Towards a unified geometric framework for finite element modeling of articulated soft robots and soft-bodied robots

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***Abstract*— In this paper we present a geometric framework for modeling soft robots using the finite element method. The framework is based on the geometrically exact Cosserat rod theory formulated on a Lie group. The method has been demonstrated to be able to simulate the dynamics of a generic robotic mechanism, consisting of an articulated robot with soft arms and soft joints.**

***Index Terms*— Soft robotics, Cosserat rod theory, differential geometry, mathematical modeling.**

NOMENCLATURE

$\dot{(\cdot)}$	derivative with respect to time
$(\cdot)'$	derivative with respect to space
$\widetilde{(\cdot)}$	Lie algebra operator
$(\cdot,\cdot)$	Linear operator
$\widehat{(\cdot)}$	Lie bracket operator
t	$\in \mathbb{R}$, time
α	$\in \mathbb{R}$, reference curve parametrization
$\boldsymbol{u}$	$\in \mathbb{R}^3$, position vector
$\mathbf{R}$	$\in SO(3)$, rotation matrix
$\mathbf{H}_I$	$\in SE(3)$, nodal frame
$\mathbf{H}_{J,I}$	$\in$ subgroup of $SE(3)$, relative frame
$\boldsymbol{\eta}_I$	$\in \mathbb{R}^6$, velocity vector
$\boldsymbol{\eta}_{j,I}$	$\in \mathbb{R}^{k_I}$, $k_I \leq 6$ relative velocity vector
$\boldsymbol{f}$	$\in \mathbb{R}^6$, deformation vector
$\boldsymbol{\sigma}$	$\in \mathbb{R}^6$, stress vector

I. INTRODUCTION

Mathematical modeling of soft robots is a an open challenge in the robotics community. Several approaches have been proposed in the recent years [1], [2], [3], [4], [5]. In this short paper we formulate the dynamics of articulated soft robots and soft-bodied robots using a novel screw-based nonlinear finite element method. The method combines the *Cosserat rod theory* with the *finite element method* in a *multibody dynamics* framework, using techniques from the *differential geometry* of Lie groups and Lie algebras.

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A. Motivations

Most of current methods can be used to model either articulated soft robots [1], [6], [7] or soft-bodied robots [2], [5], [8]. One of the first attempts in modeling multi-body systems of soft arms is made by Renda *et al.* in [9], where they model a rigid joint as a special case of immaterial soft body with unitary length. In this paper we focus on a more generic approach involving *absolute variables* for describing the absolute motion of the rigid and soft bodies and *relative variables* for describing the relative motion between the bodies. In this framework, the joints behave as algebraic constraints which prevent the non-allowed relative motion between two or more adiacent bodies. The benefits of this geometric framework are: (i) it allows modeling all low-pair joints using concepts of Lie subalgebra; (ii) it allows modeling robotic manipulators structured in a serial and parallel topology; (iii) it allows the simulation of both active and passive joints; (iv) it allows to model both rigid and soft joints. The rest of the paper summarizes the main aspects of the framework, which was introduced for the first time in [10] and it is exhaustively explained in [11].

II. SYSTEM MODELING

A generic multibody system of soft and rigid arms is seen as composed by:

- *Nodal frames* $\in SE(3)$, for the description of the rigid-body transformations of the nodes of the finite element mesh of the system;
- *Relative frames* $\in$ subgroup of $SE(3)$, for the description of the relative transformations between the nodes.

In the following we show the equations governing the modeling of rigid and soft bodies, joints and articulated systems.

A. Rigid body

A single rigid body is represented by a single six degrees-of-freedom node to which a local frame $\mathbf{H}$ is associated. The equations of motion of a rigid body are given by

$$\dot{\mathbf{H}} = \mathbf{H}\widetilde{\boldsymbol{\eta}} \tag{1}$$

$$\mathbf{M}\dot{\boldsymbol{\eta}} - \widehat{\boldsymbol{\eta}}^T\mathbf{M}\boldsymbol{\eta} = \mathbf{g}_{ext} \tag{2}$$

where $\mathbf{M} \in \mathbb{R}^{6\times 6}$ is the mass matrix of the rigid body and $\mathbf{g}_{ext} \in \mathbb{R}^6$ is the vector of external forces.

