

User-Defined Accuracy in the Augmented Task Space Approach for Redundant Manipulators

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This paper shows how singularity robust redundancy resolution can be performed using the augmented task space approach with user-defined weighted damped least-squares. The resulting scheme achieves a task priority strategy between the end-effector task and the constraint task. The minimum singular value of the augmented Jacobian is exploited to calculate appropriate values for the damping and weighting factors. The scheme is successfully implemented in simulation studies on a two-joint planar arm and on a seven-joint manipulator with a kinematic design derived from the PUMA geometry.

1. INTRODUCTION

Many applications of manipulators with redundant degrees of freedom are conveniently described in the framework of the so-called *augmented task space* approach independently introduced by Egeland [1] and by Sciavicco and Siciliano [2], and later used by Seraji [3] under the name of configuration control.

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The simple common idea is to augment the end-effector task with a suitable constraint task that is aimed at specifying the internal motion of the arm; one typical constraint is to require the manipulator to avoid kinematic singularities. On the other hand, this formulation suffers from the drawback of algorithmic or *artificial singularities* which are introduced in addition to the kinematic singularities, due to conflicts between the two tasks.

Conceptually similar to the above approach is the use of inverse kinematic functions, defined on a singularity-free workspace proposed by Baker and Wampler [4]; however, the definition of closed-form inverse kinematic functions is feasible only for manipulators with few joints. Baillieul [5] included a constraint task vector in his extended Jacobian scheme which was derived through the optimization of a scalar objective function. This method is rather simple but fails when the extended Jacobian becomes singular even though the end-effector Jacobian has full rank; this logically corresponds to the occurrence of the above artificial singularities.

Nakamura, Hanafusa, and Yoshikawa [6] proposed the so-called task priority strategy that establishes an order of priority between the end-effector task and the constraint task. This method is computationally more expensive than the previous ones, but remarkably gives a correct primary end-effector solution as long as the end-effector Jacobian maintains full-rank. Nevertheless, the solution is ill-conditioned close to artificial singularities due to the use of a pseudoinverse of a matrix that becomes near

rank-deficient. The drawback was addressed by Maciejewski and Klein [7] who devised an effective solution obtained by treating the above matrix as singular in the neighborhood of the artificial singularity.

A remedy for the handling of artificial singularities was proposed by Chiacchio and Siciliano [8] using the transpose of the ill-conditioned matrix together with a feedback correction term. The technique was later refined by Chiacchio *et al.* [9].

The above discussion suggests that it would be desirable to have a solution that combines the computational simplicity of the augmented task space technique with the effectiveness of the task priority scheme, and at the same time gives acceptable trajectories in the neighborhood of (artificial) singularities. This is achieved in the present work, where a method based on *weighted damped least-squares* solution [10] with *user-defined accuracy* [11] is presented. The damped least-squares yields singularity robustness [10, 12] and the use of proper weighting allows the shaping of the solution along given task space directions [11]. In particular, a true task priority strategy is performed by giving lower weight to the constraint task which is imposed for keeping the manipulator off kinematic singularities [13]. The singular value decomposition is invoked to analyze the features of the method; an estimate of the smallest singular value [14], which gives a measure of closeness to singularities, is used to compute suitable values for the damping and weighting factors.

The scheme is first applied to the simple case of a two-joint planar arm, and then is tested on a seven-joint manipulator derived from the PUMA geometry. Simulation results are reported in the paper.

2. AUGMENTED TASK SPACE

Let \mathbf{q} denote the n -dimensional vector of joint coordinates of the manipulator and \mathbf{x}_E the m -dimensional vector of end-effector task coordinates. The differential task space motion is described by $\delta\mathbf{x}_E = \dot{\mathbf{x}}_E \delta t$ where δt is the time increment. The $(m \times n)$ end-effector Jacobian matrix $\mathbf{J}_E(\mathbf{q})$ is defined by $\dot{\mathbf{x}}_E = \mathbf{J}_E(\mathbf{q})\dot{\mathbf{q}}$ which, in terms of incremental motions, gives

$$\delta\mathbf{x}_E = \mathbf{J}_E(\mathbf{q})\delta\mathbf{q} \quad (1)$$

where $\delta\mathbf{q} = \dot{\mathbf{q}}\delta t$.

In configurations where \mathbf{J}_E is rank deficient, that is, when $\text{rank}(\mathbf{J}_E) = r$ with $r < m$, the manipulator is in a *singular configuration* and there exist directions of the end-effector task space along which motion is not feasible. Further, when $m < n$ the manipulator is *kinematically redundant* with respect to the given

end-effector task, and $(n - m)$ degrees of freedom become available to specify the motion of the manipulator in all n -dimensional joint space.

