Direct Adaptive Control of a One-Link Flexible Arm with Tracking

Bau-San Yuan and Wayne J. Book
George W. Woodruff School of Mechanical Engineering
Georgia Institute of Technology
Atlanta, GA 30332

Bruno Siciliano
Dipartimento di Informatica e Sistemistica
Universitá degli Studî di Napoli "Federico II"
Via Claudio 21; 80125 Napoli, Italy

Received August 31, 1987; accepted July 10, 1989

A robust tracking controller for a one-link flexible arm based on a model reference adaptive control approach is proposed. In order to satisfy the model matching conditions, the reference model is chosen to be the optimally controlled linearized model of the system. The resulting controller overcomes the fundamental limitation in previously published research on direct adaptive control of flexible robots that required additional actuators solely to control the flexible degrees of freedom. The nominal trajectory is commanded by means of a tracking control. Simulation results for the prototype in the laboratory show improvements obtained with the outer adaptive feedback loop compared to a pure optimal regulator control. Robustness is tested by varying the payload mass.

本論文では、1リンクフレキシブルアームのためのモデル参照型適応制御に基づくロバストな追跡コントローラが提案される。モデルとの適合条件を満足させるために、最適に制御される線形化モデルとなるように参照モデルが選択される。その結果選択されたコントローラは、フレキシブルな自由度を制御するためには付加的なアクチュエータを必要とする過去に発表されたフレキシブルロボットの直接適応制御が持つ基本的な限界に打ち勝つことができる。ノミナルな軌道が軌道制御によって命令される。研究室の試作機のシミュレーション結果は純粋な最適レギュレータ制御に比べ、外部適応フィードバックループによる改善を示した。また付加質量を変化させることによりロバスト性がテストされた。

INTRODUCTION

Lightweight arms are a challenging research topic with potential to improve today's robot performance. Control is one key to effective use of lighter arms,[1,2]

Journal of Robotic Systems 6(6), 663–680 (1989)
 CCC 0741-2223/89/060663-18$4.00

but it is limited by uncertainties in the arm's behavior and in the environment. The main problem with lightweight structures is the flexible vibrations that are naturally excited as the arm is commanded to move.[3]

The first step in designing a control system consists of developing a dynamic model for the flexible arm. A general dynamic modeling technique was established by Book,[4] based on a recursive Lagrangian-assumed modes method. If one is interested in the regulator control problem requiring that the arm reach a prespecified nominal state with satisfactory response, the approach of linearizing the dynamic equations by assuming small motions around the nominal state and neglecting terms of higher order, proves effective. An optimal control for a one-link flexible arm was experimentally tested by Hastings and Book.[5] Also, experimental results with linear models were reported by Cannon and Schmitz,[6] by Fukuda,[7] by Sakawa et al.,[8] and by Chalhoub and Ulsoy.[9] Frequency domain techniques, instead, were adopted by Book and Majette[10] and recently revisited by Ower and Van de Vegte.[11]

On the other hand, if one is concerned with controlling the arm while it is moving along a predefined path with given velocity and acceleration of the joint variables, the technique of linearizing the system is likely to fail. Furthermore, linearization around a sequence of nominal states, as done by Sunada and Dubowski[12] for instance, seem expensive computationally and not necessarily very robust when applied to the overall nonlinear dynamics.

This article describes research on control for a one-link flexible arm moving along predefined trajectories. The resulting controller overcomes the fundamental limitation in previously published research on direct adaptive control of flexible robots that required additional actuators solely to control the flexible degrees of freedom. Previous efforts aimed at designing tracking controllers for flexible arms have been produced by Singh and Schy[13] with a nonlinear inversion control, and by Davis and Hirschorn[14] with a linear control. They have both taken advantage, however, of additional active tip actuators. A nonlinear joint tracking controller has been devised by DeLuca and Siciliano.[15] A singular perturbation approach has been pursued, instead, by Siciliano and Book.[16]

The approach adopted here is based on model reference adaptive control (MRAC),[17] as recently proposed by Siciliano et al.[18] In order to assure the satisfaction of the so-called "model matching conditions", the reference model is chosen as the linearized system (second-order terms neglected) as optimally controlled. Integral type adaptive actions guarantee the stability of the overall system, as is proved via the Lyapunov direct method. However, since the reference model turns out not to be decoupled, the reference trajectory is forced on the system by means of a tracking controller.[19] A direct adaptive controller for a linear model of a flexible arm was also designed by Meldrum and Balas,[20] but stability was guaranteed only for a special class of trajectories. An indirect adaptive control conversely, with dynamic parameter identification was proposed by Canudas, De Wit, and Van den Bossche.[21]

A case study based on a laboratory prototype, whose dynamic model is described in Hastings and Book[22] shows that the control performs well when tracking a fast trajectory. The whole nonlinear system is considered for simulation purposes. Moreover, the control proves robust to parameter variations such as payload changes.

It must be mentioned that full state availability is assumed for control synthesis. While the state variables representing deflection can be obtained from strain gauge measurements,[5] their derivatives need to be reconstructed by means of an observer.[23]

PROBLEM FORMULATION

Nonlinear equations of motion for a flexible arm can be derived using the Lagrangian approach.[4] The deflection of the elastic members is represented as a linear combination of admissible functions multiplied by time dependent generalized coordinates.[24] The flexible motion of a link is then described by

$$u(\eta, t) = \sum_{i=1}^{n} \phi_i(\eta)\delta_i(t) \tag{1}$$

where the $\phi_i(\eta)$ are assumed in this article to be eigenfunctions of a clamped-free beam, $\delta_i(t)$ are the generalized coordinates, and η is any point along the undeformed link (Fig. 1). Furthermore, assuming that the amplitudes of the higher modes of the flexible link are very small compared to the lower modes, $n = 2$ will be accurate enough to describe the flexible motion.[22,25]

The derivation of the dynamic equations for the one-link arm follows then as in Book[4] and Siciliano and Book[16] (i.e., dropping the explicit reference to time dependence).

$$M(\theta, \delta)\begin{bmatrix}\ddot{\theta}\\ \ddot{\delta}\end{bmatrix} + \begin{bmatrix}f_1\\ f_2\end{bmatrix} + \begin{bmatrix}0\\ K\delta\end{bmatrix} = \begin{bmatrix}u\\ 0\end{bmatrix} \tag{2}$$

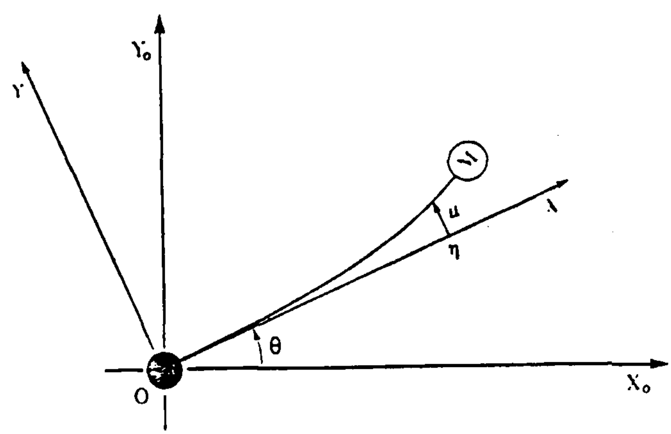

Figure 1. The one-link flexible arm.

where θ is the joint angle, M is the inertia matrix, f_1 and f_2 are vectors containing nonlinear dynamic terms (interactions of angular rates and deflections), K is the effective spring matrix, u is the net input torque.

