-DOF haptic device model was studied using Linear Matrix Inequality (LMI). Nonlinear inertia of the HD is reduced in [33], by employing force-torque sensing. Influence of the vibration modes of a 2-DOF HD is also studied in [34], which shows internal vibration modes can reduce stability boundary of a HD.

In all mentioned references that work with multi-DOF HD, numerical methods are used to predict or improve the stability boundaries of nonlinear models. In fact, there is no straightforward method which simply reduces the nonlinear and multi-DOF dynamics of a HD into a 1-DOF model. This method can also simplify the HD dynamics to accurate model with constant parameters in operation point, which can enable analytical stability/passivity analysis. Redundancy resolution is also possible through this method [35].

The current work proposes a general framework to study the stability of a HD with multiple active DOFs, by analyzing equivalent 1-DOF dynamics instead of nonlinear multi-DOF dynamics. The main contributions of the current study are summarized as follows,

- Considering quantization effect along with viscous and Coulomb friction in robot joints in stability analysis;
- Proposing a method for computing an equivalent sensor resolution;
- Employing projection method to obtain an accurate 1-DOF model in operation direction;
- Simplifying the nonlinear multi-DOF dynamic of a haptic device to a 1-DOF dynamic consisting of an effective mass, viscous friction, Coulomb friction, and sensor quantization;
- Studying multi-DOF HD stability and comparing it with the case of simplified 1-DOF.

Without simplifying the nonlinear multi-DOF dynamic of a haptic device, it will be so hard to present analytical stability or passivity analysis. Finally, the new approach is utilized for the stability analysis of the 3-DOF PHANTOM and 7-DOF KUKA as haptic interfaces. Employing advanced industrial robots like KUKA, as HD is recently a new trend [36]. These manipulators have a complicated mechanism, more dexterity, and a wide range of accurate force feedback.

## 2 Modeling Haptic Interface

### 2.1 Virtual Environment

During physical interaction with most of the objects around us, elastic and damp behaviour arises. Hence, VE entities are commonly considered as walls with virtual stiffness ( $K_w$ ) and virtual damping ( $B_w$ ). An interface stylus is usually assumed

in contact with this virtual wall. Thus the VE can be modelled as:

$$H(z) = K_w + B_w \frac{z-1}{T_s z}, \quad (1)$$

where  $T_s$  is the constant value of sampling time.

## 2.2 Time Delay

Time delay may exist in the control loop of the HD due to computation, communication, sensor delay, actuator delay and so on. Since the sampling rate for the haptic interface is usually more than 1000 Hz, some millisecond time delay may alter the system functionality. As shown in [25], a haptic interface with lower time delay may simulate virtual walls with higher stiffness. According to [37], usually, haptic interface time delay  $T_d$  is integrated.

## 2.3 Haptic Device

The dynamic model of an m-DOF haptic device with series configuration has the following dynamic equation:

$$\tau = M(q) \ddot{q} + n(q, \dot{q}) + g(q), \quad (2)$$

where  $\tau \in \mathbb{R}^{m \times 1}$  and  $M \in \mathbb{R}^{m \times m}$  are joint actuator torques vector and mass matrix, respectively.  $n \in \mathbb{R}^{m \times 1}$  include Centrifugal, Coriolis, and friction forces vector.  $q = [q_1, q_2, \dots, q_m]^T$  is the vector of joint variables, and  $g$  is the gravity vector. The terms on the right-hand side contain mass matrix, gravity vector, and  $n$  contain trigonometric terms. Frequently, the gravity vector is compensated by the internal controller of the HD.

Friction exists in the haptic device's joints, which is mainly modelled by viscous and Coulomb terms. Rewriting  $n$  in detail as

$$n = c + f_v + f_c = c + \begin{bmatrix} b_1 \dot{q}_1 \\ b_2 \dot{q}_2 \\ \vdots \\ b_n \dot{q}_m \end{bmatrix} + \begin{bmatrix} \mu_1 \text{sign}(\dot{q}_1) \\ \mu_2 \text{sign}(\dot{q}_2) \\ \vdots \\ \mu_n \text{sign}(\dot{q}_m) \end{bmatrix}, \quad (3)$$

where  $c$  includes Centrifugal and Coriolis forces besides  $f_v$  and  $f_c$  are vector of viscous and Coulomb friction, respectively. Moreover,  $b_i$  and  $\mu_i$  ( $1 \leq i \leq m$ ) are viscous and Coulomb friction coefficients of each joint. Therefore, stability analysis of (2) in contact with a VE is complicated because of multiple nonlinear equations.

## 3 Simplified Dynamic of the Haptic Device

HD should simulate various environments, from soft to hard. The ideal case of zero penetration depth corresponds to infinite stiffness. However, this penetration depth is not available since HD becomes unstable. Therefore, the higher limit of the virtual wall stiffness specifies the robot stability region.

On the other hand, the robot dynamic parameters are non-linear functions of joint position and velocity. While the stylus can move in a broad workspace, virtual objects typically are stiff and smaller than the HD workspace and restrict the stylus motion. So, studying the system's behaviour around the operating configuration is reasonable.

The effective dynamic parameters compose the equivalent robot model according to the robot stylus direction in the operating point. Then, the stability analysis can be accomplished through this simplified model to find the maximum value for the virtual wall stiffness.

### 3.1 Computing Dynamic Parameters

In this work, projecting robot dynamic parameters to the task space and computing the effective parameters is followed. The purpose is to find an equivalent system for the non-linear multi-DOF HD, with a 1-DOF HD with an effective mass, moving with effective viscous and Coulomb friction, while its position is sensed through a virtual sensor with an effective resolution. The main issue is having an accurate stability boundary in various cases. The effective parameters change in each operating point. A general formula is necessary to obtain 1-DOF dynamic model in each practical point according to the HD configuration and direction of the stylus movement. To this end, (2) is rewritten as

$$\ddot{q} = M^{-1}(J^T \Gamma - n(q, \dot{q}) - g(q)), \quad (4)$$

where  $\Gamma$  is actuator torque projection in task space directions, and parameters dependencies are omitted for brevity. Therefore, one may obtain

$$\ddot{x} - \dot{J}\dot{q} + JM^{-1}(n(q, \dot{q}) + g(q)) = JM^{-1}J^T \Gamma. \quad (5)$$