The *augmented task space approach* [1, 2] provides a natural framework to exploit redundancy in robotic systems. An additional constraint task is introduced by specifying an $(n - m)$ -dimensional vector \mathbf{x}_C as a function of the manipulator joint variables. Accordingly, an $((n - m) \times n)$ constraint Jacobian matrix \mathbf{J}_C can be defined through

$$\delta\mathbf{x}_C = \mathbf{J}_C(\mathbf{q})\delta\mathbf{q}. \quad (2)$$

$$\delta\mathbf{x} = \begin{pmatrix} \delta\mathbf{x}_E \\ \delta\mathbf{x}_C \end{pmatrix},$$

Then, defining an n -dimensional task increment equations (1, 2) can be combined into

$$\delta\mathbf{x} = \mathbf{J}(\mathbf{q})\delta\mathbf{q} = \begin{pmatrix} \mathbf{J}_E(\mathbf{q}) \\ \mathbf{J}_C(\mathbf{q}) \end{pmatrix} \delta\mathbf{q}, \quad (3)$$

where \mathbf{J} is the $(n \times n)$ augmented Jacobian matrix.

The joint increment $\delta\mathbf{q}$ can be computed from (3) as

$$\delta\mathbf{q} = \mathbf{J}^{-1}(\mathbf{q})\delta\mathbf{x}, \quad (4)$$

on condition that the matrix \mathbf{J} has full rank. To the purpose, even if the end-effector Jacobian \mathbf{J}_E is non-singular, the augmented Jacobian \mathbf{J} may be singular. This happens when the rows of the constraint Jacobian \mathbf{J}_C become linearly dependent on the rows of \mathbf{J}_E , indicating that the constraint task is in conflict with the end-effector task. In this case, the manipulator is said to be in an *artificial singularity* and no exact solution for $\delta\mathbf{q}$ will exist unless $\delta\mathbf{x}$ is in the range space of \mathbf{J} . Actually, the constraint task is often chosen to keep the manipulator off kinematic singularities; therefore, the occurrence of an artificial singularity is really an undesirable effect from a practical point of view.

An effective way to handle the conflicting task situations is offered by the task priority strategy [6], that assigns different priorities to the two tasks and ensures the correct execution of the task with higher priority. Instead of solving (3) directly as in (4), the joint space increment is computed as—dropping the dependence on \mathbf{q} —

$$\delta\mathbf{q} = \mathbf{J}_E^+ \delta\mathbf{x}_E + (\mathbf{I} - \mathbf{J}_E^+ \mathbf{J}_E) (\mathbf{J}_C (\mathbf{I} - \mathbf{J}_E^+ \mathbf{J}_E))^+ (\delta\mathbf{x}_C - \mathbf{J}_C \mathbf{J}_E^+ \delta\mathbf{x}_E), \quad (5)$$

where the symbol “+” denotes the pseudoinverse of a matrix. Solution (5) can be simplified to [7]

$$\delta\mathbf{q} = \mathbf{J}_E^+ \delta\mathbf{x}_E + (\mathbf{J}_C (\mathbf{I} - \mathbf{J}_E^+ \mathbf{J}_E))^+ (\delta\mathbf{x}_C - \mathbf{J}_C \mathbf{J}_E^+ \delta\mathbf{x}_E). \quad (6)$$

From (5) it is easily seen that $J_E \delta q = \delta x_E$, which means that if J_E has full rank the end-effector motion is correctly executed and is unaffected by the constraint task. This solves the problem of artificial singularities only in part, because the solution (5) or (6) still involves the computation of the pseudoinverse of the matrix $J_C (I - J_E^+ J_E)$ which is rank-deficient at an artificial singularity. In particular, it can be shown that this matrix has full rank $(n - m)$ if and only if the augmented Jacobian J has full rank n [9]. This in turn reveals that, when a pseudoinverse of the matrix $J_C (I - J_E^+ J_E)$ exists, the task priority solution gives the same solution as the pure augmented solution (4), but still the problem remains in the neighborhood of artificial singularities.

A first possible way to tackle the above inconvenience is to use a damped least-squares solution [10, 12] in connection with the matrix $J_C (I - J_E^+ J_E)$ in such a way that the errors due to damping will mostly affect the secondary constraint task directions. It is anticipated, however, that the computational requirements of such a solution might be impractical for on-line implementation of the technique.

A computationally inexpensive alternative is to modify the solution (6) resorting to the transpose of the constraint Jacobian together with a suitable feedback correction term for the constraint task [8, 9].

3. WEIGHTED DAMPED LEAST-SQUARES WITH USER-DEFINED ACCURACY

As pointed out above, the solution to equation (3) becomes ill-conditioned in the neighborhood of (kinematic or artificial) singularities for the augmented Jacobian matrix. In this case, solution (4) cannot be used any more and a robust inverse solution must be sought.

