Asymptotics for some nonlinear elliptic systems in Maxwell-Chern-Simons vortex theory

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Abstract

We provide a unified proof of the asymptotics of the self-dual Maxwell-Chern-Simons vortices, as the Maxwell term is neglected, in both the $U(1)$ and $CP(1)$ case. This result is achieved by identifying and analyzing a suitable class of nonlinear elliptic systems with exponential type nonlinearities.

KEY WORDS: nonlinear elliptic system, Chern-Simons vortex theory
MCS 2000 SUBJECT CLASSIFICATION: 35J60

1 Introduction and main result

The vortex solutions for the $U(1)$ Maxwell-Chern-Simons model introduced in [9], correspond to (distributional) solutions $(\widetilde{u}, v)$ for the system:

$$-\Delta\widetilde{u} = \varepsilon^{-1}(v - e^{\widetilde{u}}) - 4\pi\sum_{j=1}^{n}\delta_{p_j} \qquad \text{on } \Sigma \tag{1}$$

$$-\Delta v = \varepsilon^{-1}\left\{e^{\widetilde{u}}(1-v) - \varepsilon^{-1}(v - e^{\widetilde{u}})\right\} \qquad \text{on } \Sigma, \tag{2}$$

where Σ is a compact Riemannian 2-manifold without boundary, $n \geq 0$ is an integer, $p_j \in \Sigma$ for $j = 1, \ldots, n$, Δ denotes the Laplace-Beltrami operator and $\varepsilon > 0$ a constant. We shall be interested in the asymptotic behavior of solutions when $\varepsilon \to 0$.

Physically, $e^{\widetilde{u}}$ represents a *density* of particles; it vanishes exactly at the points p_j, $j = 1, \ldots, n$ (the *vortex points*). The function v is a neutral scalar field and $\varepsilon > 0$ is the coupling constant for the Maxwell term. In particular, letting $\varepsilon \to 0$ corresponds to dropping the Maxwell term in the Lagrangian.

The limit $\varepsilon \to 0$ is meaningful in view of the following

*Partially supported by PRIN 2000 "Variational Methods and Nonlinear Differential Equations"

Theorem 1.1 ([13]). *If $\varepsilon 4\pi n/|\Sigma|$ is sufficiently small, then there exist at least two solutions for* (1)–(2).

The proof of Theorem 1.1 is variational. The two solutions are obtained as a local minimum and a mountain pass for a suitable functional. We refer to [13] for the detailed proof.

By a formal analysis of (1)–(2), we expect that as $\varepsilon \to 0$, e^u should converge to a solution u_∞ for the equation

$$-\Delta u_\infty = e^{u_\infty}(1-e^{u_\infty}) - 4\pi\sum_{j=1}^n \delta_{p_j} \qquad \text{on } \Sigma. \tag{3}$$

We observe that solutions for (3) correspond to vortex solutions for the Chern-Simons model introduced in [7] and [6]. In [11] we provided a rigorous proof of this formal argument, in any relevant norm. Namely, we showed

Theorem 1.2 ([11]). *Suppose (u,v) are (distributional) solutions for* (1)–(2) *with $\varepsilon \to 0$. Then there exists a solution u_∞ for the equation* (3) *such that, up to subsequences, $(e^{\widetilde{u}}, v) \to (e^{u_\infty}, e^{u_\infty})$ in $C^h(\Sigma)\times C^h(\Sigma)$, for any $h \geq 0$.*

Note that $e^{\widetilde{u}}$, e^{u_∞} are smooth. Theorem 1.2 completed our previous convergence result obtained with Tarantello [13], where the asymptotics for v was established in the L^2-sense only. See also Chae and Kim [2].