Notice that in the model no actuator dynamics is considered, and no friction at the joints nor in the structural vibrations is explicitly included. Define the full state vector

$$X^T = [x^{pT}, x^{vT}] \quad \text{and} \quad x^{vT} = [\dot{\theta}, \dot{\delta}^T] = \dot{x}^{pT} \tag{3}$$

The dynamic model of the flexible arm of Figure 1 can be expressed in state variable form as

$$\frac{d}{dt}\begin{bmatrix} x^p \\ x^v \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ A_1(x^p) & A_2(x^p, x^v) \end{bmatrix}\begin{bmatrix} x^p \\ x^i \end{bmatrix} + \begin{bmatrix} 0 \\ B_2(x^p) \end{bmatrix} u \tag{4}$$

$$\dot{X} = A(X)X + b(X)u \tag{5}$$

where

$$A_1(x^p)x^p = M^{-1}\begin{bmatrix} 0 \\ K\delta \end{bmatrix}$$

$$A_2(x^p, x^v)x^v = M^{-1}\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$B_2(x^p) = M^{-1}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

At this point it becomes clear why the tracking control problem is difficult. If the goal is just to require that the arm reaches a prespecified nominal state, linearizing (5) around the nominal state leads naturally to an optimal regulator in which one can eventually specify the closed loop poles of the linearized system with an arbitrary degree of stability. However, if one desires to control the arm while it moves along a predefined trajectory, in terms of joint angle rates and accelerations, a different approach must be sought, rather than trying to linearize (5) around a sequence of nominal states.

In order to obtain good trajectory tracking and steady-state accuracy, a direct MRAC approach[17] is pursued in the following. The basic idea of this approach is to define a linear time-invariant reference model and directly synthesize a controller that assures that the error between the states of the system and those of the model tends to zero. To this purpose let

$$\dot{X}_m = A_m X_m + b_m u_m \tag{6a}$$

$$A_m = \begin{bmatrix} 0 & I \\ A_{10} & A_{20} \end{bmatrix} \qquad b_m = \begin{bmatrix} 0 \\ b_0 \end{bmatrix} \tag{6b}$$

be a linear time-invariant reference model of the same dimension as the system described by Eq. (5).

As in the work on MRAC for rigid manipulators,[26,27] it would seem appropriate to select a decoupled model for (6), i.e., $A_{10} = \mathrm{diag}(a_{11}a_{12}a_{13})$, $a_{1i} < 0$, $A_{20} = \mathrm{diag}(a_{21}a_{22}a_{23})$, $a_{2i} < 0$. However the model matching conditions that are the basis of an MRAC approach[28] cannot be satisfied independent from the particular values of A, A_m, b, b_m. This can be confirmed by observing that the system described in (5) does not have as many control inputs as nontrivial state variables (θ, δ_1, δ_2), i.e., the lower block of vector b_0 in (6b) is not a square block (a row vector in this case).

In the particular case of the system in (5), however, the nonlinear terms do not play a dominant role, thus it appears adequate to choose a reference model on the basis of the linearized model of the system (second-order terms neglected) as optimally controlled; this approach will be outlined in the next section.

CONTROL LAW DEVELOPMENT

Following the basic MRAC scheme in Landau[17] a control for the overall system (5)–(6) is proposed in the form

$$u = u_1 + u_2 \tag{7a}$$

$$u_1 = -K_x^T X + K_u u_m \qquad u_2 = -\Delta K_x^T X + \Delta K_u u_m \tag{7b}$$

where u_1 is a linear model following control and u_2 represents the adaptive control that is devoted to assuring the stability of the whole system. Under the action of control (7), the system (5) becomes

$$\dot{X} = A_s(X)X + b_s(X)u_m \tag{8a}$$

$$A_s = A - b(K_x^T + \Delta K_x^T), \qquad b_s = b(K_u + \Delta K_u). \tag{8b}$$

Let then

$$e = X_m - X \tag{9}$$

be the error between the model and system states. On reduction of (6) and (8), the error dynamics are found to be

$$\dot{e} = A_m e + (A_m - A_s)X + (b_m - b_s)u_m \tag{10}$$

In order to satisfy the model matching conditions, the following should hold:[28]

$$A_m = \tilde{A} - \tilde{b}K_x^T \qquad b_m = \tilde{b}K_u \tag{11}$$

where $\tilde{A}$ and $\tilde{b}$ are the linearized forms of A and b, respectively. Assuming that the pair $(\tilde{A}, \tilde{b})$ is stabilizable, K_x^T can be designed by means of optimal control techniques for the linearized system in (A, b). K_u is chosen to equal 1 for simplicity. Substituting (8b) and (11) into (10) gives

$$\dot{e} = A_m e + [\Delta A - \Delta b K_u^T + b\Delta K_x^T]x + [\Delta b K_u - b\Delta K_u]u_m \tag{12}$$

where

$$\tilde{A} - A = \Delta A \tag{13a}$$

and

$$\tilde{b} - b = \Delta b \tag{13b}$$

express the difference between the actual system and its linearized parts. In order to guarantee the stability of the overall system, a candidate Lyapunov function is

$$V = e^T P e + \mathrm{tr}[(A_m - A_s)^T F_a^{-1}(A_m - A_s)] + \mathrm{tr}[(b_m - b_s)^T F_b^{-1}(b_m - b_s)] \tag{14}$$

where P, F_a, F_b are positive definite matrices. The derivative of V including (12) yields:

$$\begin{aligned}\dot{V} = e^T(A_m^T P + PA_m)e &+ 2\mathrm{tr}[(\Delta A - \Delta b K_u^T + b\Delta K_x^T)^T(PeX^T - F_a^{-1}\dot{A}_s)] \\ &+ 2\mathrm{tr}[(\Delta b K_u - b\Delta K_u)^T(Peu_m - F_b^{-1}\dot{b}_s)]\end{aligned} \tag{15}$$