Nakamura and Hanafusa [10] and Wampler [12] independently proposed the adoption of the damped least-squares solution to (3) in the inverse kinematics algorithm. The solution minimizes the index

$$L = \|\delta x - J \delta q\|^2 + \lambda^2 \|\delta q\|^2, \quad (7)$$

where a trade-off between solution accuracy—first term of (7)—and solution feasibility—second term of (7)—is set through the damping factor λ .

Proper weighting [10] can be introduced in (7) concerning the accuracy of the solution along different task space directions [11]. This technique can be conveniently used also for redundant manipulators [13]. In particular, for the augmented task space

increment δx , different weighting can be imposed for the end-effector task and the constraint task. A weighted task increment is thus defined as

$$\delta \tilde{x} = W \delta x = \tilde{J} \delta q, \quad (8)$$

where

$$W = \begin{pmatrix} I & O \\ O & W_C \end{pmatrix} \quad (9)$$

and

$$\tilde{J} = WJ = \begin{pmatrix} J_E \\ W_C J_C \end{pmatrix}. \quad (10)$$

By choosing $W_C = wI$, the weighted damped least-squares solution minimizes the index

$$L = \|\delta x_E - J_E \delta q\|^2 + w^2 \|\delta x_C - J_C \delta q\|^2 + \lambda^2 \|\delta q\|^2. \quad (11)$$

It can be easily shown that the joint space increment is given by

$$\delta q = (J_E^T J_E + w^2 J_C^T J_C + \lambda^2 I)^{-1} (J_E^T \delta x_E + w^2 J_C^T \delta x_C). \quad (12)$$

In terms of the singular value decomposition, solution (12) can be written as

$$\delta q = \sum_{i=1}^n \frac{\sigma_i}{\sigma_i^2 + \lambda^2} v_i u_i^T \delta \tilde{x}, \quad (13)$$

where σ_i is the i -th singular value of matrix \tilde{J} , and v_i (u_i) is the i -th input (output) singular vector of matrix \tilde{J} , that is,

$$\tilde{J} = \sum_{i=1}^n \sigma_i u_i v_i^T. \quad (14)$$

The singular values σ_i and the singular vectors v_i and u_i will depend on the weighting matrix W . This has no impact on the solution δq as long as $\lambda \ll \sigma_n$.

Close to artificial singularities (\tilde{J} is near rank-deficient), where $\lambda > \sigma_{r+1}$ for some $r < n$, the solution can be shaped by selecting a proper weighting factor w . In particular, if $w \ll 1$ and J_E has full rank, the first m output singular vectors will be practically aligned with the end-effector task space directions, while the damping will mostly affect the constraint task space directions. In other words, a true task priority strategy is accomplished by this technique.

To study the effect of weighting and damping in detail, the error in the augmented task space coordinates can be analyzed. The joint space increment corresponding to a pure inverse solution to equation (8) can be written in terms of the singular value decomposition as

$$\delta q = \sum_{i=1}^n \frac{1}{\sigma_i} v_i u_i^T \delta \tilde{x}. \quad (15)$$

The error in the joint space due to damping, e_q , can be obtained by subtracting (13) from (15), that is,

$$e_q = \sum_{i=1}^n \frac{\lambda^2}{\sigma_i(\sigma_i^2 + \lambda^2)} v_i u_i^T \delta \bar{x}. \quad (16)$$

The resulting error in the augmented task space is given by $e_x = J e_q$, which on reduction of (14, 16) gives

$$e_x = \sum_{i=1}^n \frac{\lambda^2}{\sigma_i^2 + \lambda^2} u_i u_i^T \delta \bar{x}. \quad (17)$$

Assuming that $\sigma_m \gg \lambda$, the error can be approximated as

$$e_x = \sum_{i=m+1}^n \frac{\lambda^2}{\sigma_i^2 + \lambda^2} u_i u_i^T \delta \bar{x}, \quad (18)$$

which shows that the error essentially occurs along the constraint task space directions.

Far from artificial singularities, there is actually no need to weight the solution nor even to resort to a damped least-squares (if J_E has full rank). This fact suggests the use of both varying damping factor λ and varying weighting factor w to suitably tune the solution with respect to the distance from singularities.

On the other hand, the weighting factor w should not introduce itself excessive ill-conditioning of J : Indeed, the region inside which the manipulator is treated as singular is established by λ ; choosing a much too low value of w would unnecessarily enlarge the region where damping is active. Therefore, it is convenient to choose $w > \lambda$.

Concerning the actual determination of the weighting factor w and of the damping factor λ , the minimum singular value σ_n is significant to detect the occurrence of near-singular configurations. An estimate of σ_n can be effectively obtained via the numerical technique illustrated in [14].