B. Soft body

A single soft body is represented by the continuous assembly of two-dimensional cross sections moving upon a three-dimensional curve according to the infinite rigid-body transformations defined by internal laws of deformation. Hence, the position field of a soft arm with length L is given by: $\alpha \in [0, L] \mapsto \mathbf{H}(\alpha) \in SE(3)$. According to the Cosserat rod theory, the equations of motion of a soft body take the form of partial differential equations (PDE) as

$$\dot{\mathbf{H}} = \mathbf{H}\widetilde{\boldsymbol{\eta}} \tag{3}$$

$$\mathbf{H}' = \mathbf{H}\widetilde{\boldsymbol{f}} \tag{4}$$

$$\mathbf{M}\dot{\boldsymbol{\eta}} - \widehat{\boldsymbol{\eta}}^T\mathbf{M}\boldsymbol{\eta} - \boldsymbol{\sigma}' + \widehat{\boldsymbol{f}}^T\boldsymbol{\sigma} - \mathbf{g}_{ext} = \mathbf{0} \tag{5}$$

where we omitted the dependency of the quantities on the material abscissa α. The spatial discretization process uses an *helical shape function* connecting two nodes A and B (placed at $\alpha = 0$ and $\alpha = L$) such that the approximation of the kinematic shape of the soft arm is given by

$$\mathbf{H}(\alpha) = \mathbf{H}_A \exp_{SE(3)}(\alpha \boldsymbol{f}) \tag{6}$$

where $\exp_{SE(3)}(\cdot)$ is the exponential mapping on $SE(3)$. By applying (6) to (3)–(5), the equations of motion of the soft body take the form of ordinary differential equations (ODE)

$$\dot{\mathbf{H}}_A = \mathbf{H}_A\widetilde{\boldsymbol{\eta}}_A \tag{7}$$

$$\dot{\mathbf{H}}_B = \mathbf{H}_B\widetilde{\boldsymbol{\eta}}_B \tag{8}$$

$$\mathbb{M}(\boldsymbol{f})\dot{\boldsymbol{\eta}}_{AB} + \mathbb{C}(\boldsymbol{f}, \boldsymbol{\eta}_{AB})\boldsymbol{\eta}_{AB} + \mathbb{K}\boldsymbol{f} = \mathbf{0}_{12\times 1} \tag{9}$$

where $\mathbb{M}, \mathbb{C}$ and $\mathbb{K}$ are rispectively the discretized mass, velocity and stiffness matrices, while $\boldsymbol{\eta}_{AB} = [\boldsymbol{\eta}_A^T \;\; \boldsymbol{\eta}_B^T]^T$.

C. Rigid/Soft Joint

The relative motion between two nodes A and B of the finite element mesh of the system is described by the relative transformation frame $\mathbf{H}_{J,I} \in$ Lie subgroup of $SE(3)$ as

$$\mathbf{H}_B = \mathbf{H}_A\mathbf{H}_{J,I} \tag{10}$$

To model soft joints, internal stiffness and/or damping elements can be added between the nodal frames $\mathbf{H}_A$ and $\mathbf{H}_B$ (see Fig. 1). In this case, there exists an internal deflection of the joint I given by $\boldsymbol{\delta}_I = \boldsymbol{\alpha}_I - \boldsymbol{\alpha}_I^0$. Springer and damper add internal elastic force and internal dissipation force respectively as $\mathbf{g}_{int,I,K}(\boldsymbol{\alpha}_I) = \mathbf{K}_I\boldsymbol{\delta}_I$ and $\mathbf{g}_{int,I,D}(\boldsymbol{\alpha}_I) = \mathbf{D}_I\dot{\boldsymbol{\delta}}_I$, where $\mathbf{K}_I = \text{diag}(K_{I,1}, \ldots, K_{I,K_I})$ and $\mathbf{D}_I = \text{diag}(D_{I,1}, \ldots, D_{I,D_I})$ are the diagonal matrices of stiffness and damping coefficients of joint I.

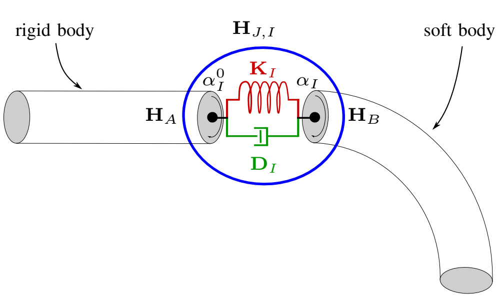

Fig. 1: Soft joint with internal stiffness and damping.