At this point it is natural to seek a more general class of systems which exhibit an asymptotic behavior as in Theorem 1.2. A further motivation to this question is provided by the analysis of the $CP(1)$ Maxwell-Chern-Simons model in [3]. In [3] the authors analyze an elliptic system, whose solutions correspond to vortex solutions for the self-dual $CP(1)$ Maxwell-Chern-Simons model introduced in [4]. Their system (in a special case) is given by:

$$\Delta U = 2Q\Big(-V + S - \frac{1-e^U}{1+e^U}\Big) + 4\pi\sum_{j=1}^n \delta_{p_j} \qquad \text{on } \Sigma \tag{4}$$

$$\Delta V = -4Q^2\Big(-V + S - \frac{1-e^U}{1+e^U}\Big) + Q\frac{4e^U}{(1+e^U)^2}V \qquad \text{on } \Sigma \tag{5}$$

where Σ and $p_1,\dots,p_n$ are as in (1)–(2), U, V are the unknown functions and $S\in\mathbb{R}$, $Q>0$ are given constants. They prove the existence of at least one solution for (4)–(5); furthermore, they derive an asymptotic behavior as $Q\to+\infty$ *analogous* to that of system (1)–(2).

With this motivation, we consider (distributional) solutions $(\widetilde{u}, v)$ for the system:

$$-\Delta\widetilde{u} = \varepsilon^{-1}(v - f(e^{\widetilde{u}})) - 4\pi\sum_{j=1}^n \delta_{p_j} \qquad \text{on } \Sigma \tag{6}$$

$$-\Delta v = \varepsilon^{-1}\Big[f'(e^{\widetilde{u}})e^{\widetilde{u}}(s-v) - \varepsilon^{-1}(v - f(e^{\widetilde{u}}))\Big] \qquad \text{on } \Sigma. \tag{7}$$

Here Σ and $p_1,\dots,p_n$ are as in (1)–(2), $f = f(t)$, $t\geq 0$ is smooth and *strictly increasing*, $s\in\mathbb{R}$ satisfies $f(0) < s < \sup_{t>0} f(t)$. Without loss of generality, we assume $\mathrm{vol}\Sigma = 1$.

Clearly, when $f(t) = t$ and $s = 1$, system (6)–(7) reduces to (1)–(2). On the other hand, setting $v := V - S$, $s := -S$, $\varepsilon^{-1} := 2Q$, system (6)–(7) reduces to system (4)–(5) with f defined by $f(t) = (t-1)/(t+1)$.

By a formal analysis of (6)–(7) we expect that, up to subsequences, $(\widetilde{u}, v)$ should converge to $(\widetilde{u}_\infty, f(e^{\widetilde{u}_\infty}))$, where $\widetilde{u}_\infty$ is a solutions for the equation for the equation:

$$-\Delta\widetilde{u}_\infty = f'(e^{\widetilde{u}_\infty})e^{\widetilde{u}_\infty}(s - f(e^{\widetilde{u}_\infty})) - 4\pi\sum_{j=1}^{n}\delta_{p_j} \qquad \text{on } \Sigma. \tag{8}$$

Our main result states that this is indeed the case, with respect to any relevant norm:

Theorem 1.3 ([12]). *Let $(\widetilde{u}, v)$ be (distributional) solutions to (6)–(7), with $\varepsilon \to 0$. There exists a (distributional) solution $\widetilde{u}_\infty$ to (8) such that a subsequence, still denoted $(\widetilde{u}, v)$, satisfies:*

$$(e^{\widetilde{u}}, v) \to \left(e^{\widetilde{u}_\infty}, f(e^{\widetilde{u}_\infty})\right) \qquad \text{in } C^h(\Sigma)\times C^h(\Sigma),\ \forall h \geq 0.$$

In the rest of this note we shall outline the proof of Theorem 1.3. The detailed proof is contianed in [12], although some arguments are proved here in a simpler form. Henceforth we denote by $C > 0$ a general constant independent of ε, which may vary from line to line. Unless otherwise specified, all equations are defined on Σ and all integrals are taken over Σ with respect to the Lebesgue measure.