The following choice for the weighting factor and the damping factor [11] can be recognized as to ensure continuity and good shaping of the solution:

$$\lambda^2 = \begin{cases} 0 & \\ \left(1 - \left(\frac{\hat{\sigma}_n}{\epsilon}\right)^2\right) \lambda_{\max}^2 & \text{when } \hat{\sigma}_n \geq \epsilon \\ \text{otherwise,} & \end{cases} \quad (19)$$

$$(w-1)^2 = \begin{cases} 0 & \\ \left(1 - \left(\frac{\hat{\sigma}_n}{\epsilon}\right)^2\right) (w_{\min}-1)^2 & \text{when } \hat{\sigma}_n \geq \epsilon \\ \text{otherwise,} & \end{cases} \quad (20)$$

where $\hat{\sigma}_n$ denotes the available estimate of the minimum singular value, ϵ sets the size of the singular region, λ_{\max} is the maximum value of the damping factor, and w_{\min} is the minimum value of the weighting factor.

Finally, it should be mentioned that an inherent advantage of the solution (12) is that it properly

works also close to kinematic singularities (J_E is near rank-deficient). To achieve the same performance using the original task priority solution (6), one should replace each pseudoinverse with a damped least-squares inverse, thus increasing the overall computational burden.

4. CASE STUDIES

In the following, the weighted damped least-squares solution with user-defined accuracy is tested in two case studies: In the first one, a simple two-joint planar arm is considered to demonstrate how the method essentially works. In the second one, a seven-joint spatial manipulator is considered to demonstrate the applicability to a real robot.

4.1 Two-Joint Planar Arm

The proposed method is applied in simulation to the two-revolute-joint planar arm depicted in Figure 1 in its initial configuration. The end-effector task x_E for the arm is to track a motion trajectory along the x_0 axis, and then the arm is redundant with respect to this task. The augmented task space technique is applied, with an additional constraint task x_c consisting of a motion trajectory along the y_0 axis.

The manipulator is kinematically singular with respect to the end-effector task if $\sin q_1 = 0$ and $\sin q_2 = 0$. Artificial singularities appear when $\sin q_2 = 0$, that is when the arm is stretched out or folded up; in this case, the reference for the constraint task cannot be tracked independently of the reference for the end-effector task.

The weighted damped least-squares solution with user-defined accuracy is used to give priority to the end-effector task over the constraint task. The damping factor λ is computed as in (19) with $\lambda_{\max} = \epsilon = 0.01$, while the estimate of the mini-

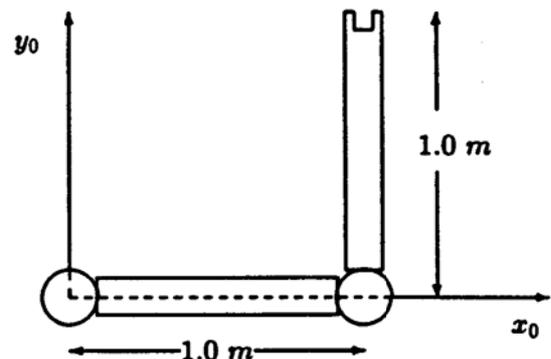


Figure 1. The two-joint planar arm.

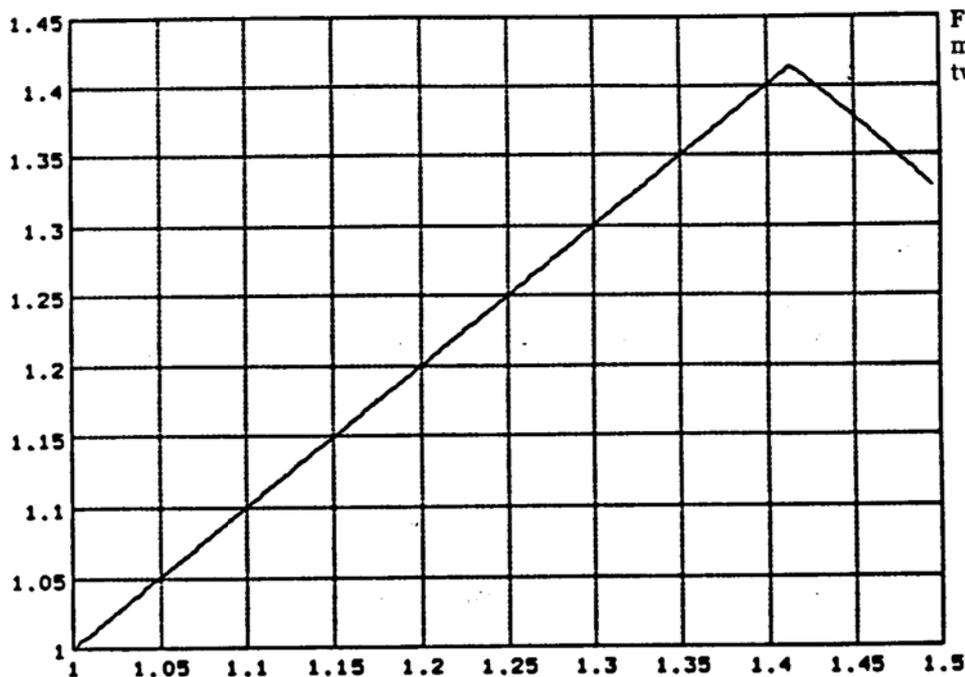


Figure 2. Space-plot of endpoint motion in the simulation of the two-joint arm.

minimum singular value δ_2 is calculated according to [14]. The weighting factor is computed as in (20) with $w_{\min} = 0.1$.