Stylus motion direction, shown by  $e_s \in \mathbb{R}^{3 \times 1}$ , is supposed as the robot task space. Considering the robot stylus in contact with the virtual wall along  $e_s$ , which is usually perpendicular to the virtual wall, effective dynamic parameters are

$$\begin{aligned} \Lambda_s(q) &= (J_s(q)M^{-1}(q)J_s^T(q))^{-1}, \\ \gamma_s(q) &= \Lambda_s(J_s(q)M^{-1}(q)n(q, \dot{q}) - \dot{J}_s(q)\dot{q}), \\ h_i(q) &= \Lambda_s(q)J_s(q)M^{-1}(q)g(q), \end{aligned} \quad (6)$$

where  $J_s(q) = e_s^T J(q)$  and  $J(q) \in \mathbb{R}^{m \times m}$  is the robot Jacobian.  $\Lambda_s$  and  $h_i$  are equivalent mass matrix and gravity vector while  $\gamma_s$  is the projection of  $n(q, \dot{q})$  in the operation space. Since  $J_s$  is  $1 \times m$ , all the effective dynamic parameters computed in (6) are scalar quantities.

Since quantization is a critical parameter in HD stability, an effective resolution should also be considered. According to the robot differential kinematic ( $\dot{x} = J\dot{q}$ ) and calculus of variations, one may write

$$\delta q = J^{-1}(q)\delta x. \quad (7)$$

where  $\delta x$  is the unit vector for the arbitrary task space motion and  $\delta q$  is the corresponding joint motion. Assuming similar resolution for all joints ( $\Delta q_0$ ) and

$$n_{\delta q} = \delta q / \|\delta q\|, \quad (8)$$

as the normal joint space motion vector, one can compute the joint space resolution multiplier as

$$\lambda = \Delta q_0 / \max(n_{\delta q}). \quad (9)$$

Hence,  $\Delta q = \lambda n_{\delta q}$  shows the marginally joint space motion, which leads to reaching a joint resolution. Consequently, effective task space resolution is suggested as

$$\Delta_{eff} = J_s(q)\Delta q. \quad (10)$$

### 3.2 Linearizing Dynamic Model

To this step, all the formulas are general and without any simplification. According to the aforementioned discussions, HD stability during interaction with the VE is critical. In reality, the stylus of the HD has a slow and slight penetration inside the stiff virtual objects. So, the equivalent 1-DOF robot can be obtained with constant dynamic parameters in the operating point ( $q_p$ ). Consequently, effective inertia is realized as

$$\Lambda_{eff} = (J_s(q_p)M^{-1}(q_p)J_s^T(q_p))^{-1}, \quad (11)$$

and effective viscous, Coulomb coefficients and sensor resolution are

$$\begin{aligned} b_{eff} &= \Lambda_s(J_s(q_p)M^{-1}(q_p)b)J^{-1}, \\ \mu_{eff} &= \Lambda_s(J_s(q_p)M^{-1}(q_p)f_c)/\text{sign}(J^{-1}\dot{s}), \\ \Delta_{eff} &= J_s(q_p)\Delta q, \end{aligned} \quad (12)$$

where  $\dot{s} = e_s v$  and  $v = J\dot{q}$ . The realized constant values of  $\Lambda_{eff}$ ,  $b_{eff}$  and  $\mu_{eff}$  are valid around operating point.

Therefore, the nonlinear multi-DOF dynamic of the HD is approximated by a simple 1-DOF system constructed by effective mass, viscous and Coulomb coefficients. Stability analysis by this model around the operating point is more effortless.

## 4 Simulations

As seen in the literature review, stability and passivity analysis of the HDs are performed through simplified 1-DOF haptic systems. Obtaining stability domain through simulation is a common approach (e.g. [14, 23, 28]). Herein, the stability of the multi-DOF HD is compared with the 1-DOF equivalent system via simulations, while Coulomb and viscous friction are considered, as well as sensor quantization.

Two different robots are used as HD in the simulations: PHANTOM 1.5 and KUKA LWR IV. The former has six DOFs, while its three first joints are active and the other three joints only have a position sensor and are not active, so the 3-DOF dynamics of this HD is extracted from [38] and used in the simulations. The latter has seven DOFs, and its dynamic model is validated in [39]. KUKA is recently employed as HD in multiple studies (see [24, 36]). Since this robot has 7-DOF, one of the joints is programmed to be inactive for the sake of avoiding null-space stability issues.

For each device, the method performance is studied through two different sets of simulations: A) Effect of time delay, and B) Effect of sampling time. In each set of simulations, stability boundary determined from the nonlinear multi-DOF dynamics is determined and compared with the stability boundary of the simplified 1-DOF system, in two different configurations for each robot.

In the case of time delay, the value of delay time in the control loop ( $T_d$ ) changed, and its effect on the accuracy of the simplified dynamics is studied. In the other set of simulations, the method performance is evaluated by changing the value of sampling time ( $T_s$ ). Moreover, another set of simulations is carried out in subsection C for the KUKA LWR to study the effect of dynamic parameters ratio. In this set of simulations, mass, viscous coefficient, Coulomb coefficient, and sensor resolution are changed and their effect on the stability domain is discussed.

In all simulations, the PHANTOM 1.5 tries to simulate a virtual object, perpendicular to its global X direction, while the KUKA LWR IV simulates a virtual environment in the Z direction of the last joint.

### 4.1 Effect of Time Delay

In this section, the method performance for finding stability domain by different time delays is studied. Simulation cases are defined in distinct operating points. So, the HD

**Table 1** Effective parameters of the PHANTom 1.5 in  $q_{case\ 1}$ 

$m_{eff}(Kg)$	$b_{eff}(Ns/m)$	$c_{eff}(N)$	$\Delta_{eff}(\mu m)$
0.200	0.033	0.003	19.433

configuration ( $q$ ) and stylus penetration direction ( $e_s$ ), which are critical in computing effective parameters of (11) and (12), are different.

#### 4.1.1 PHANTom 1.5 Haptic Device

The first simulation case is carried out in  $q_{case\ 1} = [\pi/4, \pi/6, -\pi/6]$ . In this configuration, effective parameters of mass, viscous friction, Coulomb friction, and quantization are determined, as reported in Table 1.

These parameters are used in simulations for predicting the stability boundary of 1-DOF equivalent system (Table 1). Figure 2 shows the stability boundary for  $T_d = 0ms$  and  $T_d = 1ms$ . In this figure, the maximum virtual stiffness correspond to each virtual damping for stable operation is specified. The red and green lines are realized by simulating the main robot model, while blue and cyan lines correspond to the equivalent 1-DOF model, respectively. In this figure, accuracy for even a large virtual stiffness value is acceptable.

