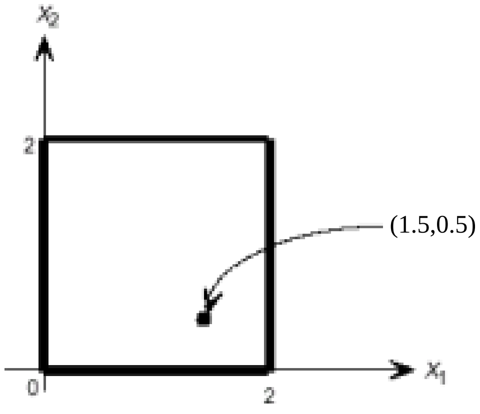
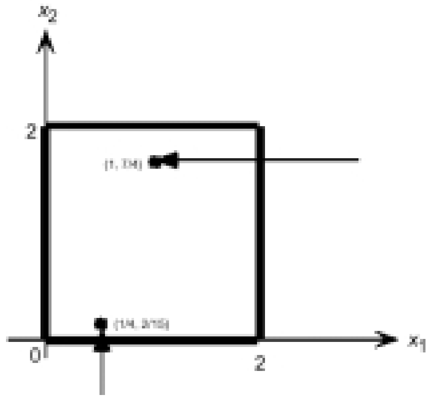
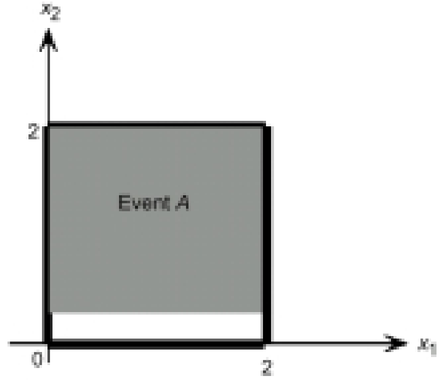
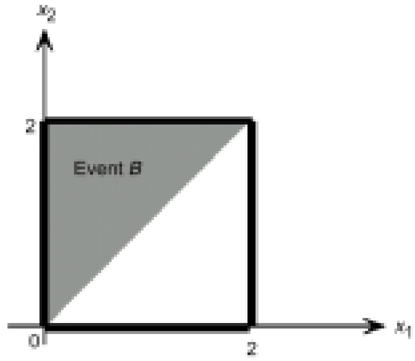
Distribuzioni congiunte e condizionate





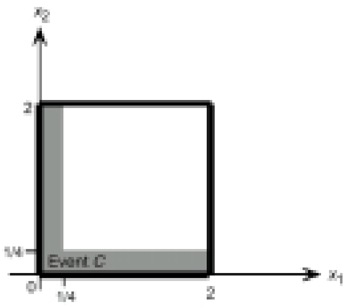
Evento A: L'autobus 2 arriva con più di 15 secondi di ritardo
Evento B: L'autobus 2 arriva prima dell' autobus 1





Evento A: L'autobus 2 arriva con più di 15 secondi di ritardo
Evento B: L'autobus 2 arriva prima dell' autobus 1

Evento C: Il primo autobus arriva entro 15 secondi.

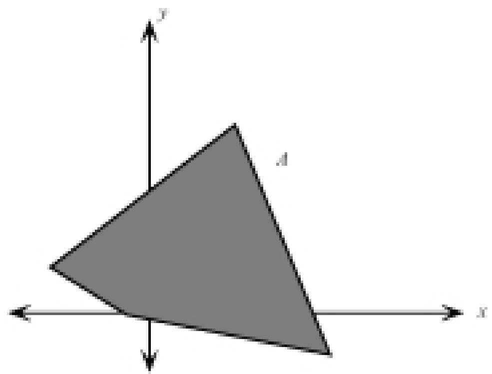


Funzione densità di probabilità congiunta:

$$f_{X,Y}(x,y)\,dx\,dy = P(x < X \le x+dx, y < Y \le y+dy)$$

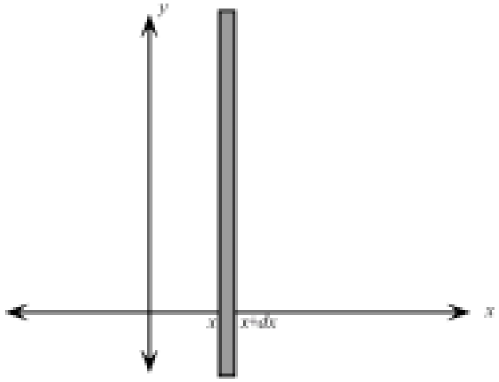
$$P(a_1 < X \le a_2, b_1 < Y \le b_2) = \int_{b_1}^{b_2}\int_{a_1}^{a_2} f_{X,Y}(x,y)\,dxdy$$