Setting, as is usual,

$$A_m^T P + PA_m = -H \tag{16}$$

where H is a positive definite matrix, and assuming that the rate of the adjustable gains is larger than that of the system, $\Delta\dot{K}_x$, $\Delta\dot{K}_u \gg \dot{A}$, $\dot{b}$, leads to

$$\begin{aligned}\dot{V} = -e^T He &+ 2\mathrm{tr}[(\Delta A - \Delta b K_u^T + b\Delta K_x^T)^T(PeX^T + F_a^{-1} b\Delta\dot{K}_x^T)] \\ &+ 2\mathrm{tr}[(\Delta b K_u - b\Delta K_u)^T(Peu_m - F_b^{-1} b\Delta\dot{K}_u)]\end{aligned} \tag{17}$$

At this point the choice of

$$\Delta\dot{K}_x^T = -(b^T F_a^{-1} b)^{-1} b^T PeX^T, \tag{18a}$$

$$\Delta K_x^T|_{t=0} = \Delta K_{x0}^T$$

$$\Delta\dot{K}_u = (b^T F_b^{-1} b)^{-1} b^T Peu_m, \tag{18b}$$

$$\Delta K_u|_{t=0} = \Delta K_{u0}$$

results in cancellation of the last two terms in (17), and assures that $\dot{V}$ is negative definite, thus guaranteeing that $e \rightarrow 0 (X \rightarrow X_m)$.

The only problem now remaining is to force the system to track a desired trajectory. This point has been addressed by Meldrum and Balas[20] but, even with an equal number of controls and output variables, only a sinusoidal reference trajectory could be commanded of the rigid body motion. An inverse model technique of the type proposed in Balestrino et al.[26] cannot be adopted since the model (6), satisfying (11), turns out not to be decoupled. However, the state-space design existing in the reference model (6) appears to provide a possible way out of this dilemma by specifying the development of systematic design procedures for both the optimal regulator and the tracking problems.[19]

TRACKING CONTROLLER

The tracking problem was initially conceived in order to extend state-space regulator methods to problems having external command inputs. Therefore, consider an output form

$$Y = CX_m \tag{19}$$

where Y is the output to be tracked, C is a constant matrix.

Meanwhile, a control system for the reference model (6) and (19) must be synthesized such that in the steady-state condition, the output Y becomes equal to some arbitrary desired constant reference output $Y_r(t) = Y_r$. In order to pursue this goal, the integral error W between the reference and the actual output is defined as follows:

$$\dot{W} = Y_r - Y \text{ or } W = \int^t (Y_r - Y)\, dt \tag{20}$$

and the tracking control law can thus be written as

$$u_m = -K_m X_m - K_I \dot{W} \tag{21}$$

where K_m, K_I are the proportional and the integral gains respectively. Adjoining (20) and (21) to (6), gives

$$\dot{Z} = A_0 Z + B_0 Y_r \tag{22}$$

where $Z^T = [X_m^T, W]$

$$A_0 = \begin{bmatrix} A_m - b_m k_m & -b_m K_I \\ -C & 0 \end{bmatrix}$$

$$B_0 = \begin{bmatrix} 0 \\ I \end{bmatrix}$$

It is claimed that the dynamic system (22) is asymptotically stable, if K_I is chosen appropriately. Then, in the steady state,

$$\lim_{t\to\infty} Z = Z_\infty = \begin{bmatrix} X_\infty \\ W_\infty \end{bmatrix} = -\begin{bmatrix} A_m + b_m K_m & -b_m K_I \\ -C & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} Y_r \tag{23}$$

where the inverse matrix exists due to the asymptotical stability. Clearly, the desired zero error between Y and Y_r is also obtained in the steady state (i.e., $\lim_{t\to\infty} Y(t) = Y_r$ or $\lim_{t\to\infty} W(t) = 0$). Now, the objective is to find the gains K_m and K_I. Define

$$\Delta X_m = X_m - X_\infty, \quad \Delta W = W - W_\infty, \quad \Delta u_m = u_m - u_\infty \tag{24}$$

where $u_\infty = -K_m X_\infty - K_I W_\infty$.

The transient response is then governed by the set of differential equations

$$\frac{d}{dt}\begin{bmatrix} \Delta X_m \\ \Delta W \end{bmatrix} = \begin{bmatrix} A_m & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix} \Delta X_m \\ \Delta W \end{bmatrix} + \begin{bmatrix} b_m \\ 0 \end{bmatrix} \Delta u_m \tag{25}$$

An LQR design is utilized to minimize the performance functional for (25)

$$J = \int_0^\infty \left([\Delta X_m^T \ \Delta W] Q \begin{bmatrix} \Delta X_m \\ \Delta W \end{bmatrix} + R \Delta u_m^2 \right) dt \tag{26}$$

This results in

$$K_m = R^{-1} b_m S_{11} \tag{27a}$$

$$K_I = R^{-1} b_m S_{12} \tag{27b}$$

where $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} > 0$ is the solution of the Riccati equation.

In summary, since the constant matrix C is determined by the output Y, one needs at least as many inputs as the number of outputs to be tracked and needs the dynamical system (25) to be controllable.[19] Therefore, K_m and K_I are simultaneously derived as in (27). With only one input, for example, the dynamical system (25) in the case of a one-link flexible arm may be uncontrollable when the joint velocity is tracked as is shown in the following example. This may result in a singular solution for the Riccati equation (27). Finally, the total control problem becomes one of choosing the feedback constant gains K_x, K_u, along with the adaptive gains ΔK_x, ΔK_u for system stability, and K_m as well as the integral gain K_I for the desired reference tracking. In other words, u is composed of (7) and (21). The block diagram of the total system is shown in Figure 2.