2 Proof of Theorem 1.3

In order to work in suitable Sobolev spaces, it is standard (see [14]) to define a "Green's function" u_0, solution for the problem

$$\begin{aligned} &-\Delta u_0 = 4\pi\left(n - \sum_{j=1}^{n}\delta_{p_j}\right) \qquad \text{on } \Sigma \\ &\int_\Sigma u_0 = 0 \end{aligned}$$

(see [1] for the unique existence of u_0). Setting $\widetilde{u} = u_0 + u$, we obtain the equivalent system for $(u, v) \in H^1(\Sigma)\times H^1(\Sigma)$:

$$-\Delta u = \varepsilon^{-1}\left(v - f(e^{u_0+u})\right) - 4\pi n \qquad \text{on } \Sigma \tag{9}$$

$$-\Delta v = \varepsilon^{-1}\left[f'(e^{u_0+u})e^{u_0+u}(s - v) - \varepsilon^{-1}\left(v - f(e^{u_0+u})\right)\right] \qquad \text{on } \Sigma, \tag{10}$$

where e^{u_0} is *smooth.*

The proof is obtained by a priori estimates and an inductive argument. It will be convenient to introduce the spaces $X^k := H^k \cap L^\infty$. By the following inequality, which follows from the well-known Sobolev-Gagliardo-Nirenberg inequality (see e.g. [10]):

$$\|D^j u\|_{L^{2k/j}} \leq C\|D^k u\|_{L^2}^{j/k}\|u\|_{L^\infty}^{1-j/k} \qquad \forall u \in C^\infty(\Sigma), \tag{11}$$

X^k is a *Banach algebra* for every $k \geq 0$, i.e.,

$$\|u_1 u_2\|_{X^k} \leq C \|u_1\|_{X^k} \|u_2\|_{X^k}.$$

It will also be convenient to set

$$w := \varepsilon^{-1}(v - f(e^{u_0+u}))$$

and to consider w as a third unknown function. Then the triple (u, v, w) satisfies a system of the following simple form:

$$-\Delta u = w - 4\pi n \tag{12}$$

$$-\varepsilon^2 \Delta v + [1 + \varepsilon c(x,u)]v = F_\varepsilon(x,u) \tag{13}$$

$$-\varepsilon^2 \Delta w + [1 + \varepsilon c(x,u)]w = G_\varepsilon(x,u,v,\nabla u) \tag{14}$$

where

$$\begin{aligned} c(x,u) =& f'(e^{u_0+u})e^{u_0+u} \\ F_\varepsilon(x,u) =& f(e^{u_0+u}) + s\varepsilon f'(e^{u_0+u})e^{u_0+u} \\ G_\varepsilon(x,u,v,\nabla u) =& f'(e^{u_0+u})e^{u_0+u}(s - v) \\ &+ \varepsilon \left(f''(e^{u_0+u})e^{u_0+u} + f'(e^{u_0+u})\right) e^{u_0+u} |\nabla(u_0+u)|^2. \end{aligned}$$

Furthermore, using (11), we obtain:

Lemma 2.1. *Let $F \in C^\infty(\Sigma \times \mathbb{R})$, $G \in C^\infty(\Sigma \times \mathbb{R} \times \mathbb{R} \times \mathbb{R}^2)$. Then for all $k \geq 0$ there exists constants $C_k = C_k(\|u\|_{L^\infty})$, $C'_k = C'_k(\|u\|_{L^\infty}, \|v\|_{L^\infty}, \|\nabla u\|_{L^\infty})$, such that:*

$$\begin{aligned} &\|F(x,u)\|_{X^k} \leq C_k(1 + \|u\|_{X^k}^k) \\ &\|G(x,u,v,\nabla u)\|_{X^{k-1}} \leq C'_k(1 + \|u\|_{X^k}^{k-1} + \|v\|_{X^{k-1}}). \end{aligned}$$

The basis for the induction is incuded in the following

Proposition 2.1. *There exists a constant $C > 0$ independent of $\varepsilon \to 0$, such that:*

(i) $\|u\|_{X^1} + \|v\|_{X^1} + \|w\|_{X^0} \leq C$

(ii) $\|u\|_{L^\infty} \leq C$

(iii) $\|\nabla u\|_{L^\infty} \leq C$

In order to prove Proposition 2.1 we need some preliminary estimates.