The same result can be seen for the second case in Table 2 and Fig. 3. This simulation set is done in  $q_{case\ 2} = [\pi/3, \pi/5, -\pi/6]$ .

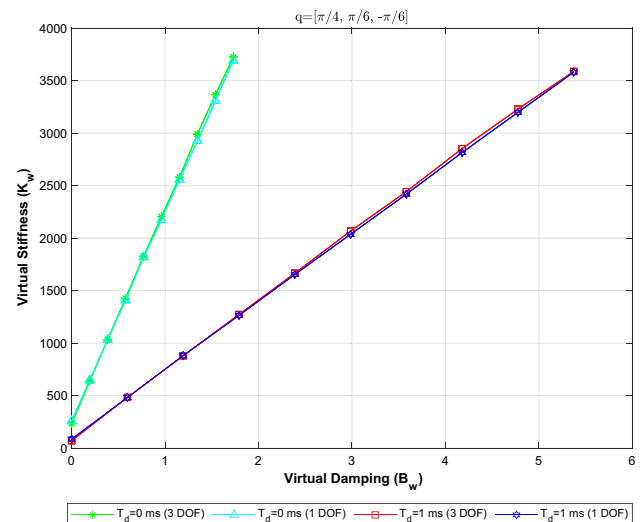
By increasing time delay, the maximum allowable virtual stiffness decreases in case one (see Fig. 4). Meanwhile, the simplified model tracks the main model behaviour acceptably and is conservative for large virtual damping.

Time delays of  $T_d = 2ms$  and  $T_d = 3ms$  are also studied for the second case. Figure 5 shows the proposed simplifying method also preserves its performance in different time delays in this configuration. Usually, the HD is utilized in the case of small virtual damping values and time delay [18, 25, 28]. From these figures, it is clear that simplifying a 3-DOF nonlinear haptic device dynamics model to a 1-DOF model by the presented equations has good accuracy in the usual HD operation domain.

It is noteworthy that a time delay of more than  $3ms$  is not common. The acquired results for both cases are precise, especially in the linear part of the diagrams, which has the most practical usage.

**Table 2** Effective parameters of the PHANTom 1.5 in  $q_{case\ 2}$ 

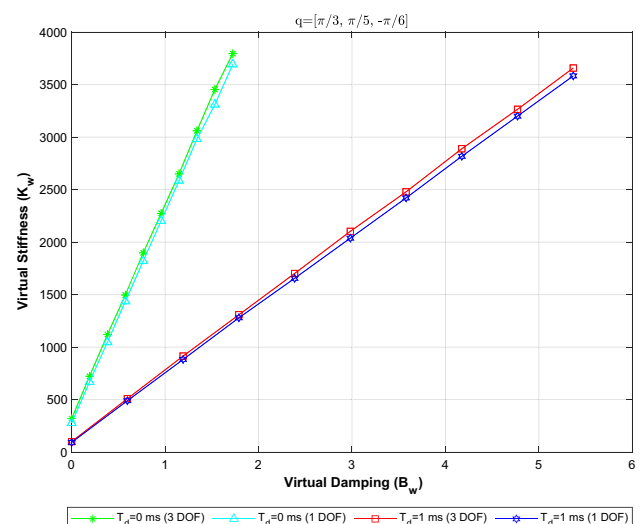
$m_{eff}(Kg)$	$b_{eff}(Ns/m)$	$c_{eff}(N)$	$\Delta_{eff}(\mu m)$
0.201	0.041	0.004	13.817

**Fig. 2** Boundary of stability with  $T_d = 0$  &  $1$  ms in case I

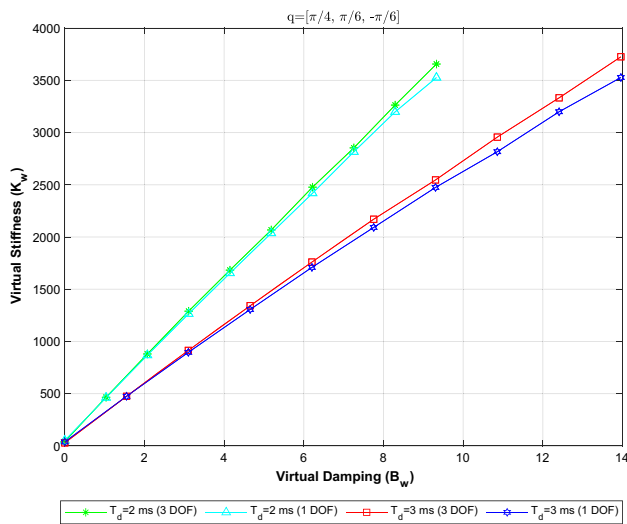
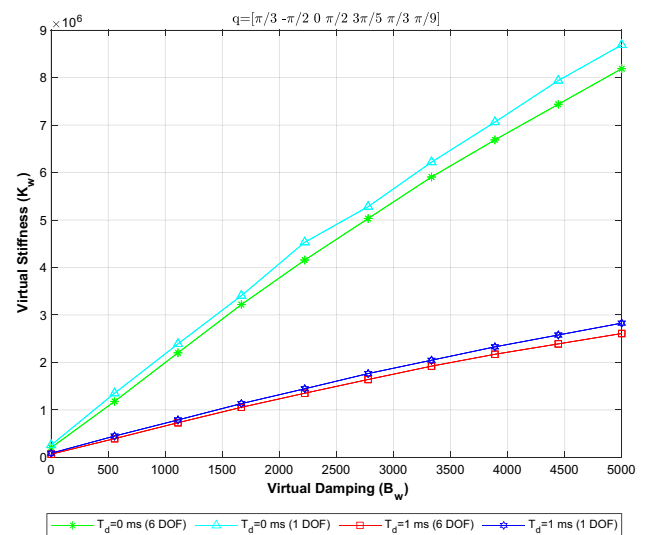
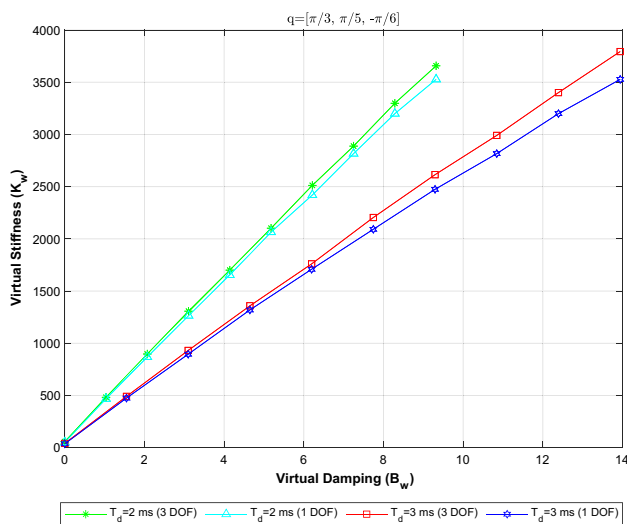
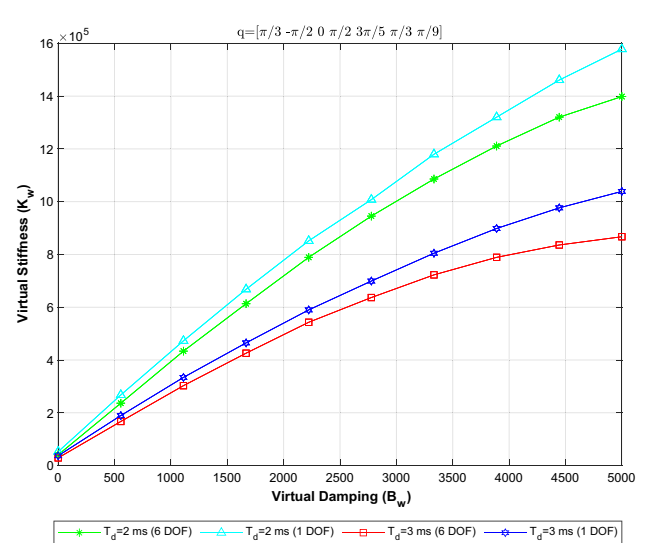
#### 4.1.2 KUKA Light Weight Robot IV