The reference trajectory is a straight line with constant velocity for both components of the augmented task space.

As illustrated in Figure 2, both references x_E and x_C are tracked until joint 2 is stretched out (artificial singularity). Then the end-effector continues to track x_E accurately, keeping joint 2 stretched out, while there is an increasing error on the secondary component x_C . The time-plots for the estimate of the smallest singular value δ_2 and the damping factor λ are shown in Figure 3, which confirm the effect of the damping when the arm is proximal to the artificial singularity.

4.2 Seven-Joint Manipulator

The proposed method is now applied in simulation to the seven-revolute-joint manipulator depicted in Figure 4 together with its Denavit-Hartenberg parameters. The kinematic structure is derived from the well-known PUMA geometry with an extra roll joint in the shoulder which was proposed in [15]. Detailed kinematic analysis for this structure can be found in [16, 17], which reveals the following kinematic singularities:

1. $\sin q_4 = 0$ (elbow singularity),
2. $\sin q_2 = 0$ and $\cos q_3 = 0$ (shoulder/shoulder singularity),

3. $\sin q_2 = 0$ and $\sin q_6 = 0$ (shoulder/wrist singularity),

4. $\sin q_6 = 0$ and $\cos q_5 = 0$ (wrist/wrist singularity).

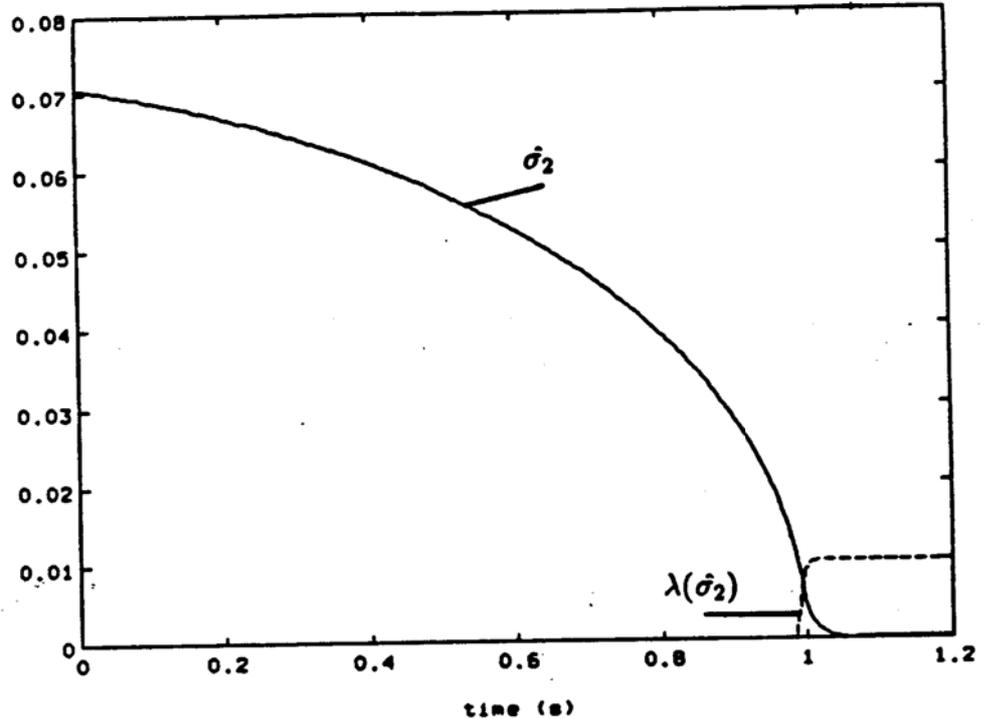
The end-effector task x_E for the arm is to track a motion trajectory for the six components (three of position and three of orientation), and then the arm is redundant with respect to this task. The augmented task space technique is applied; the additional constraint task x_C is the extra joint q_3 . This choice is very effective because the analysis of artificial singularities follows directly from the analysis of the kinematic structure of the original PUMA manipulator. In fact, the constraint Jacobian J_C is the row $(0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0)$ which is added to the end-effector Jacobian J_E to form the augmented Jacobian J ; then, a simple shift of column 3 of J to the last column gives the matrix

$$\begin{pmatrix} & & & & & & * \\ & & & & & & \vdots \\ J_{PUMA} & & & & & & \vdots \\ & & & & & & * \\ 0 & \dots & 0 & 1 & & & \end{pmatrix}$$

which has the same singularities as those of the (6×6) matrix J_{PUMA} .