Lemma 2.2. *The following estimates hold, pointwise on Σ:*

(i) $f(0) \leq f(e^{\widetilde{u}}) \leq s$

(ii) $f(0) \leq v \leq s.$

Proof. By maximum principle. The increasing monotonicity of f is essential here. □

As a consequence of Lemma 2.2, the nonlinearity f may be *truncated*. Therefore in what follows, without loss of generality, we assume that:

$$\sup_{t>0}\{|f(t)|+|f'(t)|+|f''(t)|\}\leq C. \tag{15}$$

The next identity is the main step in deriving the H^1-estimate for v and the L^2-estimate for w:

Lemma 2.3. *The following identity holds:*

$$\int|\nabla v|^2+\int w^2=\int(s-v)\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2. \tag{16}$$

Proof. We compute:

$$\Delta f(e^{\widetilde{u}})=\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2+f'(e^{\widetilde{u}})e^{\widetilde{u}}\Delta\widetilde{u}.$$

Therefore $f(e^{\widetilde{u}})$ satisfies the equation:

(17)

$$-\Delta f(e^{u})+\varepsilon^{-1}f'(e^{\widetilde{u}})e^{\widetilde{u}}\,f(e^{\widetilde{u}})=\varepsilon^{-1}f'(e^{\widetilde{u}})e^{\widetilde{u}}v-\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2.$$

Integrating (17), we obtain

$$\varepsilon^{-1}\int f'(e^{\widetilde{u}})e^{\widetilde{u}}(v-f(e^{\widetilde{u}}))=\int\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2 \tag{18}$$

Now we multiply (7) by $v-f(e^{\widetilde{u}})$ and integrate to obtain:

$$\int-\Delta v(v-f(e^{\widetilde{u}}))=\varepsilon^{-1}\int f'(e^{\widetilde{u}})e^{\widetilde{u}}(s-v)(v-f(e^{\widetilde{u}}))-\varepsilon^{-2}\int(v-f(e^{\widetilde{u}}))^2.$$

Integrating by parts and using (17) we find:

$$\begin{aligned}&\int-\Delta v(v-f(e^{\widetilde{u}}))=\int|\nabla v|^2+\int v\Delta f(e^{\widetilde{u}})\\&=\int|\nabla v|^2-\varepsilon^{-1}\int vf'(e^{\widetilde{u}})e^{\widetilde{u}}(v-f(e^{\widetilde{u}}))+\int v\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2.\end{aligned}$$

Equating left hand sides in the last two identities, we obtain

$$\begin{aligned}\int|\nabla v|^2+\varepsilon^{-2}\int(v-f(e^{\widetilde{u}}))^2+\int v\left(f''(e^{\widetilde{u}})e^{\widetilde{u}}+f'(e^{\widetilde{u}})\right)e^{\widetilde{u}}|\nabla\widetilde{u}|^2\\=s\varepsilon^{-1}\int f'(e^{\widetilde{u}})e^{\widetilde{u}}(v-f(e^{\widetilde{u}})),\end{aligned}$$

and thus identity (16) is established. □

We shall need some a priori estimates for solutions to

$$-\varepsilon^2\Delta u+(1+\varepsilon c)u=f. \tag{19}$$

Indeed, both (13) and (14) are of the form (19).

Lemma 2.4. *Let $c, f \in X^k$ and suppose that u satisfies:* (19). *For every $k \geq 0$ there exist $\varepsilon_k > 0$, $C_k > 0$ such that*

$$\|u\|_{X^k} \leq C_k \|f\|_{X^k},$$

for all $\varepsilon \leq \varepsilon_k$.

Proof. The proof is an easy consequence of the following fact Let G_ε be the Green's function for

$$-\varepsilon \Delta_x G_\varepsilon + G_\varepsilon = \delta_y \qquad \text{on } \Sigma.$$

Then $G_\varepsilon(x, y) \to \delta_y$ weakly in the sense of measures. Note that since the operator $-\varepsilon\Delta + 1$ is coercive, the Green's function G_ε is uniquely defined on Σ. By the maximum principle, $G_\varepsilon > 0$ on Σ. Integrating over Σ, we find $\int G_\varepsilon = \int |G_\varepsilon| = 1$. Therefore, there exists a Radon measure μ such that $G_\varepsilon \rightharpoonup \mu$ weakly in the sense of measures. For any $\varphi \in C^\infty(\Sigma)$ we have:

$$\varepsilon \int -\Delta G_\varepsilon \varphi + \int G_\varepsilon \varphi = \varphi(y).$$