D. Rigid/Soft Multi-body system

A generic robot is constituted by soft and rigid bodies connected through rigid and soft joints. The equations of motion of a generic robot using this geometric finite element framework take the form of differential algebraic equations (DAE) on a Lie group as

$$\dot{\mathbf{H}} = \mathbf{H}\widetilde{\mathbf{A}\boldsymbol{\eta}} \tag{11}$$

$$\mathbf{g}_{ine}(\mathbf{H}, \boldsymbol{\eta}, \dot{\boldsymbol{\eta}}) + \mathbf{g}_{int}(\mathbf{H}) + \mathbf{A}^T\boldsymbol{\varphi}_q^T(\mathbf{H})\boldsymbol{\lambda} - \mathbf{g}_{ext}(\mathbf{H}) = \mathbf{0} \tag{12}$$

$$\boldsymbol{\varphi}(\mathbf{H}) = \mathbf{0} \tag{13}$$

where $\mathbf{A}$ is a matrix taking into account the particular joints, $\mathbf{g}_{ine}, \mathbf{g}_{int}$ and $\mathbf{g}_{ext}$ embody the sum of inertia, internal and external forces, while $\boldsymbol{\varphi}$, $\boldsymbol{\varphi}_q$ and $\boldsymbol{\lambda}$ are respectively the constraint equations, the constraint gradient and the lagrange multipliers due to the presence of joints.

III. EXAMPLE

As illustrative example of the formulation, we show the dynamic analysis of a generic robotic mechanism (GRM). The GRM is composed by rigid and soft bodies, articulated in a kinematic chain with one branched tree and one closed loop. The GRM topology is illustrated in Fig. 2. The GRM comprises six rigid bodies, each one with the following mass and rotation inertia properties: $m = 0.15\,\text{kg}$; $J_{xx} = J_{yy} = J_{zz} = 1 \times 10^{-4}\,\text{kgm}^2$. Instead, each of the five soft bodies have the following mass and stiffness matrices

$$\mathbf{M} = \text{diag}(0.1\,\text{kgm}^{-1}, 0.1\,\text{kgm}^{-1}, 0.1\,\text{kgm}^{-1}, \ldots \quad \ldots 0.5\,\text{kg}\,\text{m}, 0.5\,\text{kg}\,\text{m}, 0.5\,\text{kg}\,\text{m}) \tag{14}$$

$$\mathbf{K} = \text{diag}(1 \times 10^6\,\text{N}, 1 \times 10^6\,\text{N}, 1 \times 10^6\,\text{N}, \ldots \quad \ldots 1\,\text{Nm}^2, 1\,\text{Nm}^2, 1\,\text{Nm}^2) \tag{15}$$

which correspond to a very soft arm. The initial configuration of the GRM is given in Table I, while the joints are defined in Table II. There are a total of eight joints: four passive and four actuated. All joints are revolute about the $z-$axis, except joint 5 which rotates about $x-$axis and joint 3 which is prismatic. The active joints are actuated with a bang-bang acceleration profile (triangular velocity profile) for $1\,\text{s}$, according to the data in Table III. Three actuation forces are applied on the system at points 6,7 and 8. These forces follow a $S-$shaped profile for $1\,\text{s}$ (see Table IV). The system is subject to gravity downward $z-$direction. The dynamic simulations are performed using SimSOFT©, our physics engine for soft robots [12]. Snapshots of the simulation are shown in Fig. 3a–3d. As output, we plot the 3D displacements, velocities and accelerations of the tip of the GRM (point 8) as well as of the free joint (4). The simulations are performed with the assumption of: (i) rigid joints; (ii) soft joints with internal stiffness $K = 10\,\text{Nm}^{-1}$ for the prismatic joint and $K = 10\,\text{Nmrad}^{-1}$ for the revolute joints; (iii) soft joints with internal stiffness (same as before) and damping $D = 5\,\text{Nsm}^{-1}$ for the prismatic joint and $D = 5\,\text{Nmsrad}^{-1}$ for the revolute joints.