Herein, two different configurations are considered for KUKA LWR IV as HD to evaluate the performance of the method in a more complex manipulator. Since this robot has seven degrees of freedom, the third joint of this robot assumed to be locked with the angle of zero in all configurations; thus other six joints are changed to put the robot in different configurations.

In the third set of simulations, the configuration of the KUKA LWR IV is  $q_{case\ 3} = [\pi/3, -\pi/2, 0, \pi/2, 3\pi/5, \pi/3, \pi/9]$  and effective parameters are determined from (11) and (12) and listed in Table 3.

**Fig. 3** Boundary of stability with  $T_d = 0$  &  $1$  ms in case II



Fig. 4 Boundary of stability with  $T_d = 2$  &  $3$  ms in case IFig. 6 Boundary of stability with  $T_d = 0$  &  $1$  ms in case IIIFig. 5 Boundary of stability with  $T_d = 2$  &  $3$  ms in case IIFig. 7 Boundary of stability with  $T_d = 2$  &  $3$  ms in case III**Table 3** Effective parameters of the KUKA LWR IV in  $q_{case\ 3}$ 

$m_{eff}(Kg)$	$b_{eff}(Ns/m)$	$c_{eff}(N)$	$\Delta_{eff}(\mu m)$
32.90	15.39	4.57	45.48

**Table 4** Effective parameters of the KUKA LWR IV in  $q_{case\ 4}$ 

$m_{eff}(Kg)$	$b_{eff}(Ns/m)$	$c_{eff}(N)$	$\Delta_{eff}(\mu m)$
157.59	85.85	10.76	18.95

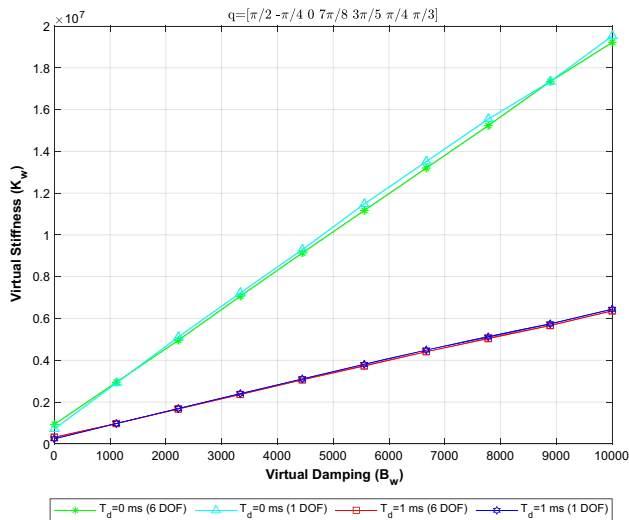


Fig. 8 Boundary of stability with  $T_d = 0$  &  $1$  ms in case IV

Figures 6 and 7 show the accurate behaviour of the simplified model for time delay from zero to  $3$  ms in this configuration.

Another set of simulation is done in  $q_{case\ 4} = [\pi/2, -\pi/4, 0, 7\pi/8, 3\pi/5, \pi/4, \pi/3]$ , and effective parameters are listed in Table 4.

The stability boundaries in this configuration for various time delays are reported in Figs. 8 and 9. Figure 8 confirms the precise prediction of the robot stability region by the 1-DOF model for  $T_d = 0$  and  $1$  ms, while the same result is shown in Fig. 9 for  $T_d = 2$  and  $3$  ms.

Hence, the proposed model properly specifies the stability region for both 3-DOF and 6-DOF HD, when various time delays exist.