At this point, it is easy to recognize that the resulting artificial singularities are:

Figure 3. Time-plot of the estimate of the smallest singular value $\hat{\sigma}_2$ and the damping factor λ in the simulation of the two-joint arm.



1. $\sin q_6 = 0$ (wrist singularity),
2. $d_3 \sin q_2 + d_5 \sin(q_2 + q_4) = 0$ and $\sin q_3 = 0$ (shoulder singularity).

Notice that the elbow singularity of the PUMA geometry is also a kinematic singularity for the seven-joint manipulator, and then is not strictly considered to be an artificial singularity.

The weighted damped least-squares solution with user-defined accuracy is used to give priority to the end-effector task over the constraint task. The damping factor λ is computed as in (19) with $\lambda_{\max} = \epsilon = 0.03$, while the estimate of the minimum singular value σ_7 is calculated according to [14]. The weighting factor is computed as in (20) with $w_{\min} = 0.1$.

First, a trajectory through an artificial singularity is assigned. The initial configuration of the manipulator is

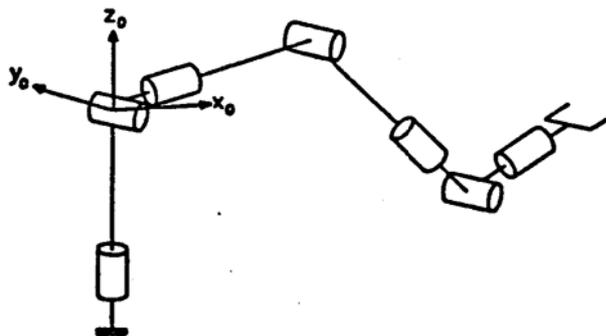
$$q = (0 \quad -\pi/8 \quad 0 \quad -3\pi/8 \quad 0 \quad 0 \quad 0)^T;$$

a saw-tooth reference is given in the y_0 direction, while the remaining components of the end-effector task space are left constant. The reference for the constraint task is $x_C = 0$ which makes the manipulator to work around the "nominal" PUMA configuration, which is known to have a good kinematic design.

The tracking of the end-effector trajectory is accurate. Figure 5 reveals an initial deviation for the constraint variable which is compensated by the wrist angles q_5 and q_7 that rotate $\pm \pi/2$; thus, the next time the trajectory passes through the artificial singularity the commanded motion is feasible.

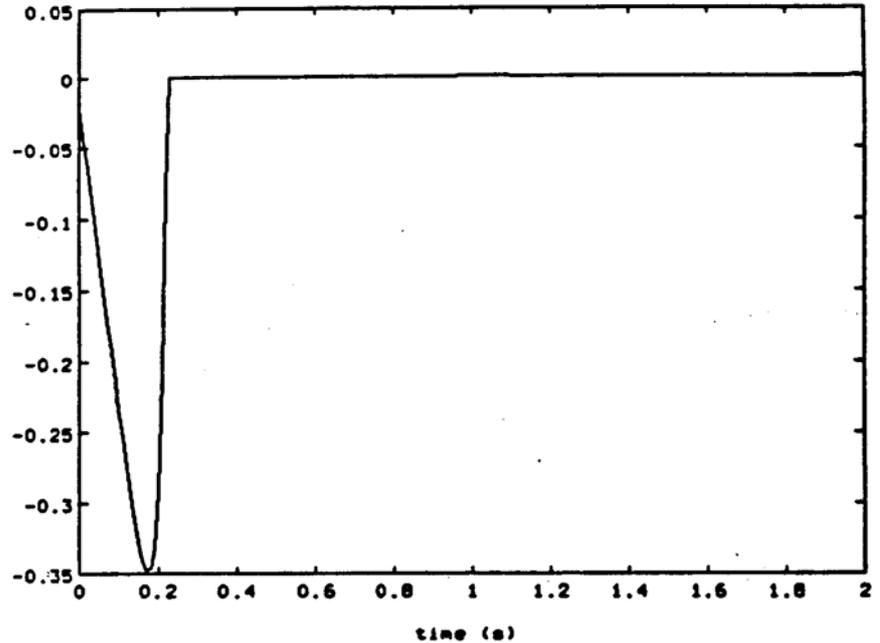
The time-plots of the estimate of the smallest singular value $\hat{\sigma}_7$, and of the damping factor λ in Figure 6 show the periodic occurrence of the artificial singularity and the handling thereof. In fact, it can be seen

Figure 4. The seven-joint manipulator.



Denavit-Hartenberg parameters				
Joint	θ	α	a	d
1	0	90	0	0
2	0	-90	0	0
3	0	90	0	1.0 m
4	0	-90	0	0
5	0	90	0	1.0 m
6	0	-90	0	0
7	0	0	0	0.3 m

Figure 5. Motion of the constraint variable q_3 in the simulation of the seven-joint manipulator going through an artificial singularity.



that the solution algorithm does not remove the artificial singularity but, thanks to the weighting, allows to align the range space of the augmented Jacobian along the directions of the commanded motion.