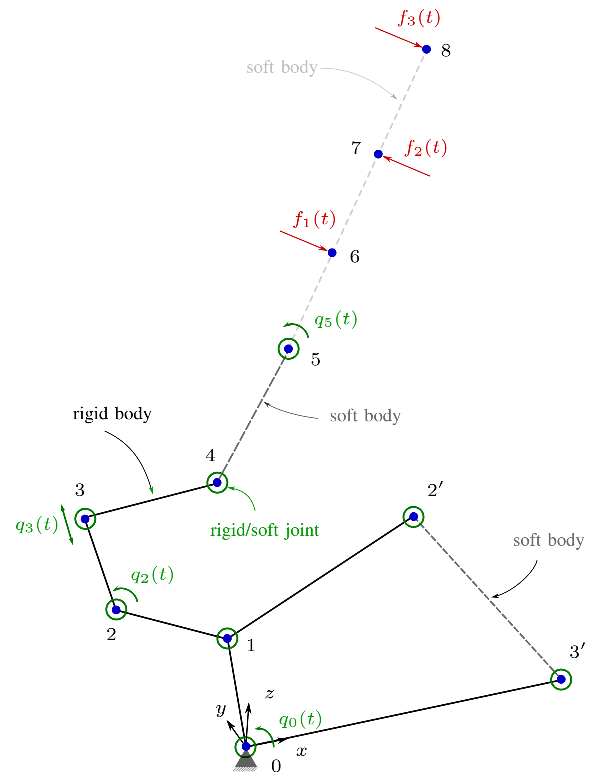

Fig. 2: Geometric description of the GRM. Solid lines: rigid bodies; Dashed lines: soft bodies.

TABLE I: Initial configuration of the GRM.

Point	x [m]	y [m]	z [m]
0	0	0	0
1	0.2	0.6	0
2'	1	1	0
3'	1.2	0	0
2	0	1	0
3	0.2	1.5	0
4	0.5	1.3	0.2
5	0.8	1.6	0.2
6	1	1.8	0.3
7	1.2	2	0.4
8	1.4	2.2	0.5

TABLE II: Kinematic joint definition of the GRM. A = actuated; P = passive. c = cos; s = sin.

Joint	e_u	e_ω	A/P
0	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	A
1	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	P
2'	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	P
3'	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	P
2	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	A
3	$[c(1.1903)\ \ s(1.1903)\ \ 0]^T$	$\mathbf{0}_{3\times1}$	A
4	$\mathbf{0}_{3\times1}$	$[0\ \ 0\ \ 1]^T$	P
5	$\mathbf{0}_{3\times1}$	$[1\ \ 0\ \ 0]^T$	A

TABLE III: Point-to-point motion of the actuated joint with bang-bang acceleration profiles. q_i, q_f = initial, final values.

Joint	q_0	q_2	q_3	q_5
q_i [m] or [rad]	0	0	0	0
q_f [m] or [rad]	$\pi/6$	$\pi/6$	0.75	$\pi/3$
$\ddot{q}$ [ms^{-2}] or [rads^{-2}]	$2/3\pi$	$2/3\pi$	3	$4\pi/3$

TABLE IV: Point-to-point force motion with $S-$shaped force profiles. f_i, f_f = initial, final values.

Actuation force	f_{1x}	f_{1y}	f_{2x}	f_{2y}	f_{3x}	f_{3y}
f_i [N]	0	0	0	0	0	0
f_f [N]	100	-100	-100	100	100	-100
$\ddot{f}$ [Ns^{-2}]	400	-400	-400	400	400	-400

IV. CONCLUSIONS AND FUTURE WORK

In this short paper we summarized a novel screw-based finite element formulation for modeling of articulated soft robots and soft-bodied robots. We tested the capabilities of the method on a hybrid mechanism, composed by rigid and soft bodies connected through rigid and soft joints, simulated using SimSOFT©, our simulation environment for soft robots. A future work is planned for building a soft robotics research testbed, to validate different models and control algorithms for soft robotic systems.

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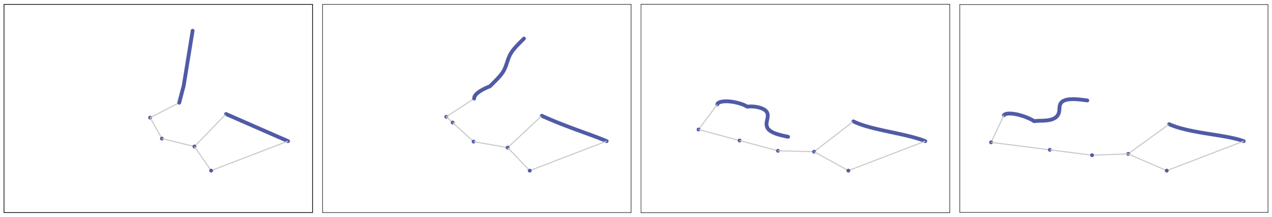