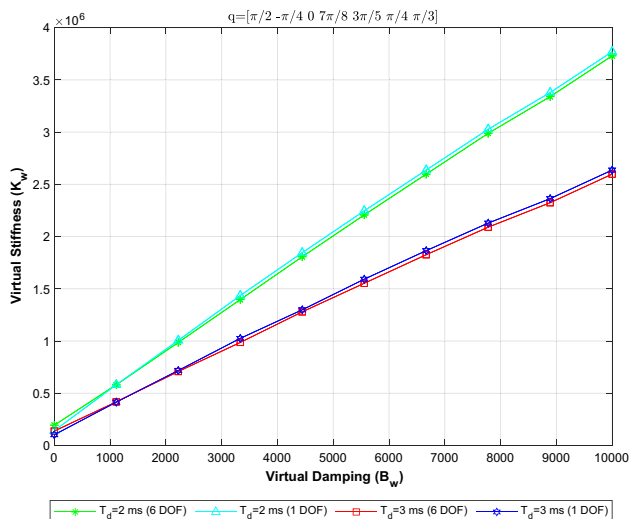


Fig. 9 Boundary of stability with  $T_d = 2$  &  $3$  ms in case IV

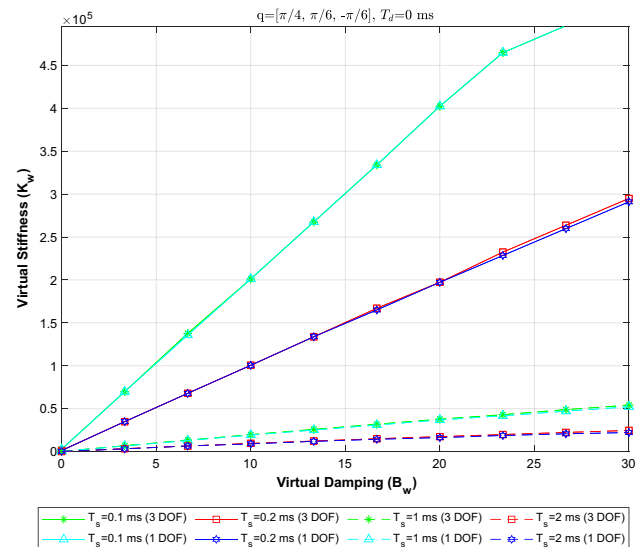


Fig. 10 Boundary of stability with  $T_s = 0.1$ ms,  $0.2$ ms,  $1$ ms, and  $2$ ms in case I

## 4.2 Effect of Sampling Time

The other critical parameter discussed in this work is sampling time. Simulations are carried out with different sampling times in various configurations. For all the simulations,  $T_d$  is  $0$ ms.

### 4.2.1 PHANTOM 1.5 Haptic Device

In Fig. 10, maximum virtual stiffness corresponds to virtual damping for stable HD in  $q_{case\ 1}$  for different sampling rates are reported. As shown in Fig. 10, by decreasing

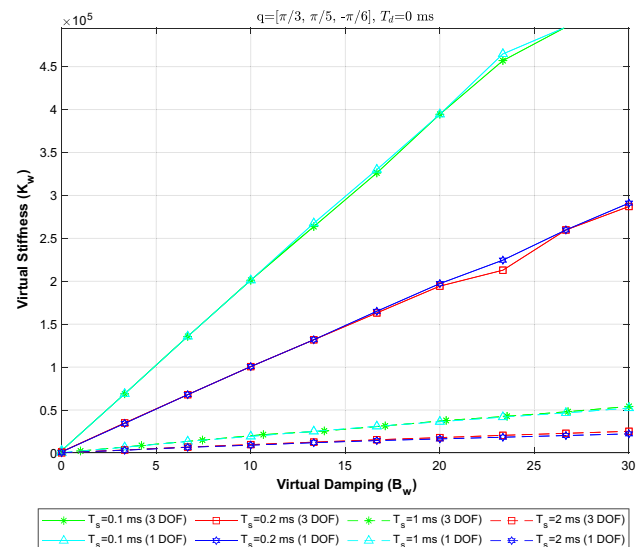
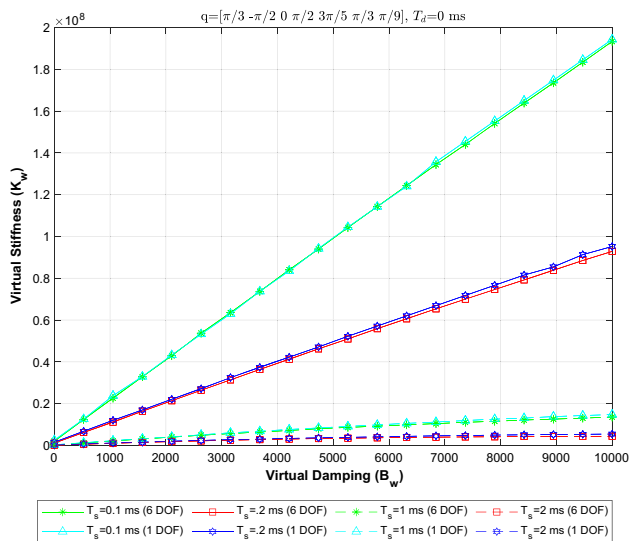


Fig. 11 Boundary of stability with  $T_s = 0.1$ ms,  $0.2$ ms,  $1$ ms, and  $2$ ms in case II

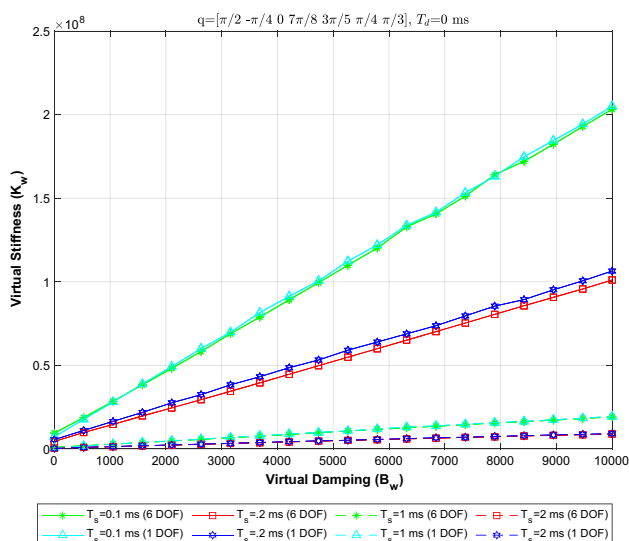


**Fig. 12** Boundary of stability with  $T_d = 0$  and  $T_s = 0.1, 0.2, 1 \& 2 \text{ ms}$  in case III

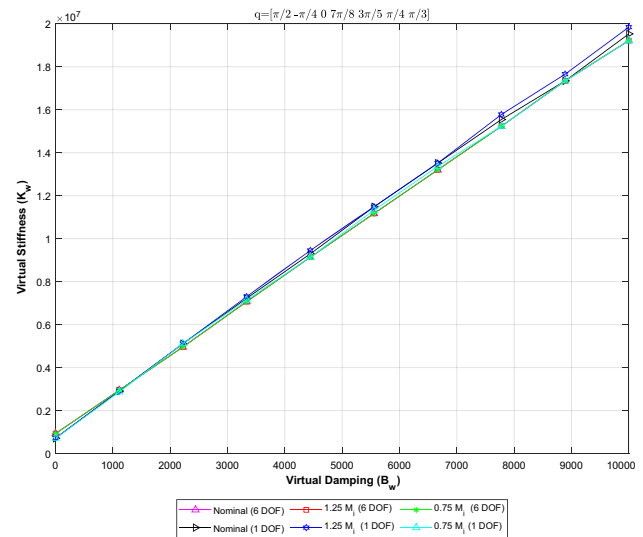
sampling time, HD has a larger stable domain and can simulate stiffer VE. Another set of simulations is reported in Fig. 11. The obtained results for case II, similar to the case I, shows acceptable accuracy of the simplification algorithm for stability analysis of the multi-DOF HD in different sampling rate.