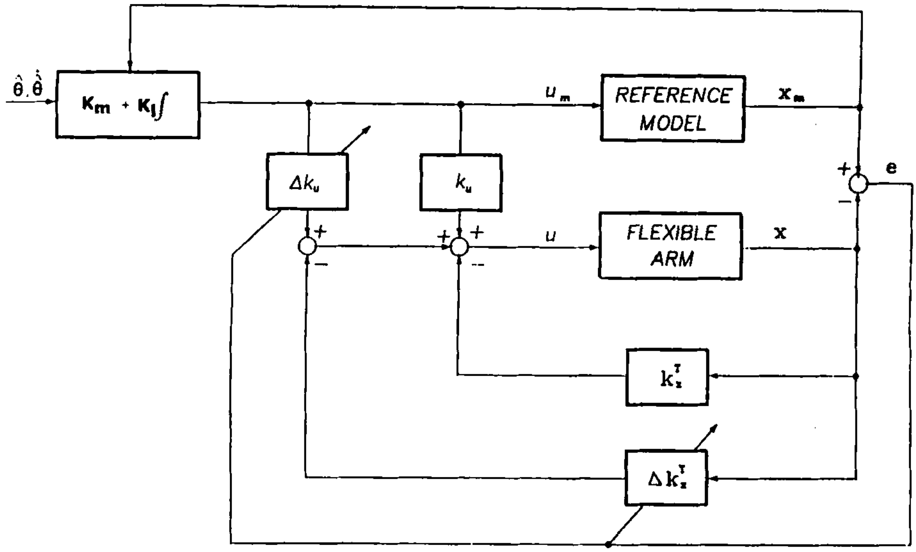

Figure 2. Block diagram of the total control scheme.

THE CASE STUDY

In the following a case study is developed for the one-link flexible arm existing in the Flexible Automation Laboratory at Georgia Tech, whose specification is fully described in Appendix A. The mode shapes (ϕ_i) used to model the arm are shown in Figure 3.

As far as the joint angle trajectory is concerned, the arm is required to move

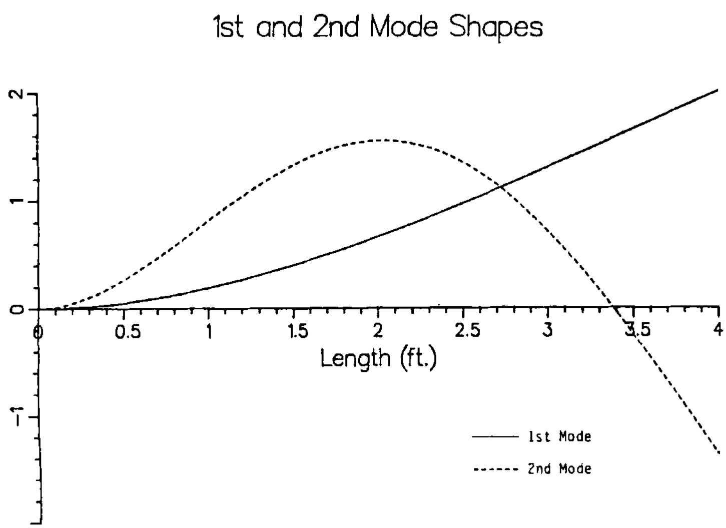

Figure 3. First and second mode shapes.

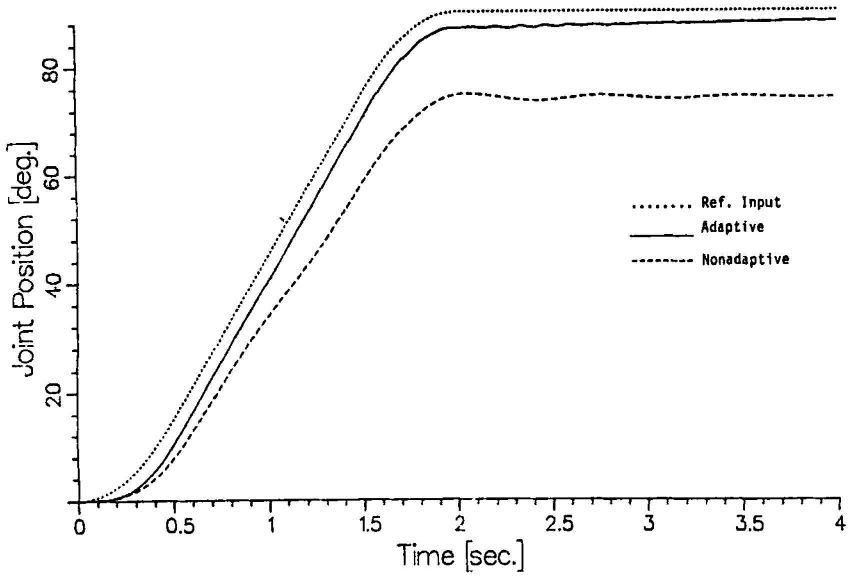

Figure 4. Joint position profiles (joint velocity to be tracked).

from $\theta_i = 0°$ to $\theta_f = 90°$ in 2 s, following a standard trapezoidal velocity profile with maximum velocity $\dot{\theta} = 60°/s$. The constant feedback gain resulting is $K_x^T = [65.27\ \ -176.13\ \ -2937.23\ \ 27.27\ \ -7.50\ \ -67.27]$ and $K_u = 1$. $\Delta \dot{K}_x^T$ and ΔK_u (18) have been chosen with $F_a = 2I$, $F_b = 0.005$, and $H = I$ in (16) such that

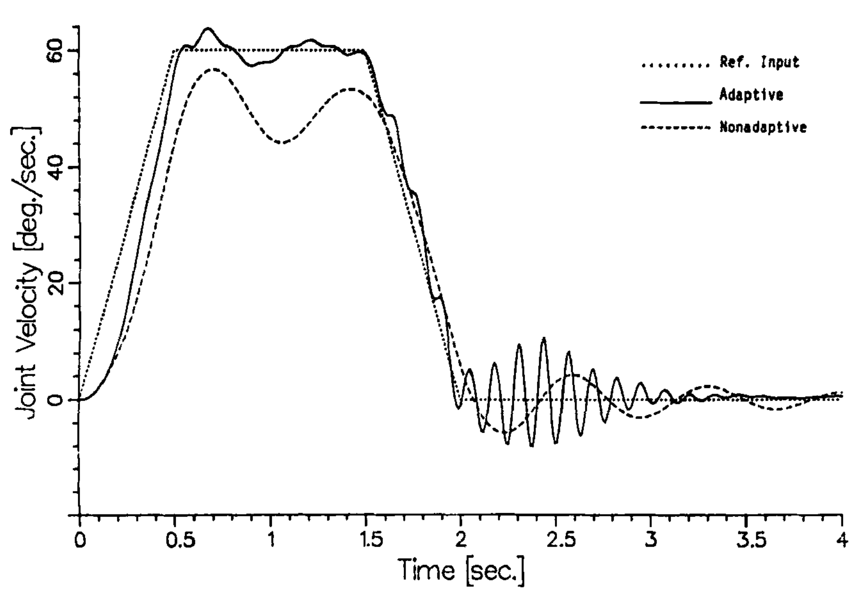

Figure 5. Joint velocity profiles (joint velocity to be tracked).