(a) t = 0 s (b) t = 0.33 s (c) t = 0.66 s (d) t = 1 s

Fig. 3: Snapshots of the GRM in SimSOFT©

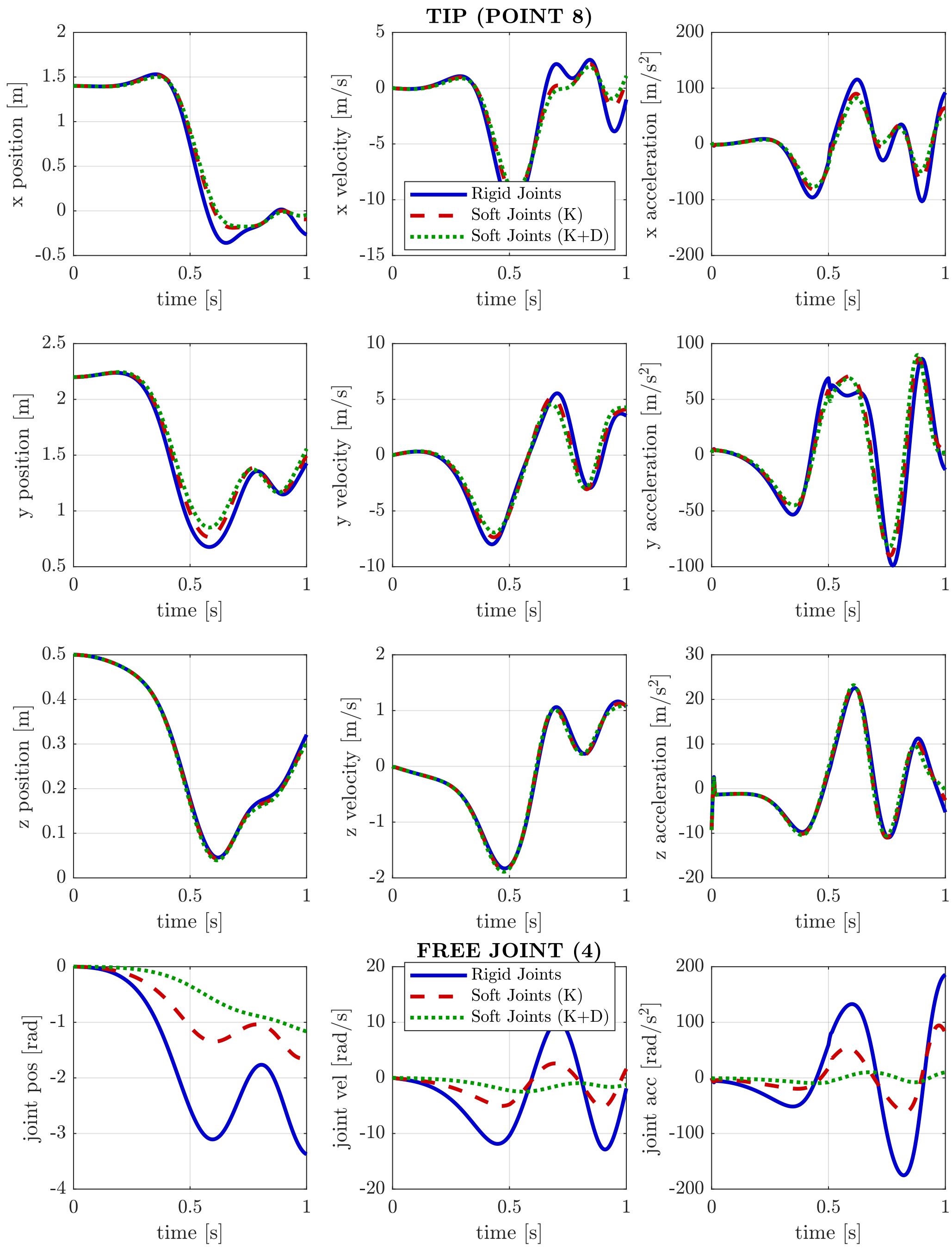

Fig. 4: Displacements, velocities and accelerations of the tip (8) and of the free joint (4) of the GRM.