Taking limits, we find $\int \varphi \, \mathrm{d}\mu = \varphi(y)$ and the statement of the lemma is established. □

Now we can provide the

Proof of Proposition 2.1. We begin by establishing

$$\int e^{\widetilde{u}} |\nabla \widetilde{u}|^2 \leq C. \tag{20}$$

Proof of the Claim. Multiplying equation (6) by $e^{\widetilde{u}}$ and integrating by parts, we obtain

$$\varepsilon^{-1} \int e^{\widetilde{u}} (v - f(e^{\widetilde{u}})) = \int e^{\widetilde{u}} |\nabla \widetilde{u}|^2 \geq 0.$$

By the pointwise estimates in Lemma 2.2, it follows that:

$$\varepsilon \int e^{\widetilde{u}} |\nabla \widetilde{u}|^2 \leq C. \tag{21}$$

Multiplying (7) by $e^{\widetilde{u}}$ and integrating, we find

$$\varepsilon^{-1} \int e^{\widetilde{u}} (v - f(e^{\widetilde{u}})) = \int e^{2\widetilde{u}} f'(e^{\widetilde{u}})(s - v) + \varepsilon \int e^{\widetilde{u}} \Delta v. \tag{22}$$

Integration by parts yields:

$$\varepsilon \int e^{\widetilde{u}} \Delta v = -\int v e^{\widetilde{u}} (v - f(e^{\widetilde{u}})) + \varepsilon \int v e^{\widetilde{u}} |\nabla \widetilde{u}|^2.$$

Hence, by the pointwise estimates as in Lemma 2.2, and taking into account (21), we conclude that

$$\varepsilon \left| \int e^{\widetilde{u}} \Delta v \right| \leq C.$$

Inserting into (22), recalling Lemma 2.2, we derive that

$$\varepsilon^{-1}\int e^{\widetilde{u}}(v-f(e^{\widetilde{u}}))\le C,$$

and thus it follows that

$$\int e^{\widetilde{u}}|\nabla\widetilde{u}|^2=\varepsilon^{-1}\int e^{\widetilde{u}}(v-f(e^{\widetilde{u}}))\le C.$$

(20) is established.

Lemma 2.2 readily implies $\|e^{\widetilde{u}}\|_{L^\infty}\le C$ and $\|v\|_{L^\infty}\le C$. In order to obtain the H^1-estimate for $e^{\widetilde{u}}$, it suffices to observe that by Lemma 2.2–(i) and by (20) we have:

$$\int|\nabla e^{\widetilde{u}}|^2=\int e^{2\widetilde{u}}|\nabla\widetilde{u}|^2\le C\int e^{\widetilde{u}}|\nabla\widetilde{u}|^2\le C.$$

Now we estimate $\|\nabla v\|_{L^2}$ and $\|\varepsilon^{-1}(v-f(e^{\widetilde{u}}))\|_{L^2}$. Using identity (16), we have:

$$\begin{aligned}\int|\nabla v|^2+\int w^2\le&\|s-v\|_\infty\|f''(e^{\widetilde{u}})e^{\widetilde{u}}+f^(\widetilde{u})\|_\infty\int e^{\widetilde{u}}|\nabla\widetilde{u}|^2\\ \le&C\int e^{\widetilde{u}}|\nabla\widetilde{u}|^2\le C,\end{aligned}$$

where we again used Lemma 2.2 and (20) in order to derive the last step. Proof of (i). Multiplying (9) by $u-\int u$ and integrating, we have:

$$\begin{aligned}\int|\nabla u|^2=&q\int(v-f(e^{\widetilde{u}}))(u-\int u)\\ \le&\|q(v-f(e^{\widetilde{u}}))\|_2\|u-\int u\|_2\le C\|\nabla u\|_2,\end{aligned}$$

where the last inequality follows by Lemma 2.2 and by the Poincaré inequality. Hence $\|\nabla u\|_2\le C$. By Lemma 2.2–(ii), we have that $e^{\widetilde{u}}\le C$, and thus we only have to show that $\int u\ge -C$. To this end, we first observe that integrating (9) and (10) we obtain:

$$\int f'(e^{u_0+u})e^{u_0+u}(s-v)=q\int(v-f(e^{u_0+u}))=4\pi n.$$

On the other hand, we have in a straightforward manner:

$$\int f'(e^{u_0+u})e^{u_0+u}(s-v)\le C\int e^{u_0+u}\le Ce^{\int u}\|e^{u_0}\|_\infty\int e^{u-\int u}\le C\int e^{u-\int u}.$$