$$\int_{y=-\infty}^{\infty}\int_{x=-\infty}^{\infty} f_{X,Y}(x,y)\,dxdy = 1$$



$$P(A) = \iint_A f_{X,Y}(x,y)$$

$$P(x < X < x + dx) = \int_{y=-\infty}^{\infty} f_{X,Y}(x,y)\,dxdy$$



$$f_X(x) = \int_{y=-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

$$f_Y(y) = \int_{x=-\infty}^{\infty} f_{X,Y}(x,y)\,dx$$

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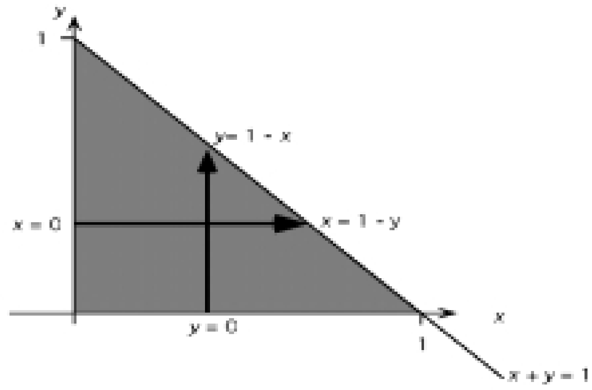
$$f_{X,Y}(x,y) = \begin{cases} 24xy & 0 < x < 1, 0 < y < 1 - x \\ 0 & \text{otherwise} \end{cases}$$



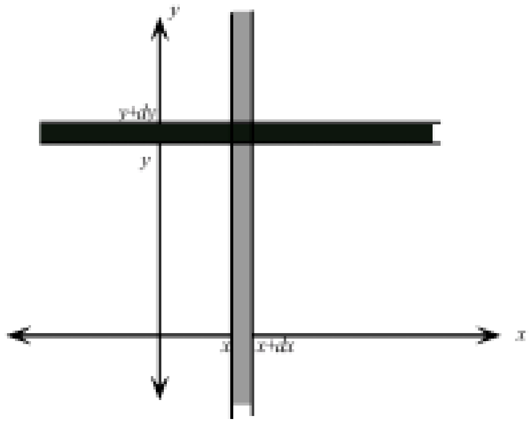
$$f_X(x) = \int_0^{1-x} 24xy\,dy = 12xy^2\Big|_0^{1-x} = 12x(1-x)^2$$

$$f_X(x) = \begin{cases} 12x(1-x)^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(y) = \int_0^{1-y} 24xy\,dx = 12yx^2\Big|_0^{1-y} = 12y(1-y)^2$$

$$f_Y(y) = \begin{cases} 12y(1-y)^2 & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Probabilità condizionata per densità congiunta



A: $x < X < x+dx$

B: $y < Y \le y+dy$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{f_{X,Y}(x,y)dxdy}{f_Y(y)dy} = \frac{f_{X,Y}(x,y)}{f_Y(y)}dx$$

Funzione densità di probabilità condizionata

$$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \qquad f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$f_{X,Y|A}(x,y \mid A) = \begin{cases} \dfrac{f_{X,Y}(x,y)}{P(A)} & (x,y) \in A \\ 0 & (x,y) \in A^c \end{cases}$$

$$f_{X|A}(x \mid A) = \begin{cases} \int\int_A f_{X,Y|A}(x,y \mid A)\,dy & x \in A \\ 0 & x \in A^c \end{cases}$$

Se invece A non dipende da y si ha:

$$f_{X|A}(x \mid A) = \begin{cases} \dfrac{f_X(x)}{P(A)} & x \in A \\ 0 & x \in A^c \end{cases}$$

Funzione densità di probabilità congiunta e variabili indipendenti Condizioni necessarie e sufficienti:

$$f_{X|Y}(x|y) = f_X(x)$$

$$f_{Y|X}(y|x) = f_Y(y)$$

$$f_{X,Y}(x,y) = f_X(x) f_Y(y)$$