#### 4.2.2 KUKA Light Weight Robot IV

The stability region of KUKA as HD is studied for different sampling rates in cases III and IV. The acquired results are shown for these two cases in Figs. 12 and 13, respectively.



**Fig. 13** Boundary of stability with  $T_d = 0$  and  $T_s = 0.1, 0.2, 1 \& 2 \text{ ms}$  in case IV

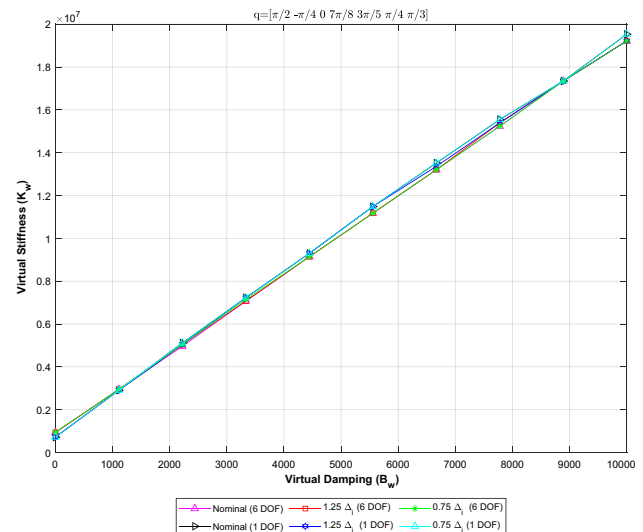


**Fig. 14** Effect of mass matrix variation on the stability boundary

The 1-DOF model specified the stability region accurately for  $T_s = 0.1, 0.2, 1 \& 2 \text{ ms}$  in both configurations. Therefore, simplified model performance seems to be independent of sampling time and it accurately specifies the stable domain.

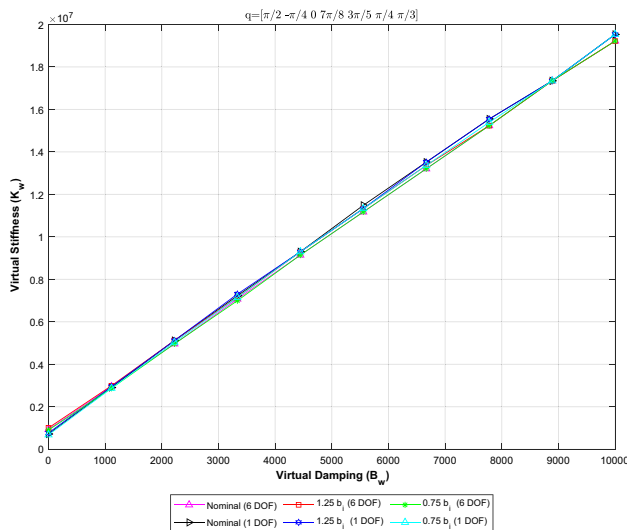
#### 4.3 Effect of Dynamic Parameter Ratio

In order to have a more general study, effect of variation of  $M, b_i, \mu_i, \Delta q_0$ , which are four dynamic parameters of the system, on the stability boundaries are studied. For the sake of brevity, results are reported just for one KUKA configuration. To this end, these four parameters are increased/decreased 25%, and the realized stable VE domain is reported. As shown in Fig. 14, by varying mass matrix parameter, the



**Fig. 15** Effect of sensor resolution variation on the stability boundary





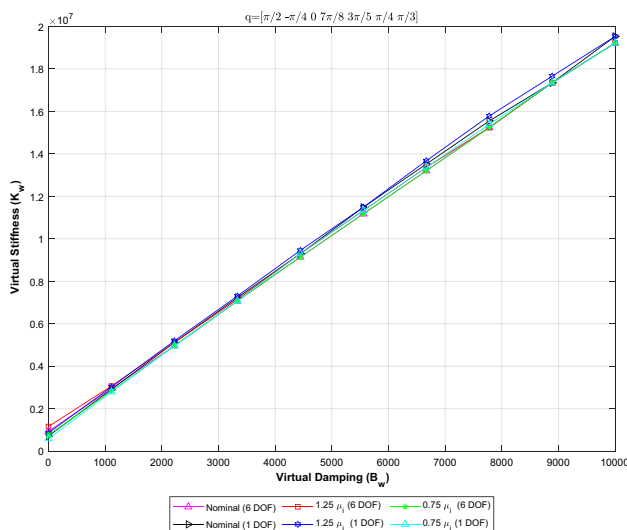
**Fig. 16** Effect of viscous friction coefficient variation on the stability boundary

accuracy of the equivalent system is similar to the nominal system.

Moreover, sensor resolution, joint damping and Coulomb coefficients are changed 25% with respect to the nominal system. The obtained stability domains are shown in Figs. 15, 16, and 17 respectively. The equivalent system in all cases behaves almost similar to the main multi-DOF system which admits the proposed method performance.

#### 4.4 Discussion

The reported results in this brief confirmed that the suggested approach can be employed as an equivalent model



**Fig. 17** Effect of Coulomb friction variation on the stability boundary

instead of multi-DOF HD. The method accurately computes the stability boundary in various configurations for different robots. Even by increasing sampling time and time delay, the method performance is acceptable. Moreover, variation of other dynamic parameters of the system does not affect the method performance. Consequently, there is no considerable limitation in employing the method, and its compatible with HD usage.