Hence, recalling the Moser-Trudinger inequality (see [1]) and the estimate for $\|\nabla u\|_2$, we conclude that

$$4\pi n\le Ce^{\int u}\int e^{u-\int u}\le Ce^{\int u}e^{\gamma\int|\nabla u|^2}\le Ce^{\int u},$$

which establishes (i). Proof of (ii). Since $\|w\|_{L^2}\le C$, by (i) and elliptic regularity we obtain $\|u\|_{H^2}\le C$. Then Sobolev embeddings yield $\|\nabla u\|_{L^p}\le C$, for any $1\le p<+\infty$ and $\|u\|_{L^\infty}\le C$, which establishes (ii). Proof of (iii). By (14), $\|\nabla u\|_{L^p}\le C$ and Lemma 2.4 imply that $\|w\|_{L^p}\le C$, for any $1\le p<+\infty$. Then (12) and Sobolev embeddings yield $\|u\|_{W^{2,p}}\le C$, for any $1\le p<+\infty$. For $p>2$, the Sobolev embeddings yield (iii). □

Proposition 2.2. *For all $k \geq 0$ there exists a constant $C > 0$ (possibly depending on k) such that:*

$$\|\widetilde{u} - u_0\|_{H^k} + \|v\|_{H^k} \leq C.$$

Proof of Proposition 2.2. We argue by induction on $k \in \mathbb{N}_0$.

CLAIM: Suppose:

$$\|u\|_{X^k} + \|v\|_{X^k} + \|w\|_{X^{k-1}} \leq C_k.$$

Then:

$$\|u\|_{X^{k+1}} + \|v\|_{X^{k+1}} + \|w\|_{X^k} \leq C_{k+1}.$$

Indeed,

$$\begin{aligned} \|w\|_{X^{k-1}} \leq C \Rightarrow \|u\|_{X^{k+1}} \leq C \quad & \text{by (12) and standard elliptic regularity} \\ \Rightarrow \|v\|_{X^{k+1}} \leq C \quad & \text{by (13), Lemma 2.1 and Lemma 2.4} \\ \Rightarrow \|w\|_{X^k} \leq C \quad & \text{by (14), Lemma 2.1 and Lemma 2.4.} \end{aligned}$$

Now Proposition 2.1, the Claim and a standard induction argument conclude the proof. □

Finally, we can prove our main result:

Proof of Theorem 1.3. Let (u, v) be solutions to system (9)–(10), with $\varepsilon \to 0$. By the a priori estimates as stated in Proposition 2.2 and by standard compactness arguments, there exist u_∞, v_∞ such that up to subsequences $u \to u_\infty$ and $v \to v_\infty$ in C^h, for all $h \geq 0$. We write (9) in the form:

$$v = f(e^{u_0+u}) + \varepsilon(-\Delta u + 4\pi n).$$

Taking limits, we find $v_\infty = f(e^{u_0+u_\infty})$. Furthermore, taking limits in (10), we obtain

$$\varepsilon^{-1}(v - f(e^{u_0+u})) \to f'(e^{u_0+u_\infty})e^{u_0+u_\infty}(s - f(e^{u_0+u_\infty})),$$

where the convergence holds in C^h, for any $h \geq 0$. Consequently, taking limits in (9), we find that u_∞ satisfies:

$$-\Delta u_\infty = f'(e^{u_0+u_\infty})e^{u_0+u_\infty}(s - f(e^{u_0+u_\infty})) - 4\pi \sum_{j=1}^{n} \delta_{p_j}. \tag{23}$$

Setting $\widetilde{u}_\infty = u_0 + u_\infty$, we conclude the proof of Theorem 1.3. □

Acknowledgement

I should like to thank the Organizers of the ICM2002 Satellite Conference on Nonlinear Functional Analysis, Taiyuan, August 14–18, as well as Professor Yanyan Li, for the invitation.

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