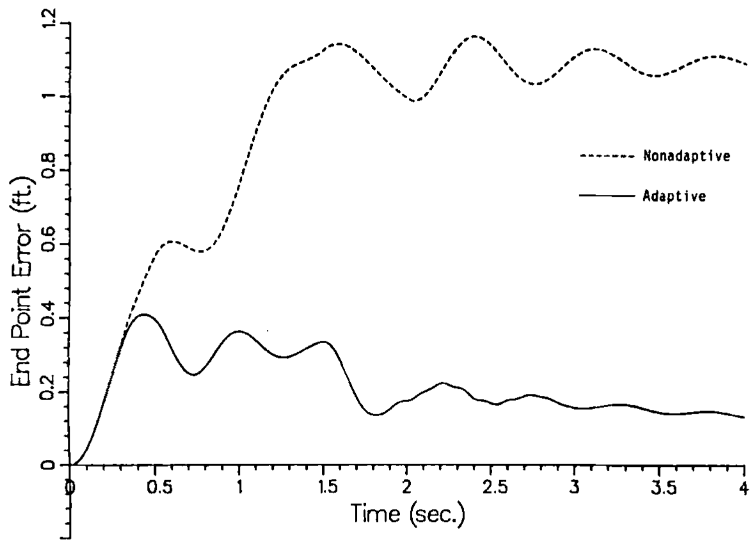

Figure 6. End point position errors (joint velocity to be tracked).

the system under adaptive control is guaranteed to be stable. ΔK_{x0}^T and ΔK_{u0} are null here. An LQR design with $Q = 2I$ and $R = 1$, which is used to derive the tracking controller, results in $K_m^T = [0.0\ -0.635\ -8.591\ 0.06\ -0.056\ 0.046]$, $K_I = 0.031$ for the joint angular velocity to be tracked (Figs. 4–7) (i.e., $C^T = [0\,0\,0\,1\,0\,0]$). For the joint angular position to be tracked (Figs. 8–11) (i.e., $C^T = [1\,0\,0\,0\,0\,0]$), the tracking controller is $K_m^T = [0.616\ -0.793\ -10.004\ 0.1335$

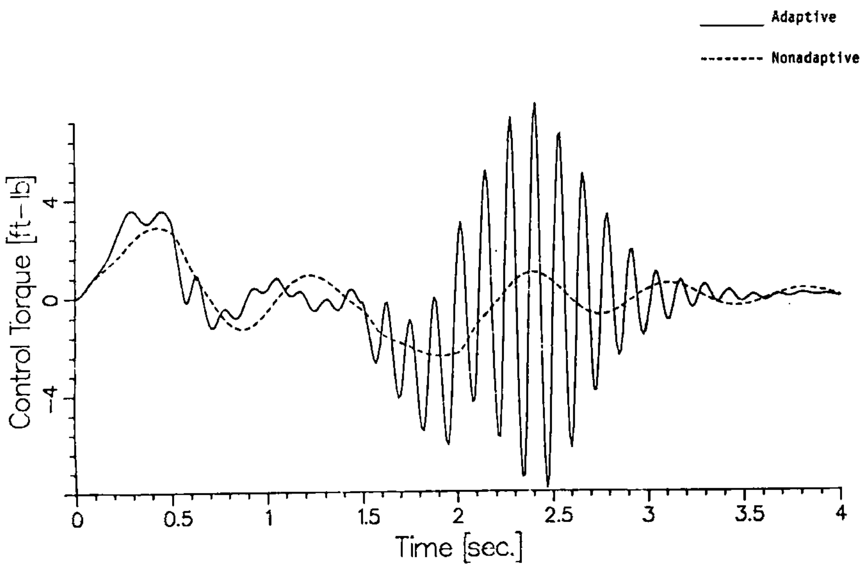

Figure 7. Control torques (joint position to be tracked).

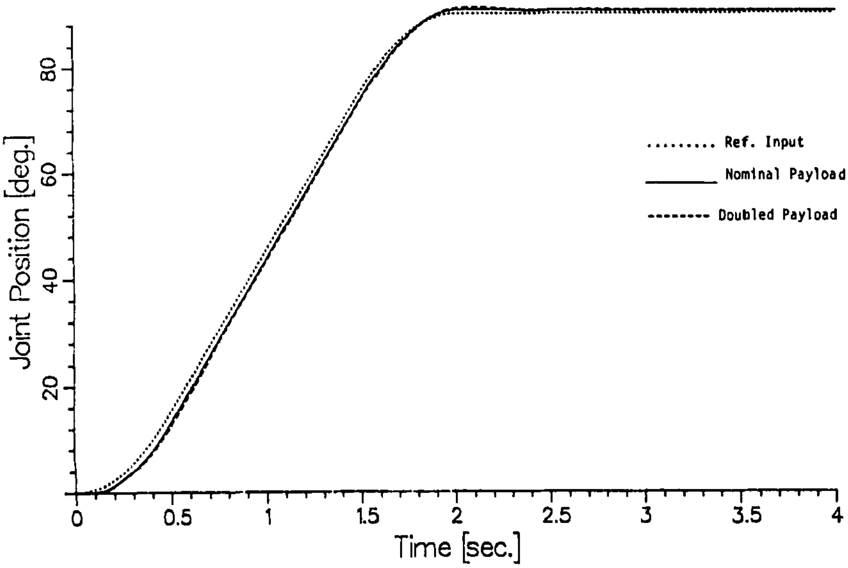

Figure 8. Joint position profiles (joint position to be tracked).

$-0.034\ 0.05]$, $K_I = 1.414$. For the end-point position to be tracked (Figs. 12–15) (i.e., $C^T = [4\ 2.02\ -1.365\ 0\ 0\ 0]$), K_m^T and K_I become $[2.41\ -1.27\ -14.32\ 0.396\ 0.05\ 0.0058]$ and 1.4142. Also notice that the dynamic system that is linearized around zero states from (4) is used to derive the optimal (constant)

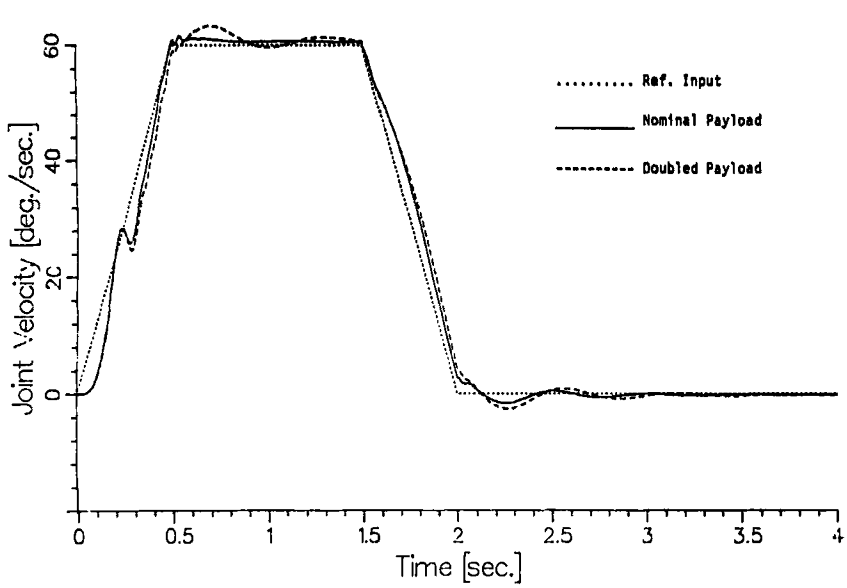

Figure 9. Joint velocity profiles (joint position to be tracked).