## 5 Conclusion

Considering multi-DOF serial manipulator is a critical issue for broadly employing haptic systems. In this work, a new method is presented for simplifying the nonlinear multi-DOF haptic device dynamic model. By employing the presented method, an equivalent 1-DOF model is obtained, consisting constant parameters of an effective mass, effective viscous damping coefficient, and effective Coulomb friction, which its motion is sensed with an effective resolution which accurately specifies the stability region of the main system. Sensor quantization, viscous, and Coulomb friction are considered in stability analysis. The method performance is evaluated for different configurations, as well as different haptic devices. The realized results properly coincide with the nonlinear multi-DOF model for a wide range of time delay and sampling time. Furthermore, the method performance for different dynamic parameters are studied. Consequently, complex dexterous manipulators can be utilized as haptic devices, and stability regions can be studied fast and with a low computation burden in any arbitrary situations, which is critical for HD broad usage.

**Author Contributions** Ahmad Mashayekhi: Substantial contributions to the design of the work and revising it critically for important intellectual content. Abbas Karami: Substantial contributions to the conception of the work and drafting the work. Bruno Siciliano: Final approval of the version to be published.

**Funding** No funding was received for conducting this study.

**Code availability** The codes employed for the findings of this study are available on request from the corresponding author.

## Declarations

**Ethics approval and consent to participate** The submitted work is original and is not published elsewhere in any form or language.

**Consent for publication** Informed consent was obtained from all individual participants included in the study.

**Conflict of interest** We certify that there is no actual or potential conflict of interest in relation to this article.

## References

- Mayetin, U., Kucuk, S.: Design and experimental evaluation of a low cost, portable, 3-dof wrist rehabilitation robot with high physical human-robot interaction. *J. Intell. Robot. Syst.* **106**(3), 65 (2022)
- Roldan, J.R.U., Milutinović, D.: Suture looping task pose planner in a constrained surgical environment. *J. Intell. Robot. Syst.* **106**(4), 78 (2022)
- Liu, Z., Wang, S., Feng, F., Xie, L.: A magnetorheological fluid based force feedback master robot for vascular interventional surgery. *J. Intell. Robot. Syst.* **106**(1), 20 (2022)
- Valenzuela-Urrutia, D., Muñoz-Riffo, R., Ruiz-del-Solar, J.: Virtual reality-based time-delayed haptic teleoperation using point cloud data. *J. Intell. Robot. Syst.* **96**, 387–400 (2019)
- Saini, S., Orlando, M.F., Pathak, P.M.: Intelligent control of a master-slave based robotic surgical system. *J. Intell. Robot. Syst.* **105**(4), 94 (2022)
- Chicaiza, F.A., Slawiński, E., Salinas, L.R., Mut, V.A.: Evaluation of path planning with force feedback for bilateral teleoperation of unmanned rotorcraft systems. *J. Intell. Robot. Syst.* **105**(2), 34 (2022)
- Naceri, A., Mazzanti, D., Bimbo, J., Tefera, Y.T., Prattichizzo, D., Caldwell, D.G., Mattos, L.S., Deshpande, N.: The vicarios virtual reality interface for remote robotic teleoperation: teleporting for intuitive tele-manipulation. *J. Intell. Robot. Syst.* **101**, 1–16 (2021)
- You, B., Li, J., Ding, L., Xu, J., Li, W., Li, K., Gao, H.: Semi-autonomous bilateral teleoperation of hexapod robot based on haptic force feedback. *J. Intell. Robot. Syst.* **91**, 583–602 (2018)
- Nahri, S.N.F., Du, S., Van Wyk, B.J.: A review on haptic bilateral teleoperation systems. *J. Intell. Robot. Syst.* **104**, 1–23 (2022)
- Hogan, N.: Controlling impedance at the man/machine interface. In: *IEEE International Conference on Robotics and Automation*, pp. 1626–1631. IEEE (1989)
- Gillespie, R.B., Cutkosky, M.R.: Stable user-specific haptic rendering of the virtual wall. In: *Proceedings of the ASME International Mechanical Engineering Congress and Exhibition*, vol. 58, pp. 397–406 (1996)
- Dong, R.G., Dong, J.H., Wu, J.Z., Rakheja, S.: Modeling of biodynamic responses distributed at the fingers and the palm of the human hand-arm system. *J. Biomech.* **40**(10), 2335–2340 (2007)
- Yoshikawa, T., Ichino, Y.: Impedance identification of human fingers using virtual task environment. In: *Proceedings of the 2003 IEEE/RSJ International Conference on Intelligent Robots and Systems*, vol. 4, pp. 3094–3099. IEEE (2003)
- Mashayekhi, A., Behbahani, S., Ficuciello, F., Siciliano, B.: Influence of human operator on stability of haptic rendering: a closed-form equation. *Int. J. Intell. Robot. Appl.* **4**, 403–415 (2020)
- Mashayekhi, A., Nahvi, A., Meghdari, A., Shad, H.M.: A new haptic interaction with a visual tracker: implementation and stability analysis. *Int. J. Intell. Robot. Appl.* **5**, 37–48 (2021)
- Zeng, C., Li, Y., Guo, J., Huang, Z., Wang, N., Yang, C.: A unified parametric representation for robotic compliant skills with adaptation of impedance and force. *IEEE/ASME Transactions on Mechatronics* **27**(2), 623–633 (2021)
- Minsky, M., Ming, O.-y., Steele, O., Brooks Jr, F.P., Behensky, M.: Feeling and seeing: issues in force display. In: *Proceedings of the 1990 Symposium on Interactive 3D Graphics*, pp. 235–241 (1990)
- Gil, J.J., Avello, A., Rubio, A., Florez, J.: Stability analysis of a 1 dof haptic interface using the Routh-Hurwitz criterion. *IEEE Trans. Control Syst. Technol.* **12**(4), 583–588 (2004)
- Diolaiti, N., Niemeyer, G., Barbagli, F., Salisbury, J.K.: Stability of haptic rendering: discretization, quantization, time delay, and coulomb effects. *IEEE Trans. Robot.* **22**(2), 256–268 (2006)
- Mashayekhi, A., Boozarjomehry, R.B., Nahvi, A., Meghdari, A., Asgari, P.: Improved passivity criterion in haptic rendering: influence of coulomb and viscous friction. *Adv. Robot.* **28**(10), 695–706 (2014)
- Colgate, J.E., Schenkel, G.: Passivity of a class of sampled-data systems: application to haptic interfaces. In: *Proceedings of the 1994 American Control Conference*, vol. 3, pp. 3236–3240. IEEE (1994)
- Abbott, J.J., Okamura, A.M.: Effects of position quantization and sampling rate on virtual-wall passivity. *IEEE Trans. Robot.* **21**(5), 952–964 (2005)
- Gil, J.J., Sánchez, E., Hulin, T., Preusche, C., Hirzinger, G.: Stability boundary for haptic rendering: influence of damping and delay. *J. Comput. Inf. Sci. Eng.* **9**(1), 011005 (2009)
- Mashayekhi, A., Behbahani, S., Ficuciello, F., Siciliano, B.: Analytical stability criterion in haptic rendering: the role of damping. *IEEE/ASME Trans. Mechatron.* **23**(2), 596–603 (2018)
- Mashayekhi, A., Behbahani, S., Ficuciello, F., Siciliano, B.: Delay-dependent stability analysis in haptic rendering. *J. Intell. Robot. Syst.* **97**, 33–45 (2019)
- Mashayekhi, A., Behbahani, S., Nahvi, A., Keshmiri, M., Shakeri, M.: Analytical describing function of lugre friction model. *Int. J. Intell. Robot. Appl.* **6**, 437–448 (2022)
- Koul, M., Manivannan, M., Saha, S.K.: Effect of dual-rate sampling on the stability of a haptic interface. *J. Intell. Robot. Syst.* **91**, 479–491 (2018)
- Hulin, T., Albu-Schaffer, A., Hirzinger, G.: Passivity and stability boundaries for haptic systems with time delay. *IEEE Trans. Control Syst. Technol.* **22**(4), 1297–1309 (2014)
- Li, J., Tavakoli, M., Huang, Q.: Absolute stability of multi-dof multilateral haptic systems. *IEEE Trans. Control Syst. Technol.* **22**(6), 2319–2328 (2014)
- Li, J., Tavakoli, M., Huang, Q.: 3-dof trilateral teleoperation using a pair of 1-dof and 2-dof haptic devices: stability analysis. *IFAC Proceedings Volumes* **46**(20), 443–448 (2013)
- Li, J., Tavakoli, M., Huang, Q.: Stability of cooperative teleoperation using haptic devices with complementary degrees of freedom. *IET Control Theory Appl.* **8**(12), 1062–1070 (2014)
- Bianchini, G., Prattichizzo, D.: Virtual coupling design for stability and transparency of multi-device haptic systems with delays. In: *2013 World Haptics Conference (WHC)*, pp. 223–228. IEEE (2013)
- Fahmi, S., Hulin, T.: Inertial properties in haptic devices: non-linear inertia shaping vs force feedforward. *Force Feedforward. IFAC-PapersOnLine* **51**(22), 79–84 (2018)
- Diaz, I., Gil, J.J.: Influence of internal vibration modes on the stability of haptic rendering. In: *IEEE International Conference on Robotics and Automation*, pp. 2884–2889. IEEE (2008)
- Karami, A., Mashayekhi, A.: Improving haptic device stability through redundancy resolution. In: *2022 10th RSI International Conference on Robotics and Mechatronics (ICRoM)*, pp. 527–532. IEEE (2022)
- Panariello, D., Caporaso, T., Grazioso, S., Di Gironimo, G., Lanzotti, A., Knopp, S., Pelliccia, L., Lorenz, M., Klimant, P.: Using the kuka lbr iiwa robot as haptic device for virtual reality training of hip replacement surgery. In: *2019 Third IEEE International Conference on Robotic Computing (IRC)*, pp. 449–450 (2019). <https://doi.org/10.1109/IRC.2019.00094>
- Ogata, K.: *Modern control engineering*, 2nd edn. Prentice Hall PTR (1990)
- Çavuşoğlu, M.C., Feygin, D., Tendick, F.: A critical study of the mechanical and electrical properties of the phantom haptic interface and improvements for high-performance control. *Presence* **11**(6), 555–568 (2002). <https://doi.org/10.1162/105474602321050695>