Next, a trajectory starting from a kinematic singularity is assigned. The initial configuration of the manipulator is

$$q = (0 \quad 0 \quad 0 \quad -\pi/2 \quad \pi/2 \quad 0 \quad 0)^T;$$

notice that this is not an artificial singularity ($\sin q_6 = 0$) because also $\cos q_5 = 0$. The same saw-tooth reference as above is given in the y_0 direction while the remaining components of the end-effector task space are left constant. The reference for the constraint task is again $x_C = 0$.

The tracking of the end-effector trajectory is accurate. The first 0.5 seconds of the motion in Figure 7 show that a very small initial deviation occurs for the constraint variable; incidentally, observe that the motion of q_3 cannot help the manipulator exit the singularity. In any case, q_3 is kept close to the reference value along the rest of the commanded trajectory.

The time-plots of the estimate smallest singular value σ_7 , its estimate $\hat{\sigma}_7$, and of the damping factor λ in Figure 8 for the same 0.5 seconds clarify the exit from the singularity; in detail, the singular value associated with the wrist/shoulder singularity increases fast, and the singular value associated with

the constraint variable becomes the smallest. Also in the Figure, the deviation of the actual singular value σ_7 from its estimate $\hat{\sigma}_7$, can be appreciated.

5. CONCLUSIONS

An inverse differential kinematic scheme for redundant manipulators has been presented in this work. The solution has been derived in the framework of the augmented task space approach, according to which an augmented Jacobian square matrix is obtained by adding a suitable constraint Jacobian to the ordinary end-effector Jacobian. The damped least-squares method has been used to handle the occurrence of both kinematic singularities and artificial singularities, the latter being originated by conflicts between the end-effector task and the constraint task. A user-defined strategy has been shown to offer the possibility of achieving task priority between the two tasks by weighting the relative task increments; usually, the end-effector task prevails over the constraint task. Singular value decomposition has been invoked to analyze the features of the solution and an estimate of the minimum singular value has been used for the determination of suitable damping and weighting factors. The two simulation studies for a two-joint planar arm and a seven-joint manipulator have validated the results anticipated in theory.

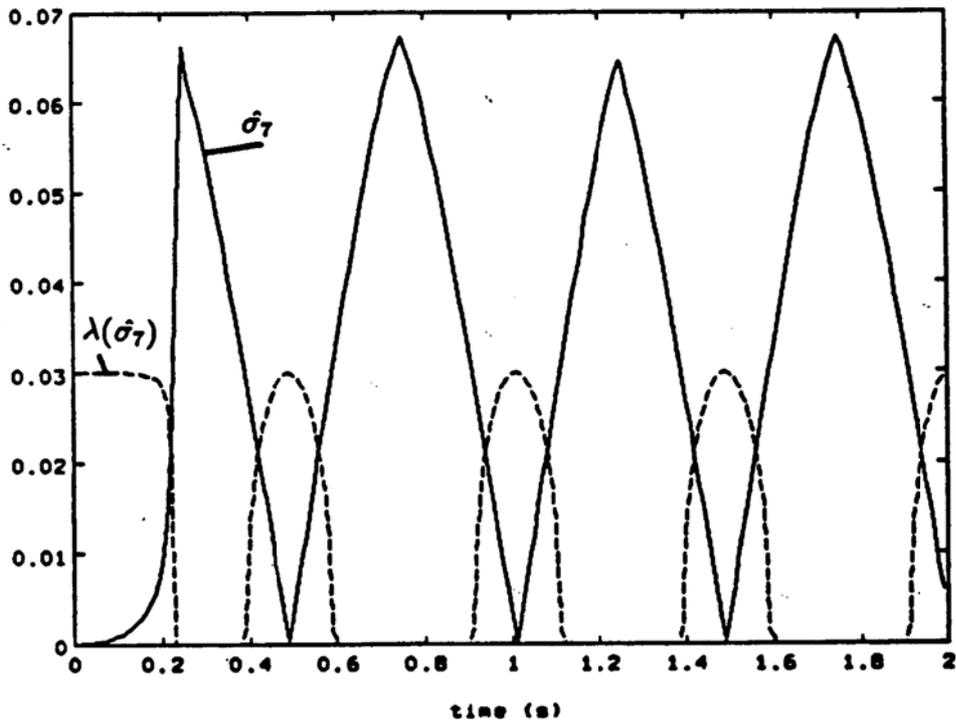


Figure 6. Time-plot of the estimate of the smallest singular value σ_7 and the damping factor λ in the simulation of the seven-joint manipulator going through an artificial singularity.