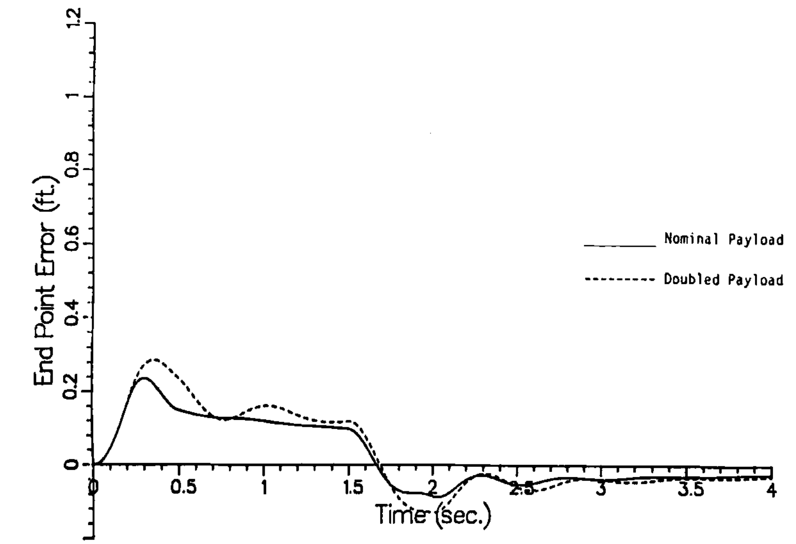

Figure 10. End point position errors (joint position to be tracked).

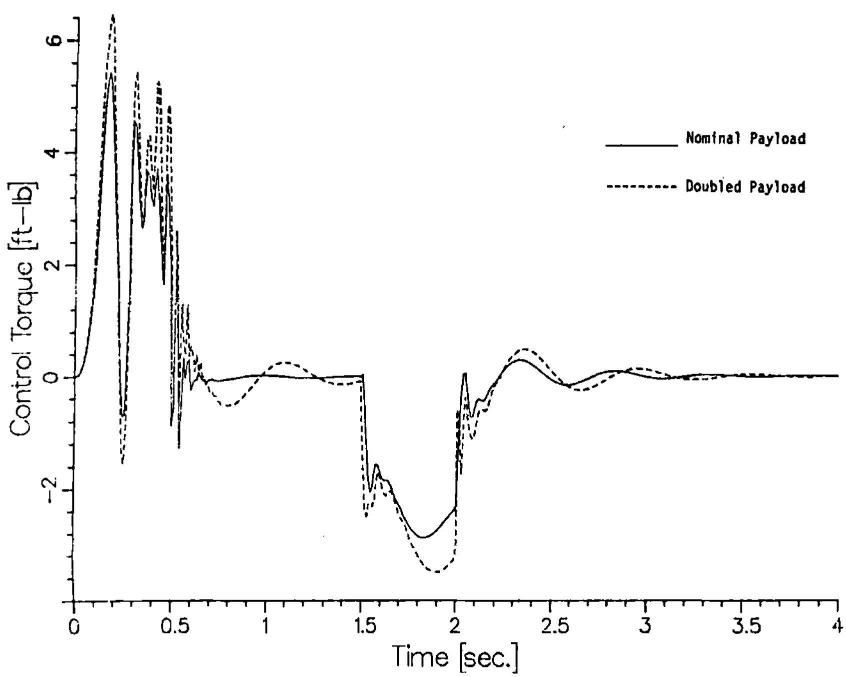

Figure 11. Control torques (joint position to be tracked).

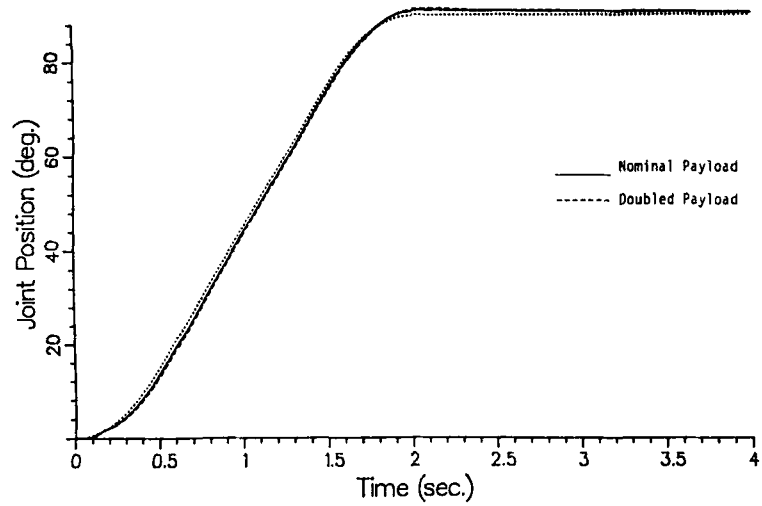

Figure 12. Joint position profiles (end-point position to be tracked).

gains K_x. This results in unstable responses for the constant (nonadaptive) feedback control system, when the arm travels at high velocity.

Different sets of simulations have been carried out, one with the above design parameters, and another one just with the constant feedback gains K_x^T and K_u, without any outer adaptive control. In order to analyze the control performance the whole nonlinear model has been simulated for the system (5) in both cases.