39. Gaz, C., Flacco, F., De Luca, A.: Identifying the dynamic model used by the kuka lwr: a reverse engineering approach. In: 2014 IEEE International Conference on Robotics and Automation (ICRA), pp. 1386–1392 (2014). <https://doi.org/10.1109/ICRA.2014.6907033>

**Publisher's Note** Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Springer Nature or its licensor (e.g. a society or other partner) holds exclusive rights to this article under a publishing agreement with the author(s) or other rightsholder(s); author self-archiving of the accepted manuscript version of this article is solely governed by the terms of such publishing agreement and applicable law.

**Ahmad Mashayekhi** received the B.Sc. degree in Mechanical Engineering from the K. N. Toosi University of Technology, Tehran, Iran, in 2008 and the M.Sc. degree in Mechatronics Engineering from Sharif University of Technology, Tehran, Iran, in 2011 and Ph.D. degree in Mechanical Engineering from Isfahan University of Technology, Isfahan, Iran. Currently, he is with department of Mechanical Engineering of Sirjan University of Technology. He was a visiting scholar with the PRISMA Lab, University of Naples Federico II, Naples, Italy, from 2016 to 2017. His current research interests include mechatronics, robotics, human-robot interaction, surgical robotics, and haptics. Dr. Mashayekhi was the recipient of a bronze medal in Physics Olympiad in 2002.

**Abbas Karami** received the Ph.D. degree in mechanical engineering from the Isfahan University of Technology in 2018. From March 2016 to March 2017, he was a visiting scholar with the DIAG Robotics Lab, Sapienza University of Rome, Italy. Starting from 2020, he is an assistant professor of Control and Robotics at Shiraz University of Technology. His research interest includes human-robot interaction, surgical robotics, non-linear control, and modelling of biomechanical systems.

**Bruno Siciliano** is professor of robotics and control at the University of Naples Federico II. He is also Honorary Professor at the University of Óbuda where he holds the Kálmán Chair. His research interests in robotics include manipulation and control, human–robot cooperation, and service robotics. Fellow of the scientific societies IEEE, ASME, IFAC, he received numerous international prizes and awards, including the 2022 Engelberger Award for Education. He was President of the IEEE Robotics and Automation Society from 2008 to 2009. He has delivered more than 150 keynotes and has published more than 300 papers and 7 books. His book “Robotics” is among the most adopted academic texts worldwide, while his edited volume “Springer Handbook of Robotics” received the highest recognition for scientific publishing: the 2008 PROSE Award for Excellence in Physical Sciences & Mathematics. His team has received more than 18 million Euro funding in the last 15 years from competitive European research projects. More details are available at <http://wpage.unina.it/sicilian/>.