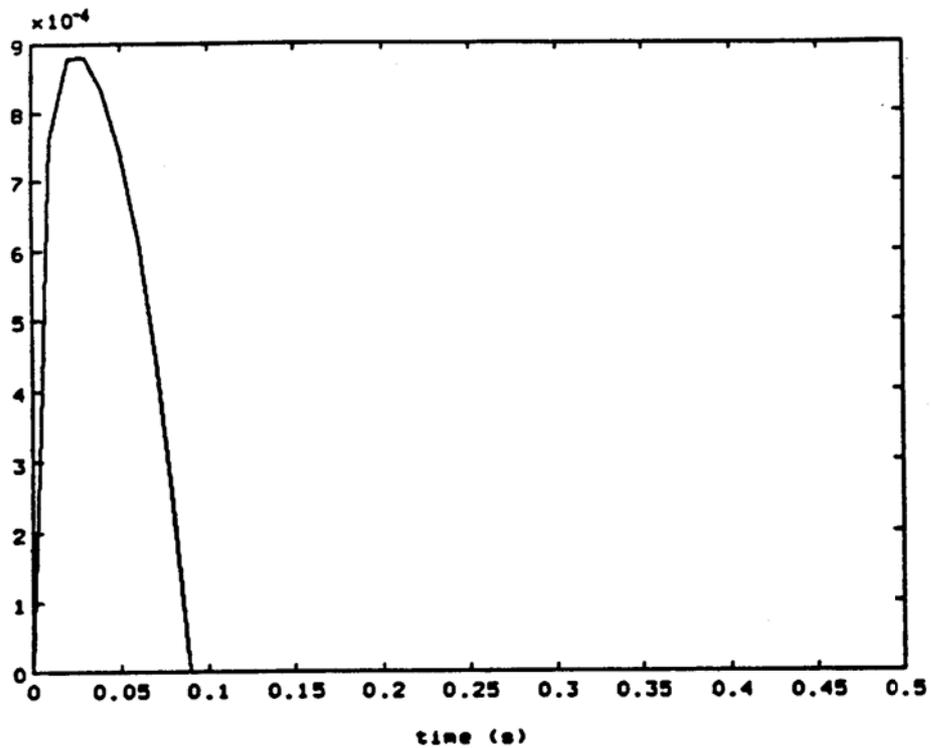


Figure 7. Motion of the constraint variable q_3 in the simulation of the seven-joint manipulator started from a kinematic singularity.

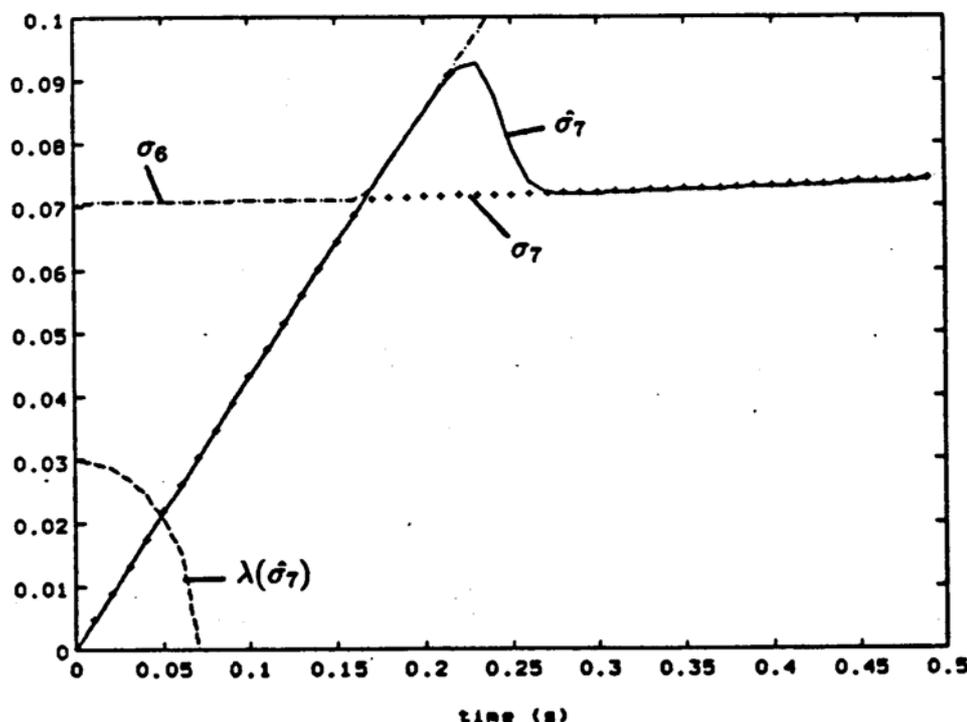


Figure 8. Time-plot of the smallest singular value σ_7 , its estimate $\hat{\sigma}_7$, and the damping factor λ in the simulation of the seven-joint manipulator started from a kinematic singularity.

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