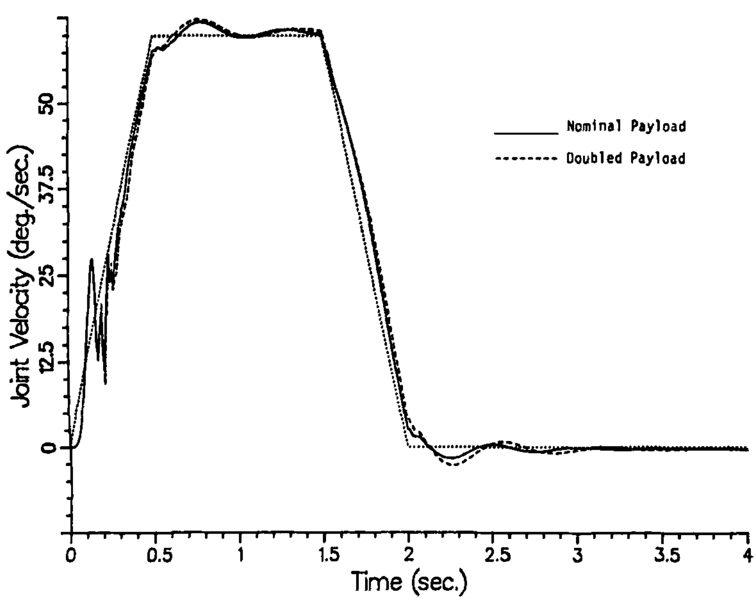

Figure 13. Joint velocity profiles (end-point position to be tracked).

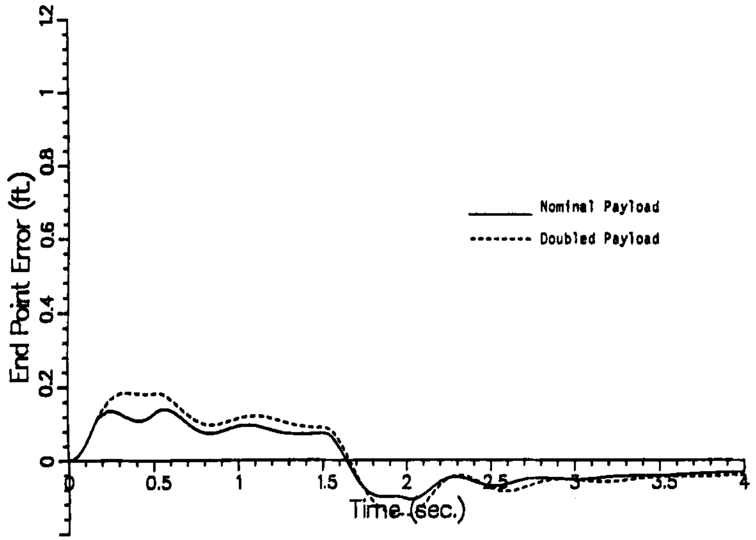

Figure 14. End point position errors (end-point position to be tracked).

A sampling rate of 0.1 m s has been adopted. Furthermore, the robustness of the system control to parameter variations has been tested by doubling the payload mass, without changing the constant control gains. Figures 4 through 15 illustrate the results obtained. It can be recognized that the adaptive control performs better than the simple optimal control, as it results in better tracking accuracy.

First consider the case (Figs. 4–7) of joint velocity tracking. Figure 4 shows

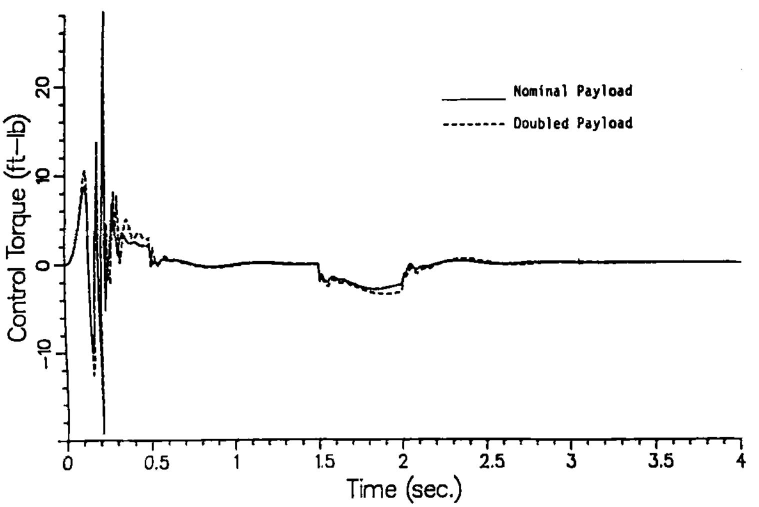

Figure 15. Control torques (end-point position to be tracked).

the joint position response with and without adaptive control and corresponding reference input. Figure 5 shows the joint velocity. Note better tracking occurs with adaptation but at the expense of some oscillations as gains adapt. Figure 6 shows differences in the end-point position error with respect to the reference signal. Figure 7 shows the joint torque. It should be pointed out that the dynamical system (25) does not satisfy the criteria of controllability. Therefore, the solution of the Riccati equation is singular, which causes undesirable response with inaccurate tracking and oscillations. However, such problems do not arise for joint position and end-point tracking.

When the system is used to track a joint position command (Figs. 8–11), the nonadaptive control is unstable due to uncompensated nonlinearities and thus not plotted. The joint position response of the adaptive control is shown in Figure 8 with the reference joint position command and responses for a nominal payload as well as twice the payload used in the design. The low steady-state error and the low effect of payload change illustrate the robust properties of the controller. Joint velocity, end-point position error, and control torques are illustrated in Figures 9–11.

Another quantity tracked in this analysis is the end point position. Figures 12–15 show the time responses for this simulation. The results are almost identical to the above joint position case, except that the end point position error is comparatively small during this control process. Note that this requires that the reference model predict the end-point position.

CONCLUSIONS

A model reference adaptive control has been presented for a one-link flexible arm, which is based on the preliminary results obtained in Siciliano et al.[18] In order to comply with the model matching conditions, the reference model has been set up to be the linearized arm model of the system as optimally controlled. Since the resulting reference model is not decoupled, the desired joint angle trajectory is commanded through a tracking controller preceeding the overall system. Full state availability has been supposed for control synthesis. The extension of this work to the use of an observer has been initiated and described in Yuan and Book.[23]

A case study has been developed for a prototype in the laboratory. Simulation results have shown the advantage of using an outer adaptive feedback control with respect to the pure optimal control and the robustness of the system control to payload variations. Furthermore, for the tracking controller, only the joint velocity command is not recommended based on the results of this work.

It must be emphasized, however, that for multiple-link flexible manipulators the results obtained in this article appear only partially satisfactory. In the case of more degrees of freedom, the nonlinear coupling terms in the joint variables (which are not present in the one link case) may become dominant, particularly at high speed, and control performance is likely to be derated.

This point, along with the problem of state reconstruction, or eventually considering output feedback, constitute two challenging research issues needing additional investigation.

The authors would like to acknowledge that this material is based in part on work supported by the Computer Integrated Manufacturing Systems Program at Georgia Tech, and by the NATO Science Program (Special Panel on Sensory Systems for Robotic Control) under grant no. 687/86.

APPENDIX A: SPECIFICATION OF EXPERIMENTAL PROPERTIES

Beam

Length: 48 in.
Section: $3/16 \times 3/4$ in.^{2}
EI: 4120
Material: Aluminum
Alloy: 6065-T6

Payload

Weight: 0.1 lb
Material: Aluminum
Alloy: 6065-T6

Torque Motor

Manufacturer: Inland Motor
Type: T-5730 (Permanent Magnet DC)
Rotor Inertia: 0.06 in.-lb-s^2.

References

1. W. J. Book, O. Maizza-Neto, and D. E. Whitney, "Feedback control of two beam, two joint systems with distributed flexibility," *ASME J. Dyn. Syst., Measur. Contr.*, **97**, 424–431 (1975).
2. M. J. Balas, "Feedback control of flexible systems," *IEEE Trans. Automat. Contr.*, **AC-23**, 673–679 (1978).
3. A. Truckenbrodt, "Modelling and control of flexible manipulator structures," *Proceedings of the 4th CISM-IFToMM Symposium on Theory and Practice of Robots and Manipulators*, A. Morecki, G. Bianchi and K. Kedzior, Eds., Zaborow, Poland, Sept. 8–12, 1981, pp. 90–101.
4. W. J. Book, "Recursive Lagrangian dynamics of flexible manipulator arms," *Int. J. of Robotics Research*, **3**, 87–101 (1984).
5. G. G. Hastings and W. J. Book, "Experiments in optimal control of a flexible arm," *Proceedings of the American Control Conference*, Boston, MA, June 10–12, 1985, pp. 728–729.
6. R. H. Cannon, Jr. and E. Schmitz, "Initial experiments on the end-point control of a flexible one-link robot," *Int. J. Robotics Research*, **3**, 62–75 (1984).
7. T. Fukuda, "Flexibility control of elastic robotic arms," *J. Robotic Systems*, **2**, 73–88 (1985).
8. Y. Sakawa, R. Matsumo, and S. Fukushima, "Modeling and feedback control of a flexible arm," *J. Robotic Systems*, **2**, 453–472 (1985).

9. N. G. Chalhoub and A. G. Ulsoy, "Control of a flexible robot arm: experimental and theoretical results," *ASME J. Dyn. Syst. Meas. Contr.*, **109**, 299–309 (1987).
10. W. J. Book and M. Majette, "Controller design for flexible, distributed parameter mechanical arms via combined state space and frequencey domain techniques," *ASME J. Dyn, Syst. Meas. Contr.*, **105**, 245–254 (1983).
11. J. C. Ower and J. Van de Vegte, "Classical control design for a flexible manipulator: modeling and control system design," *IEEE J. Robotics, Automation*, **RA-3**, 485–489 (1987).
12. W. H. Sunada and S. Dubowski, "On the dynamics analysis and behavior of industrial robotic manipulators with elastic members," *ASME J. Dyn. Syst., Meas., Contr.*, **105**, 42–51 (1983).
13. S. N. Singh and A. A. Schy, "Control of elastic robotic systems by nonlinear inversion and modal damping," *ASME J. Dyn. Syst., Meas., Contr.*, **108**, 180–189 (1986).
14. J. H. Davis and R. M. Hirschorn, "Tracking Control of a Flexible Robot Link," *IEEE Trans. Automat. Contr.* **AC-33**, 238–248 (1988).
15. A. De Luca and B. Siciliano, "Joint-based control of a nonlinear model of a flexible arm," *Proceedings of the American Control Conference*, Atlanta, GA, June 15–17, 1988, pp. 935–940.
16. B. Siciliano and W. J. Book, "A singular perturbation approach to control of light-weight flexible manipulators," *Int. J. Robotics Research*, **7**, 79–90 (1988).
17. Y. D. Landau, *Adaptive Control: The Model Reference Approach*, Marcel Dekker, New York, 1979.
18. B. Siciliano, B. S. Yuan, and W. J. Book, "Model reference adaptive control of a one link flexible arm," in *Proceedings of the 25th IEEE Conf. on Decision and Control*, Athens, Greece, Dec. 10–12, 1986, pp. 91–95.
19. P. C. Young and J. C. Willems, "An approach to the linear multivariable servomechanism problem," *Int. J. Control.* **15**, 961–979 (1972).
20. D. R. Meldrum and M. J. Balas, "Direct adaptive control of a remote manipulator arm," *Robotics and Manufacturing Automation* (Proceedings of the ASME Winter Annual Meeting), H. Donath and M. Leu, eds., Miami, FL, Nov. 17–22, 1985, pp. 115–120.
21. C. Canudas De Wit and Van den Bossche, "Adaptive control of a flexible arm with explicit estimation of the payload mass and friction," in *Preprints of the IFAC/IFIP/IMACS Int. Symposium on Theory of Robots*, P. Kopacek, I. Troch, and K. Desoyer, Eds., Vienna, Austria, Dec. 3–5, 1985, pp. 315–320.
22. G. G. Hastings and W. J. Book, "A linear dynamic model for flexible robotic manipulators," *IEEE Control Systems Magazine* **7**, 61–64 (1987).
23. B. S. Yuan and W. J. Book, "A robust scheme for direct adaptive control of flexible arms," in *Modeling and Control of Robotic Manipulators and Manufacturing Processes*, R. Shoureshi, K. Youcef-Toumi, and H. Kazerooni, Eds., ASME DSC-Vol. 6, 261–269 (1987).
24. L. Meirovitch, *Analytical Methods in Vibrations*, Macmillian, New York, 1967.
25. P. C. Hughes, "Space structure vibration modes: How many exists? Which ones are important?" *IEEE Control System Magazine*, **7**, 22–28 (1987).
26. A. Balestrino, G. De Maria and L. Sciavicco, "An adaptive model following control for robotic manipulators," *ASME J. Dyn. Syst., Meas., Contr.* **105**, 143–151 (1983).
27. K. L. Lim and M. Eslami, "Adaptive controller designs for robot manipulator systems using Lyapunov direct method," *IEEE Trans. Automat. Control.*, **AC-30**, 1229–1233 (1985).
28. H. Erzberger, "Analysis and design of model following control systems by state space techniques," in *Proceedings of the Joint American Control Conference*, Ann Arbor, MI, June 26–28, 1968, pp. 572–581.